

Gleichungen - Nullstellen ganzrationaler Funktionen

Aufgaben und Lösungen

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Inhaltsverzeichnis

1 Eigene Aufgaben lösen	1
1.1 Grundwissen	2
1.1.1 Lineare Gleichungen	2
1.1.2 Quadratischer Gleichungen	2
2 Lineare Gleichungen $ax + b = 0$	3
2.1 Aufgaben	3
2.2 Lösungen	4
3 Quadratische Gleichungen $ax^2 + bx + c = 0$	7
3.1 Aufgaben	7
3.2 Lösungen	8
4 Kubische Gleichungen $ax^3 + bx^2 + cx + d = 0$	23
4.1 Aufgaben	23
4.2 Lösungen	24
5 Funktionen 4. Grades $ax^4 + bx^3 + cx^2 + dx + e = 0$	33
5.1 Aufgaben	33
5.2 Lösungen	34
6 Funktionen höheren Grades	40
6.1 Aufgaben	40
6.2 Lösungen	41

1 Eigene Aufgaben lösen

Dieses PDF-Dokument wurde interaktiv auf <http://www.fersch.de> erstellt.

Um eigene Aufgaben zu lösen, klicken Sie hier:

[lineare Gleichungen](#)

[quadratische Gleichungen](#)

1.1 Grundwissen

1.1.1 Lineare Gleichungen

1.1.2 Quadratischer Gleichungen

a-b-c Formel (Mitternachtsformel)
$ax^2 + bx + c = 0$
$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$
Diskriminante:
$D = b^2 - 4 \cdot a \cdot c$
$D = 0$ eine Lösung
$D > 0$ zwei Lösungen
$D < 0$ keine Lösung

p-q Formel
$x^2 + px + q = 0$
$x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$
Diskriminante:
$D = \left(\frac{p}{2}\right)^2 - q$
$D = 0$ eine Lösung
$D > 0$ zwei Lösungen
$D < 0$ keine Lösung

Umformen
$ax^2 + c = 0$
$ax^2 + c = 0 \quad / -c$
$ax^2 = -c \quad / : a$
$x_{1/2} = \pm \sqrt{\frac{-c}{a}}$

x-Ausklammern
$ax^2 + bx = 0$
$x(ax + b) = 0$
$x_1 = 0 \quad \vee \quad x_2 = \frac{-b}{a}$

Satz von Vieta
$x^2 + px + q = 0$
$(x - x_1) \cdot (x - x_2) = 0$
$x_1 + x_2 = -p$
$x_1 \cdot x_2 = q$

2 Lineare Gleichungen $ax + b = 0$

2.1 Aufgaben

(1) $2x = 0$

(2) $-\frac{1}{2}x = 0$

(3) $6x - 36 = 0$

(4) $3x + 3 = 0$

(5) $-\frac{1}{2}x + 4\frac{1}{2} = 0$

(6) $-\frac{2}{3}x + \frac{1}{6} = 0$

(7) $\frac{1}{4}x - 2 = 0$

(8) $\frac{1}{4}x - 3 = 0$

(9) $-2x + 4 = 0$

(10) $x - 2 = 0$

(11) $-1\frac{1}{4}x - 10 = 0$

(12) $4x - 8 = 0$

(13) $-\frac{24}{49}x + 2\frac{22}{49} = 0$

(14) $\frac{8}{27}x + 2\frac{2}{3} = 0$

(15) $\frac{20}{81}x + 2\frac{3}{9} = 0$

(16) $1\frac{11}{25}x + 10\frac{2}{25} = 0$

2.2 Lösungen

Aufgabe (1)

$$2x = 0$$

$$x = 0 \Rightarrow x = 0$$

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (2)

$$-\frac{1}{2}x = 0$$

$$x = 0 \Rightarrow x = 0$$

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (3)

$$6x - 36 = 0$$

$$6x - 36 = 0 \quad / + 36$$

$$6x = 36 \quad / : 6$$

$$x = \frac{36}{6}$$

$$x = 6$$

$$\underline{x_1 = 6; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (4)

$$3x + 3 = 0$$

$$3x + 3 = 0 \quad / - 3$$

$$3x = -3 \quad / : 3$$

$$x = \frac{-3}{3}$$

$$x = -1$$

$$\underline{x_1 = -1; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (5)

$$-\frac{1}{2}x + 4\frac{1}{2} = 0$$

$$-\frac{1}{2}x + 4\frac{1}{2} = 0 \quad / - 4\frac{1}{2}$$

$$-\frac{1}{2}x = -4\frac{1}{2} \quad / : \left(-\frac{1}{2}\right)$$

$$x = \frac{-4\frac{1}{2}}{-\frac{1}{2}}$$

$$x = 9$$

$$\underline{x_1 = 9; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (6)

$$-\frac{2}{3}x + \frac{1}{6} = 0$$

$$-\frac{2}{3}x + \frac{1}{6} = 0 \quad / - \frac{1}{6}$$

$$-\frac{2}{3}x = -\frac{1}{6} \quad / : \left(-\frac{2}{3}\right)$$

$$x = \frac{-\frac{1}{6}}{-\frac{2}{3}}$$

$$x = \frac{1}{4}$$

$$\underline{x_1 = \frac{1}{4}; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (7)

$$\frac{1}{4}x - 2 = 0$$

$$\frac{1}{4}x - 2 = 0 \quad / + 2$$

$$\frac{1}{4}x = 2 \quad / : \frac{1}{4}$$

$$x = \frac{2}{\frac{1}{4}}$$

$$x = 8$$

$$\underline{x_1 = 8; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (8)

$$\frac{1}{4}x - 3 = 0$$

$$\frac{1}{4}x - 3 = 0 \quad / + 3$$

$$\frac{1}{4}x = 3 \quad / : \frac{1}{4}$$

$$x = \frac{3}{\frac{1}{4}}$$

$$x = 12$$

$$\underline{x_1 = 12; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (9)

$$-2x + 4 = 0$$

$$-2x + 4 = 0 \quad / - 4$$

$$-2x = -4 \quad / : (-2)$$

$$x = \frac{-4}{-2}$$

$$x = 2$$

$$\underline{x_1 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (10)

$$x - 2 = 0$$

$$x - 2 = 0 \quad / + 2$$

$$x = 2$$

$$\underline{x_1 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (11)

$$-1\frac{1}{4}x - 10 = 0$$

$$-1\frac{1}{4}x - 10 = 0 \quad / + 10$$

$$-1\frac{1}{4}x = 10 \quad / : (-1\frac{1}{4})$$

$$x = \frac{10}{-1\frac{1}{4}}$$

$$x = -8$$

$$\underline{x_1 = -8; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (12)

$$4x - 8 = 0$$

$$4x - 8 = 0 \quad / + 8$$

$$4x = 8 \quad / : 4$$

$$x = \frac{8}{4}$$

$$x = 2$$

$$\underline{x_1 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (13)

$$-\frac{24}{49}x + 2\frac{22}{49} = 0$$

$$-\frac{24}{49}x + 2\frac{22}{49} = 0 \quad / - 2\frac{22}{49}$$

$$-\frac{24}{49}x = -2\frac{22}{49} \quad / : \left(-\frac{24}{49}\right)$$

$$x = \frac{-2\frac{22}{49}}{-\frac{24}{49}}$$

$$x = 5$$

$$\underline{x_1 = 5; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (14)

$$\frac{8}{27}x + 2\frac{2}{3} = 0$$

$$\frac{8}{27}x + 2\frac{2}{3} = 0 \quad / - 2\frac{2}{3}$$

$$\frac{8}{27}x = -2\frac{2}{3} \quad / : \frac{8}{27}$$

$$x = \frac{-2\frac{2}{3}}{\frac{8}{27}}$$

$$x = -9$$

$$\underline{x_1 = -9; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (15)

$$\frac{20}{81}x + 2\frac{2}{9} = 0$$

$$\frac{20}{81}x + 2\frac{2}{9} = 0 \quad / - 2\frac{2}{9}$$

$$\frac{20}{81}x = -2\frac{2}{9} \quad / : \frac{20}{81}$$

$$x = \frac{-2\frac{2}{9}}{\frac{20}{81}}$$

$$x = -9$$

$$\underline{x_1 = -9; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (16)

$$1\frac{11}{25}x + 10\frac{2}{25} = 0$$

$$1\frac{11}{25}x + 10\frac{2}{25} = 0 \quad / - 10\frac{2}{25}$$

$$1\frac{11}{25}x = -10\frac{2}{25} \quad / : 1\frac{11}{25}$$

$$x = \frac{-10\frac{2}{25}}{1\frac{11}{25}}$$

$$x = -7$$

$$\underline{x_1 = -7; \quad 1\text{-fache Nullstelle}}$$

3 Quadratische Gleichungen $ax^2 + bx + c = 0$

3.1 Aufgaben

- | | |
|--|---|
| (1) $2x^2 = 0$ | (27) $x^2 + 6x - 2 = 0$ |
| (2) $-\frac{1}{2}x^2 = 0$ | (28) $-\frac{1}{3}x^2 + 2x + 5 = 0$ |
| (3) $6x^2 - 36 = 0$ | (29) $\frac{1}{2}x^2 - x + 4 = 0$ |
| (4) $3x^2 + 3 = 0$ | (30) $-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$ |
| (5) $-\frac{1}{2}x^2 + 4\frac{1}{2} = 0$ | (31) $-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$ |
| (6) $-\frac{2}{3}x^2 + \frac{1}{6} = 0$ | (32) $-1\frac{1}{4}x^2 + 5x = 0$ |
| (7) $\frac{1}{4}x^2 - 2 = 0$ | (33) $-\frac{3}{4}x^2 - 3x = 0$ |
| (8) $\frac{1}{4}x^2 - 3 = 0$ | (34) $\frac{5}{9}x^2 - 5 = 0$ |
| (9) $-2x^2 + 4 = 0$ | (35) $12x^2 + 12x = 0$ |
| (10) $x^2 - 2 = 0$ | (36) $-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$ |
| (11) $-\frac{1}{3}x^2 + 2x = 0$ | (37) $-\frac{2}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$ |
| (12) $-2x^2 - 8x = 0$ | (38) $-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$ |
| (13) $x^2 - x = 0$ | (39) $\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$ |
| (14) $\frac{1}{2}x^2 - \frac{2}{3}x = 0$ | (40) $-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$ |
| (15) $2x^2 - 5x = 0$ | (41) $-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0$ |
| (16) $x^2 + 2x - 24 = 0$ | (42) $-\frac{7}{9}x^2 + 4\frac{2}{3}x = 0$ |
| (17) $x^2 + 3x - 10 = 0$ | (43) $\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0$ |
| (18) $x^2 - x = 0$ | (44) $\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$ |
| (19) $x^2 - 8x - 20 = 0$ | (45) $-1\frac{1}{4}x^2 - 10x - 15 = 0$ |
| (20) $x^2 - 8x + 15 = 0$ | (46) $4x^2 - 8x = 0$ |
| (21) $-\frac{1}{3}x^2 - 2x + 3 = 0$ | (47) $-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$ |
| (22) $x^2 - 4x + 7 = 0$ | (48) $\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$ |
| (23) $-x^2 + 4x - 7 = 0$ | (49) $\frac{20}{81}x^2 + 2\frac{3}{9}x = 0$ |
| (24) $2x^2 + 4x = 0$ | (50) $1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$ |
| (25) $-\frac{1}{2}x^2 + 2x + 5 = 0$ | |
| (26) $-2x^2 + 3x + 4 = 0$ | |

3.2 Lösungen

Aufgabe (1)

$$2x^2 = 0$$

Ablesen $2x^2 = 0 \quad / : 2$ $x_{1/2} = 0$
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Aufgabe (2)

$$-\frac{1}{2}x^2 = 0$$

Ablesen $-\frac{1}{2}x^2 = 0 \quad / : (-\frac{1}{2})$ $x_{1/2} = 0$
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Aufgabe (3)

$$6x^2 - 36 = 0$$

Umformen $6x^2 - 36 = 0 \quad / + 36$ $6x^2 = 36 \quad / : 6$ $x^2 = \frac{36}{6}$ $x = \pm\sqrt{6}$ $x_1 = 2,449 \quad x_2 = -2,449$	a-b-c Formel $6x^2 + 0x - 36 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 6 \cdot (-36)}}{2 \cdot 6}$ $x_{1/2} = \frac{-0 \pm \sqrt{864}}{12}$ $x_{1/2} = \frac{0 \pm 29,394}{12}$ $x_1 = \frac{0 + 29,394}{12} \quad x_2 = \frac{0 - 29,394}{12}$ $x_1 = 2,449 \quad x_2 = -2,449$	p-q Formel $6x^2 + 0x - 36 = 0 \quad / : 6$ $x^2 + 0x - 6 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-6)}$ $x_{1/2} = 0 \pm \sqrt{6}$ $x_{1/2} = 0 \pm 2,449$ $x_1 = 2,449 \quad x_2 = -2,449$
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Aufgabe (4)

$$3x^2 + 3 = 0$$

Umformen $3x^2 + 3 = 0 \quad / - 3$ $3x^2 = -3 \quad / : 3$ $x^2 = \frac{-3}{3}$ keine Lösung	a-b-c Formel $3x^2 + 0x + 3 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$ $x_{1/2} = \frac{-0 \pm \sqrt{-36}}{6}$ Diskriminante negativ keine Loesung	p-q Formel $3x^2 + 0x + 3 = 0 \quad / : 3$ $x^2 + 0x + 1 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - 1}$ $x_{1/2} = 0 \pm \sqrt{-1}$ Diskriminante negativ keine Loesung
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Aufgabe (5)

$$-\frac{1}{2}x^2 + 4\frac{1}{2} = 0$$

Umformen $-\frac{1}{2}x^2 + 4\frac{1}{2} = 0 \quad / - 4\frac{1}{2}$ $-\frac{1}{2}x^2 = -4\frac{1}{2} \quad / : (-\frac{1}{2})$ $x^2 = \frac{-4\frac{1}{2}}{-\frac{1}{2}}$ $x = \pm\sqrt{9}$ $x_1 = 3 \quad x_2 = -3$	a-b-c Formel $-\frac{1}{2}x^2 + 0x + 4\frac{1}{2} = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-\frac{1}{2}) \cdot 4\frac{1}{2}}}{2 \cdot (-\frac{1}{2})}$ $x_{1/2} = \frac{-0 \pm \sqrt{9}}{-1}$ $x_{1/2} = \frac{0 \pm 3}{-1}$ $x_1 = \frac{0 + 3}{-1} \quad x_2 = \frac{0 - 3}{-1}$ $x_1 = -3 \quad x_2 = 3$	p-q Formel $-\frac{1}{2}x^2 + 0x + 4\frac{1}{2} = 0 \quad / : -\frac{1}{2}$ $x^2 + 0x - 9 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)}$ $x_{1/2} = 0 \pm \sqrt{9}$ $x_{1/2} = 0 \pm 3$ $x_1 = 3 \quad x_2 = -3$
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Aufgabe (6)

$$-\frac{2}{3}x^2 + \frac{1}{6} = 0$$

<p>Umformen</p> <hr/> $-\frac{2}{3}x^2 + \frac{1}{6} = 0 \quad / \cdot -\frac{1}{6}$ $-\frac{2}{3}x^2 = -\frac{1}{6} \quad / : \left(-\frac{2}{3}\right)$ $x^2 = \frac{-\frac{1}{6}}{-\frac{2}{3}}$ $x = \pm \sqrt{\frac{1}{4}}$ $x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$	<p>a-b-c Formel</p> <hr/> $-\frac{2}{3}x^2 + 0x + \frac{1}{6} = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \left(-\frac{2}{3}\right) \cdot \frac{1}{6}}}{2 \cdot \left(-\frac{2}{3}\right)}$ $x_{1/2} = \frac{-0 \pm \sqrt{\frac{4}{9}}}{-1\frac{1}{3}}$ $x_{1/2} = \frac{0 \pm \frac{2}{3}}{-1\frac{1}{3}}$ $x_1 = \frac{0 + \frac{2}{3}}{-1\frac{1}{3}} \quad x_2 = \frac{0 - \frac{2}{3}}{-1\frac{1}{3}}$ $x_1 = -\frac{1}{2} \quad x_2 = \frac{1}{2}$	<p>p-q Formel</p> <hr/> $-\frac{2}{3}x^2 + 0x + \frac{1}{6} = 0 \quad / : -\frac{2}{3}$ $x^2 + 0x - \frac{1}{4} = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - \left(-\frac{1}{4}\right)}$ $x_{1/2} = 0 \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = 0 \pm \frac{1}{2}$ $x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$
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Aufgabe (7)

$$\frac{1}{4}x^2 - 2 = 0$$

<p>Umformen</p> <hr/> $\frac{1}{4}x^2 - 2 = 0 \quad / + 2$ $\frac{1}{4}x^2 = 2 \quad / : \frac{1}{4}$ $x^2 = \frac{2}{\frac{1}{4}}$ $x = \pm \sqrt{8}$ $x_1 = 2,828 \quad x_2 = -2,828$	<p>a-b-c Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 2 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-2)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{2}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm \sqrt{1,414}}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,414}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,414}{\frac{1}{2}}$ $x_1 = 2,828 \quad x_2 = -2,828$	<p>p-q Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 2 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 8 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-8)}$ $x_{1/2} = 0 \pm \sqrt{8}$ $x_{1/2} = 0 \pm 2,828$ $x_1 = 2,828 \quad x_2 = -2,828$
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Aufgabe (8)

$$\frac{1}{4}x^2 - 3 = 0$$

<p>Umformen</p> <hr/> $\frac{1}{4}x^2 - 3 = 0 \quad / + 3$ $\frac{1}{4}x^2 = 3 \quad / : \frac{1}{4}$ $x^2 = \frac{3}{\frac{1}{4}}$ $x = \pm \sqrt{12}$ $x_1 = 3,464 \quad x_2 = -3,464$	<p>a-b-c Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 3 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-3)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{3}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm \sqrt{1,732}}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,732}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,732}{\frac{1}{2}}$ $x_1 = 3,464 \quad x_2 = -3,464$	<p>p-q Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 3 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 12 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-12)}$ $x_{1/2} = 0 \pm \sqrt{12}$ $x_{1/2} = 0 \pm 3,464$ $x_1 = 3,464 \quad x_2 = -3,464$
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Aufgabe (9)

$$-2x^2 + 4 = 0$$

Umformen	a-b-c Formel	p-q Formel
$\begin{aligned} -2x^2 + 4 &= 0 & / -4 \\ -2x^2 &= -4 & / : (-2) \\ x^2 &= \frac{-4}{-2} \\ x &= \pm\sqrt{2} \\ x_1 &= 1, 414 & x_2 = -1, 414 \end{aligned}$	$\begin{aligned} -2x^2 + 0x + 4 &= 0 \\ x_{1/2} &= \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)} \\ x_{1/2} &= \frac{-0 \pm \sqrt{32}}{-4} \\ x_{1/2} &= \frac{0 \pm 5,657}{-4} \\ x_1 &= \frac{0 + 5,657}{-4} & x_2 = \frac{0 - 5,657}{-4} \\ x_1 &= -1,414 & x_2 = 1,414 \end{aligned}$	$\begin{aligned} -2x^2 + 0x + 4 &= 0 & / : -2 \\ x^2 + 0x - 2 &= 0 \\ x_{1/2} &= -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)} \\ x_{1/2} &= 0 \pm \sqrt{2} \\ x_{1/2} &= 0 \pm 1,414 \\ x_1 &= 1,414 & x_2 = -1,414 \end{aligned}$

Aufgabe (10)

$$x^2 - 2 = 0$$

Umformen	a-b-c Formel	p-q Formel
$\begin{aligned} 1x^2 - 2 &= 0 & / + 2 \\ 1x^2 &= 2 & / : 1 \\ x^2 &= \frac{2}{1} \\ x &= \pm\sqrt{2} \\ x_1 &= 1,414 & x_2 = -1,414 \end{aligned}$	$\begin{aligned} 1x^2 + 0x - 2 &= 0 \\ x_{1/2} &= \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \\ x_{1/2} &= \frac{-0 \pm \sqrt{8}}{2} \\ x_{1/2} &= \frac{0 \pm 2,828}{2} \\ x_1 &= \frac{0 + 2,828}{2} & x_2 = \frac{0 - 2,828}{2} \\ x_1 &= 1,414 & x_2 = -1,414 \end{aligned}$	$\begin{aligned} x^2 + 0x - 2 &= 0 \\ x_{1/2} &= -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)} \\ x_{1/2} &= 0 \pm \sqrt{2} \\ x_{1/2} &= 0 \pm 1,414 \\ x_1 &= 1,414 & x_2 = -1,414 \end{aligned}$

Aufgabe (11)

$$-\frac{1}{3}x^2 + 2x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$\begin{aligned} -\frac{1}{3}x^2 + 2x &= 0 \\ x\left(-\frac{1}{3}x + 2\right) &= 0 \\ -\frac{1}{3}x + 2 &= 0 & / - 2 \\ -\frac{1}{3}x &= -2 & / : \left(-\frac{1}{3}\right) \\ x &= \frac{-2}{-\frac{1}{3}} \\ x_1 &= 0 \\ x_2 &= 6 \end{aligned}$	$\begin{aligned} -\frac{1}{3}x^2 + 2x + 0 &= 0 \\ x_{1/2} &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot \left(-\frac{1}{3}\right) \cdot 0}}{2 \cdot \left(-\frac{1}{3}\right)} \\ x_{1/2} &= \frac{-2 \pm \sqrt{4}}{-\frac{2}{3}} \\ x_{1/2} &= \frac{-2 \pm 2}{-\frac{2}{3}} \\ x_1 &= \frac{-2 + 2}{-\frac{2}{3}} & x_2 = \frac{-2 - 2}{-\frac{2}{3}} \\ x_1 &= 0 & x_2 = 6 \end{aligned}$	$\begin{aligned} -\frac{1}{3}x^2 + 2x + 0 &= 0 & / : -\frac{1}{3} \\ x^2 - 6x + 0 &= 0 \\ x_{1/2} &= -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0} \\ x_{1/2} &= 3 \pm \sqrt{9} \\ x_{1/2} &= 3 \pm 3 \\ x_1 &= 6 & x_2 = 0 \end{aligned}$

Aufgabe (12)

$$-2x^2 - 8x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$\begin{aligned} -2x^2 - 8x &= 0 \\ x(-2x - 8) &= 0 \\ -2x - 8 &= 0 & / + 8 \\ -2x &= 8 & / : (-2) \\ x &= \frac{8}{-2} \\ x_1 &= 0 \\ x_2 &= -4 \end{aligned}$	$\begin{aligned} -2x^2 - 8x + 0 &= 0 \\ x_{1/2} &= \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot (-2) \cdot 0}}{2 \cdot (-2)} \\ x_{1/2} &= \frac{+8 \pm \sqrt{64}}{-4} \\ x_{1/2} &= \frac{8 \pm 8}{-4} \\ x_1 &= \frac{8 + 8}{-4} & x_2 = \frac{8 - 8}{-4} \\ x_1 &= -4 & x_2 = 0 \end{aligned}$	$\begin{aligned} -2x^2 - 8x + 0 &= 0 & / : -2 \\ x^2 + 4x + 0 &= 0 \\ x_{1/2} &= -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0} \\ x_{1/2} &= -2 \pm \sqrt{4} \\ x_{1/2} &= -2 \pm 2 \\ x_1 &= 0 & x_2 = -4 \end{aligned}$

Aufgabe (13)

$x^2 - x = 0$

x-Ausklammern	a-b-c Formel	p-q Formel
$1x^2 - x = 0$ $x(1x - 1) = 0$ $1x - 1 = 0 \quad / + 1$ $1x = 1 \quad / : 1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	$1x^2 - x + 0 = 0$ $+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1+1}{2} \quad x_2 = \frac{1-1}{2}$ $x_1 = 1 \quad x_2 = 0$	$x^2 - x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{(-1)}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$

Aufgabe (14)

$\frac{1}{2}x^2 - \frac{2}{3}x = 0$

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{1}{2}x^2 - \frac{2}{3}x = 0$ $x(\frac{1}{2}x - \frac{2}{3}) = 0$ $\frac{1}{2}x - \frac{2}{3} = 0 \quad / + \frac{2}{3}$ $\frac{1}{2}x = \frac{2}{3} \quad / : \frac{1}{2}$ $x = \frac{\frac{2}{3}}{\frac{1}{2}}$ $x_1 = 0$ $x_2 = 1\frac{1}{3}$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0$ $+\frac{2}{3} \pm \sqrt{\left(-\frac{2}{3}\right)^2 - 4 \cdot \frac{1}{2} \cdot 0}$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9}}}{2 \cdot \frac{1}{2}}$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9}}}{1}$ $x_{1/2} = \frac{\frac{2}{3} \pm \frac{1}{3}}{1}$ $x_1 = \frac{\frac{2}{3} + \frac{1}{3}}{1} \quad x_2 = \frac{\frac{2}{3} - \frac{1}{3}}{1}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0 \quad / : \frac{1}{2}$ $x^2 - 1\frac{1}{3}x + 0 = 0$ $x_{1/2} = -\frac{-1\frac{1}{3}}{2} \pm \sqrt{\left(\frac{(-1\frac{1}{3})}{2}\right)^2 - 0}$ $x_{1/2} = \frac{2}{3} \pm \sqrt{\frac{4}{9}}$ $x_{1/2} = \frac{2}{3} \pm \frac{2}{3}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$

Aufgabe (15)

$2x^2 - 5x = 0$

x-Ausklammern	a-b-c Formel	p-q Formel
$2x^2 - 5x = 0$ $x(2x - 5) = 0$ $2x - 5 = 0 \quad / + 5$ $2x = 5 \quad / : 2$ $x = \frac{5}{2}$ $x_1 = 0$ $x_2 = 2\frac{1}{2}$	$2x^2 - 5x + 0 = 0$ $+5 \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 0}$ $x_{1/2} = \frac{+5 \pm \sqrt{25}}{2 \cdot 2}$ $x_{1/2} = \frac{+5 \pm 5}{4}$ $x_{1/2} = \frac{5 \pm 5}{4}$ $x_1 = \frac{5+5}{4} \quad x_2 = \frac{5-5}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$	$2x^2 - 5x + 0 = 0 \quad / : 2$ $x^2 - 2\frac{1}{2}x + 0 = 0$ $x_{1/2} = -\frac{-2\frac{1}{2}}{2} \pm \sqrt{\left(\frac{(-2\frac{1}{2})}{2}\right)^2 - 0}$ $x_{1/2} = 1\frac{1}{4} \pm \sqrt{1\frac{9}{16}}$ $x_{1/2} = 1\frac{1}{4} \pm 1\frac{1}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$

Aufgabe (16)

$x^2 + 2x - 24 = 0$

a-b-c Formel	p-q Formel
$1x^2 + 2x - 24 = 0$ $-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-24)}$ $x_{1/2} = \frac{-2 \pm \sqrt{100}}{2 \cdot 1}$ $x_{1/2} = \frac{-2 \pm 10}{2}$ $x_{1/2} = \frac{-2 \pm 10}{2}$ $x_1 = \frac{-2+10}{2} \quad x_2 = \frac{-2-10}{2}$ $x_1 = 4 \quad x_2 = -6$	$x^2 + 2x - 24 = 0$ $x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-24)}$ $x_{1/2} = -1 \pm \sqrt{25}$ $x_{1/2} = -1 \pm 5$ $x_1 = 4 \quad x_2 = -6$

Aufgabe (17)

$$x^2 + 3x - 10 = 0$$

a-b-c Formel	p-q Formel
$1x^2 + 3x - 10 = 0$ $x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$ $x_{1/2} = \frac{-3 \pm \sqrt{49}}{2}$ $x_{1/2} = \frac{-3 \pm 7}{2}$ $x_1 = \frac{-3 + 7}{2} \quad x_2 = \frac{-3 - 7}{2}$ $x_1 = 2 \quad x_2 = -5$	$x^2 + 3x - 10 = 0$ $x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$ $x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = 2 \quad x_2 = -5$

Aufgabe (18)

$$x^2 - x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$1x^2 - x = 0$ $x(1x - 1) = 0$ $1x - 1 = 0 \quad / +1$ $1x = 1 \quad / :1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	$1x^2 - x + 0 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1 + 1}{2} \quad x_2 = \frac{1 - 1}{2}$ $x_1 = 1 \quad x_2 = 0$	$x^2 - x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$

Aufgabe (19)

$$x^2 - 8x - 20 = 0$$

a-b-c Formel	p-q Formel
$1x^2 - 8x - 20 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (-20)}}{2 \cdot 1}$ $x_{1/2} = \frac{+8 \pm \sqrt{144}}{2}$ $x_{1/2} = \frac{8 \pm 12}{2}$ $x_1 = \frac{8 + 12}{2} \quad x_2 = \frac{8 - 12}{2}$ $x_1 = 10 \quad x_2 = -2$	$x^2 - 8x - 20 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - (-20)}$ $x_{1/2} = 4 \pm \sqrt{36}$ $x_{1/2} = 4 \pm 6$ $x_1 = 10 \quad x_2 = -2$

Aufgabe (20)

$$x^2 - 8x + 15 = 0$$

a-b-c Formel	p-q Formel
$1x^2 - 8x + 15 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$ $x_{1/2} = \frac{+8 \pm \sqrt{4}}{2}$ $x_{1/2} = \frac{8 \pm 2}{2}$ $x_1 = \frac{8 + 2}{2} \quad x_2 = \frac{8 - 2}{2}$ $x_1 = 5 \quad x_2 = 3$	$x^2 - 8x + 15 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - 15}$ $x_{1/2} = 4 \pm \sqrt{1}$ $x_{1/2} = 4 \pm 1$ $x_1 = 5 \quad x_2 = 3$

Aufgabe (21)

$$-\frac{1}{3}x^2 - 2x + 3 = 0$$

a-b-c Formel	p-q Formel
$-\frac{1}{3}x^2 - 2x + 3 = 0$ $x_{1/2} = \frac{+2 \pm \sqrt{(-2)^2 - 4 \cdot (-\frac{1}{3}) \cdot 3}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{+2 \pm \sqrt{8}}{-\frac{2}{3}}$ $x_{1/2} = \frac{2 \pm 2,828}{-\frac{2}{3}}$ $x_1 = \frac{2 + 2,828}{-\frac{2}{3}} \quad x_2 = \frac{2 - 2,828}{-\frac{2}{3}}$ $x_1 = -7,243 \quad x_2 = 1,243$	$-\frac{1}{3}x^2 - 2x + 3 = 0 \quad / : -\frac{1}{3}$ $x^2 + 6x - 9 = 0$ $x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-9)}$ $x_{1/2} = -3 \pm \sqrt{18}$ $x_{1/2} = -3 \pm 4,243$ $x_1 = 1,243 \quad x_2 = -7,243$

Aufgabe (22)

$$x^2 - 4x + 7 = 0$$

a-b-c Formel	p-q Formel
$1x^2 - 4x + 7 = 0$ $x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2}$ $x_{1/2} = \frac{+4 \pm \sqrt{-12}}{2}$ <p>Diskriminante negativ keine Loesung</p>	$x^2 - 4x + 7 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$ $x_{1/2} = 2 \pm \sqrt{-3}$ <p>Diskriminante negativ keine Loesung</p>

Aufgabe (23)

$$-x^2 + 4x - 7 = 0$$

a-b-c Formel	p-q Formel
$-x^2 + 4x - 7 = 0$ $x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-7)}}{2 \cdot (-1)}$ $x_{1/2} = \frac{-4 \pm \sqrt{-12}}{-2}$ <p>Diskriminante negativ keine Loesung</p>	$-x^2 + 4x - 7 = 0 \quad / : -1$ $x^2 - 4x + 7 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$ $x_{1/2} = 2 \pm \sqrt{-3}$ <p>Diskriminante negativ keine Loesung</p>

Aufgabe (24)

$$2x^2 + 4x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$2x^2 + 4x = 0$ $x(2x + 4) = 0$ $2x + 4 = 0 \quad / -4$ $2x = -4 \quad / : 2$ $x = \frac{-4}{2}$ $x_1 = 0$ $x_2 = -2$	$2x^2 + 4x + 0 = 0$ $x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 0}}{2 \cdot 2}$ $x_{1/2} = \frac{-4 \pm \sqrt{16}}{4}$ $x_{1/2} = \frac{-4 \pm 4}{4}$ $x_1 = \frac{-4 + 4}{4} \quad x_2 = \frac{-4 - 4}{4}$ $x_1 = 0 \quad x_2 = -2$	$2x^2 + 4x + 0 = 0 \quad / : 2$ $x^2 + 2x + 0 = 0$ $x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 0}$ $x_{1/2} = -1 \pm \sqrt{1}$ $x_{1/2} = -1 \pm 1$ $x_1 = 0 \quad x_2 = -2$

Aufgabe (25)

$$-\frac{1}{2}x^2 + 2x + 5 = 0$$

a-b-c Formel	p-q Formel
$-\frac{1}{2}x^2 + 2x + 5 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{2}) \cdot 5}}{2 \cdot (-\frac{1}{2})}$ $x_{1/2} = \frac{-2 \pm \sqrt{14}}{-1}$ $x_{1/2} = \frac{-2 \pm 3,742}{-1}$ $x_1 = \frac{-2 + 3,742}{-1} \quad x_2 = \frac{-2 - 3,742}{-1}$ $x_1 = -1,742 \quad x_2 = 5,742$	$-\frac{1}{2}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{2}$ $x^2 - 4x - 10 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - (-10)}$ $x_{1/2} = 2 \pm \sqrt{14}$ $x_{1/2} = 2 \pm 3,742$ $x_1 = 5,742 \quad x_2 = -1,742$

Aufgabe (26)

$$-2x^2 + 3x + 4 = 0$$

a-b-c Formel	p-q Formel
$-2x^2 + 3x + 4 = 0$ $x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)}$ $x_{1/2} = \frac{-3 \pm \sqrt{41}}{-4}$ $x_{1/2} = \frac{-3 \pm 6,403}{-4}$ $x_1 = \frac{-3 + 6,403}{-4} \quad x_2 = \frac{-3 - 6,403}{-4}$ $x_1 = -0,851 \quad x_2 = 2,351$	$-2x^2 + 3x + 4 = 0 \quad / : -2$ $x^2 - 1\frac{1}{2}x - 2 = 0$ $x_{1/2} = -\frac{-1\frac{1}{2}}{2} \pm \sqrt{\left(\frac{-1\frac{1}{2}}{2}\right)^2 - (-2)}$ $x_{1/2} = \frac{3}{4} \pm \sqrt{2\frac{9}{16}}$ $x_{1/2} = \frac{3}{4} \pm 1,601$ $x_1 = 2,351 \quad x_2 = -0,851$

Aufgabe (27)

$$x^2 + 6x - 2 = 0$$

a-b-c Formel	p-q Formel
$1x^2 + 6x - 2 = 0$ $x_{1/2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$ $x_{1/2} = \frac{-6 \pm \sqrt{44}}{2}$ $x_{1/2} = \frac{-6 \pm 6,633}{2}$ $x_1 = \frac{-6 + 6,633}{2} \quad x_2 = \frac{-6 - 6,633}{2}$ $x_1 = 0,317 \quad x_2 = -6,317$	$x^2 + 6x - 2 = 0$ $x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-2)}$ $x_{1/2} = -3 \pm \sqrt{11}$ $x_{1/2} = -3 \pm 3,317$ $x_1 = 0,317 \quad x_2 = -6,317$

Aufgabe (28)

$$-\frac{1}{3}x^2 + 2x + 5 = 0$$

a-b-c Formel	p-q Formel
$-\frac{1}{3}x^2 + 2x + 5 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{3}) \cdot 5}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{-2 \pm \sqrt{10\frac{2}{3}}}{-\frac{2}{3}}$ $x_{1/2} = \frac{-2 \pm 3,266}{-\frac{2}{3}}$ $x_1 = \frac{-2 + 3,266}{-\frac{2}{3}} \quad x_2 = \frac{-2 - 3,266}{-\frac{2}{3}}$ $x_1 = -1,899 \quad x_2 = 7,899$	$-\frac{1}{3}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{3}$ $x^2 - 6x - 15 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - (-15)}$ $x_{1/2} = 3 \pm \sqrt{24}$ $x_{1/2} = 3 \pm 4,899$ $x_1 = 7,899 \quad x_2 = -1,899$

Aufgabe (29)

$$\frac{1}{2}x^2 - x + 4 = 0$$

a-b-c Formel	p-q Formel
$\frac{1}{2}x^2 - x + 4 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{2} \cdot 4}}{2 \cdot \frac{1}{2}}$ $x_{1/2} = \frac{+1 \pm \sqrt{-7}}{1}$ Diskriminante negativ keine Loesung	$\frac{1}{2}x^2 - x + 4 = 0 \quad / : \frac{1}{2}$ $x^2 - 2x + 8 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - 8}$ $x_{1/2} = 1 \pm \sqrt{-7}$ Diskriminante negativ keine Loesung

Aufgabe (30)

$$-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$$

a-b-c Formel	p-q Formel
$-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$ $x_{1/2} = \frac{+\frac{24}{49} \pm \sqrt{\left(-\frac{24}{49}\right)^2 - 4 \cdot \left(-\frac{8}{49}\right) \cdot 1\frac{31}{49}}}{2 \cdot \left(-\frac{8}{49}\right)}$ $x_{1/2} = \frac{+\frac{24}{49} \pm \sqrt{1\frac{15}{49}}}{-\frac{16}{49}}$ $x_{1/2} = \frac{\frac{24}{49} \pm 1\frac{1}{7}}{-\frac{16}{49}}$ $x_1 = \frac{\frac{24}{49} + 1\frac{1}{7}}{-\frac{16}{49}} \quad x_2 = \frac{\frac{24}{49} - 1\frac{1}{7}}{-\frac{16}{49}}$ $x_1 = -5 \quad x_2 = 2$	$-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0 \quad / : -\frac{8}{49}$ $x^2 + 3x - 10 = 0$ $x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$ $x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = 2 \quad x_2 = -5$

Aufgabe (31)

$$-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$$

a-b-c Formel	p-q Formel
$-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$ $x_{1/2} = \frac{+\frac{32}{81} \pm \sqrt{\left(-\frac{32}{81}\right)^2 - 4 \cdot \left(-\frac{32}{81}\right) \cdot 7\frac{73}{81}}}{2 \cdot \left(-\frac{32}{81}\right)}$ $x_{1/2} = \frac{+\frac{32}{81} \pm \sqrt{12\frac{52}{81}}}{-\frac{64}{81}}$ $x_{1/2} = \frac{\frac{32}{81} \pm 3\frac{5}{9}}{-\frac{64}{81}}$ $x_1 = \frac{\frac{32}{81} + 3\frac{5}{9}}{-\frac{64}{81}} \quad x_2 = \frac{\frac{32}{81} - 3\frac{5}{9}}{-\frac{64}{81}}$ $x_1 = -5 \quad x_2 = 4$	$-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0 \quad / : -\frac{32}{81}$ $x^2 + x - 20 = 0$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-20)}$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$ $x_{1/2} = -\frac{1}{2} \pm 4\frac{1}{2}$ $x_1 = 4 \quad x_2 = -5$

Aufgabe (32)

$$-1\frac{1}{4}x^2 + 5x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$-1\frac{1}{4}x^2 + 5x = 0$ $x(-1\frac{1}{4}x + 5) = 0$ $-1\frac{1}{4}x + 5 = 0 \quad / -5$ $-1\frac{1}{4}x = -5 \quad / : (-1\frac{1}{4})$ $x = \frac{-5}{-1\frac{1}{4}}$ $x_1 = 0$ $x_2 = 4$	$-1\frac{1}{4}x^2 + 5x + 0 = 0$ $x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-1\frac{1}{4}) \cdot 0}}{2 \cdot (-1\frac{1}{4})}$ $x_{1/2} = \frac{-5 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{-5 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{-5 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{-5 - 5}{-2\frac{1}{2}}$ $x_1 = 0 \quad x_2 = 4$	$-1\frac{1}{4}x^2 + 5x + 0 = 0 \quad / : -1\frac{1}{4}$ $x^2 - 4x + 0 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - 0}$ $x_{1/2} = 2 \pm \sqrt{4}$ $x_{1/2} = 2 \pm 2$ $x_1 = 4 \quad x_2 = 0$

Aufgabe (33)

$$-\frac{3}{4}x^2 - 3x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$-\frac{3}{4}x^2 - 3x = 0$ $x(-\frac{3}{4}x - 3) = 0$ $-\frac{3}{4}x - 3 = 0 \quad / +3$ $-\frac{3}{4}x = 3 \quad / : (-\frac{3}{4})$ $x = \frac{3}{-\frac{3}{4}}$ $x_1 = 0$ $x_2 = -4$	$-\frac{3}{4}x^2 - 3x + 0 = 0$ $x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot (-\frac{3}{4}) \cdot 0}}{2 \cdot (-\frac{3}{4})}$ $x_{1/2} = \frac{+3 \pm \sqrt{9}}{-1\frac{1}{2}}$ $x_{1/2} = \frac{3 \pm 3}{-1\frac{1}{2}}$ $x_1 = \frac{3 + 3}{-1\frac{1}{2}} \quad x_2 = \frac{3 - 3}{-1\frac{1}{2}}$ $x_1 = -4 \quad x_2 = 0$	$-\frac{3}{4}x^2 - 3x + 0 = 0 \quad / : -\frac{3}{4}$ $x^2 + 4x + 0 = 0$ $x_{1/2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0}$ $x_{1/2} = -2 \pm \sqrt{4}$ $x_{1/2} = -2 \pm 2$ $x_1 = 0 \quad x_2 = -4$

Aufgabe (34)

$$\frac{5}{9}x^2 - 5 = 0$$

Umformen	a-b-c Formel	p-q Formel
$\frac{5}{9}x^2 - 5 = 0 \quad / +5$ $\frac{5}{9}x^2 = 5 \quad / : \frac{5}{9}$ $x^2 = \frac{5}{\frac{5}{9}}$ $x = \pm\sqrt{9}$ $x_1 = 3 \quad x_2 = -3$	$\frac{5}{9}x^2 + 0x - 5 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{5}{9} \cdot (-5)}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{-0 \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{0 \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{0 + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{0 - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 3 \quad x_2 = -3$	$\frac{5}{9}x^2 + 0x - 5 = 0 \quad / : \frac{5}{9}$ $x^2 + 0x - 9 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)}$ $x_{1/2} = 0 \pm \sqrt{9}$ $x_{1/2} = 0 \pm 3$ $x_1 = 3 \quad x_2 = -3$

Aufgabe (35)

$$12x^2 + 12x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$12x^2 + 12x = 0$ $x(12x + 12) = 0$ $12x + 12 = 0 \quad / -12$ $12x = -12 \quad / : 12$ $x = \frac{-12}{12}$ $x_1 = 0$ $x_2 = -1$	$12x^2 + 12x + 0 = 0$ $x_{1/2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 12 \cdot 0}}{2 \cdot 12}$ $x_{1/2} = \frac{-12 \pm \sqrt{144}}{24}$ $x_{1/2} = \frac{-12 \pm 12}{24}$ $x_1 = \frac{-12 + 12}{24} \quad x_2 = \frac{-12 - 12}{24}$ $x_1 = 0 \quad x_2 = -1$	$12x^2 + 12x + 0 = 0 \quad / : 12$ $x^2 + x + 0 = 0$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 0}$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = -\frac{1}{2} \pm \frac{1}{2}$ $x_1 = 0 \quad x_2 = -1$

Aufgabe (36)

$$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$$

a-b-c Formel	p-q Formel
$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{1\frac{23}{25}^2 - 4 \cdot \left(-\frac{6}{25}\right) \cdot 2\frac{4}{25}}}{2 \cdot \left(-\frac{6}{25}\right)}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{5\frac{19}{25}}}{-\frac{12}{25}}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = \frac{-1\frac{23}{25} + 2\frac{2}{5}}{-\frac{12}{25}} \quad x_2 = \frac{-1\frac{23}{25} - 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = -1 \quad x_2 = 9$	$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0 \quad / : -\frac{6}{25}$ $x^2 - 8x - 9 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - (-9)}$ $x_{1/2} = 4 \pm \sqrt{25}$ $x_{1/2} = 4 \pm 5$ $x_1 = 9 \quad x_2 = -1$

Aufgabe (37)

$$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$$

a-b-c Formel	p-q Formel
$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{\left(-2\frac{22}{25}\right)^2 - 4 \cdot \left(-\frac{9}{25}\right) \cdot 3\frac{6}{25}}}{2 \cdot \left(-\frac{9}{25}\right)}$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{12\frac{24}{25}}}{-\frac{18}{25}}$ $x_{1/2} = \frac{2\frac{22}{25} \pm 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = \frac{2\frac{22}{25} + 3\frac{3}{5}}{-\frac{18}{25}} \quad x_2 = \frac{2\frac{22}{25} - 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = -9 \quad x_2 = 1$	$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0 \quad / : -\frac{9}{25}$ $x^2 + 8x - 9 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - (-9)}$ $x_{1/2} = -4 \pm \sqrt{25}$ $x_{1/2} = -4 \pm 5$ $x_1 = 1 \quad x_2 = -9$

Aufgabe (38)

$$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$$

a-b-c Formel	p-q Formel
$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$ $x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{\frac{1}{4}^2 - 4 \cdot (-\frac{1}{8}) \cdot 7\frac{7}{8}}}{2 \cdot (-\frac{1}{8})}$ $x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{4}}{-\frac{1}{4}}$ $x_{1/2} = \frac{-\frac{1}{4} \pm 2}{-\frac{1}{4}}$ $x_1 = \frac{-\frac{1}{4} + 2}{-\frac{1}{4}} \quad x_2 = \frac{-\frac{1}{4} - 2}{-\frac{1}{4}}$ $x_1 = -7 \quad x_2 = 9$	$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0 \quad /: -\frac{1}{8}$ $x^2 - 2x - 63 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-63)}$ $x_{1/2} = 1 \pm \sqrt{64}$ $x_{1/2} = 1 \pm 8$ $x_1 = 9 \quad x_2 = -7$

Aufgabe (39)

$$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$$

a-b-c Formel	p-q Formel
$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$ $x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{3\frac{33}{49}^2 - 4 \cdot \frac{20}{49} \cdot 3\frac{13}{49}}}{2 \cdot \frac{20}{49}}$ $x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{8\frac{8}{49}}}{\frac{40}{49}}$ $x_{1/2} = \frac{-3\frac{33}{49} \pm 2\frac{6}{7}}{\frac{40}{49}}$ $x_1 = \frac{-3\frac{33}{49} + 2\frac{6}{7}}{\frac{40}{49}} \quad x_2 = \frac{-3\frac{33}{49} - 2\frac{6}{7}}{\frac{40}{49}}$ $x_1 = -1 \quad x_2 = -8$	$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0 \quad /: \frac{20}{49}$ $x^2 + 9x + 8 = 0$ $x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 8}$ $x_{1/2} = -4\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = -4\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = -1 \quad x_2 = -8$

Aufgabe (40)

$$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$$

a-b-c Formel	p-q Formel
$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$ $x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{\frac{4}{9}^2 - 4 \cdot (-\frac{4}{9}) \cdot \frac{8}{9}}}{2 \cdot (-\frac{4}{9})}$ $x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{1\frac{7}{9}}}{-\frac{8}{9}}$ $x_{1/2} = \frac{-\frac{4}{9} \pm 1\frac{1}{3}}{-\frac{8}{9}}$ $x_1 = \frac{-\frac{4}{9} + 1\frac{1}{3}}{-\frac{8}{9}} \quad x_2 = \frac{-\frac{4}{9} - 1\frac{1}{3}}{-\frac{8}{9}}$ $x_1 = -1 \quad x_2 = 2$	$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0 \quad /: -\frac{4}{9}$ $x^2 - x - 2 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-2)}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{2\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm 1\frac{1}{2}$ $x_1 = 2 \quad x_2 = -1$

Aufgabe (41)

$$-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0$$

a-b-c Formel	p-q Formel
$-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0$ $x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{(-2\frac{2}{9})^2 - 4 \cdot (-2\frac{2}{9}) \cdot 4\frac{4}{9}}}{2 \cdot (-2\frac{2}{9})}$ $x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{44\frac{4}{9}}}{-4\frac{4}{9}}$ $x_{1/2} = \frac{2\frac{2}{9} \pm 6\frac{2}{3}}{-4\frac{4}{9}}$ $x_1 = \frac{2\frac{2}{9} + 6\frac{2}{3}}{-4\frac{4}{9}} \quad x_2 = \frac{2\frac{2}{9} - 6\frac{2}{3}}{-4\frac{4}{9}}$ $x_1 = -2 \quad x_2 = 1$	$-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0 \quad / : -2\frac{2}{9}$ $x^2 + x - 2 = 0$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-2)}$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{2\frac{1}{4}}$ $x_{1/2} = -\frac{1}{2} \pm 1\frac{1}{2}$ $x_1 = 1 \quad x_2 = -2$

Aufgabe (42)

$$-\frac{7}{9}x^2 + 4\frac{2}{3}x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$-\frac{7}{9}x^2 + 4\frac{2}{3}x = 0$ $x(-\frac{7}{9}x + 4\frac{2}{3}) = 0$ $-\frac{7}{9}x + 4\frac{2}{3} = 0 \quad / -4\frac{2}{3}$ $-\frac{7}{9}x = -4\frac{2}{3} \quad / : (-\frac{7}{9})$ $x = \frac{-4\frac{2}{3}}{-\frac{7}{9}}$ $x_1 = 0$ $x_2 = 6$	$-\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 = 0$ $x_{1/2} = \frac{-4\frac{2}{3} \pm \sqrt{4\frac{2}{3}^2 - 4 \cdot (-\frac{7}{9}) \cdot 0}}{2 \cdot (-\frac{7}{9})}$ $x_{1/2} = \frac{-4\frac{2}{3} \pm \sqrt{21\frac{7}{9}}}{-1\frac{5}{9}}$ $x_{1/2} = \frac{-4\frac{2}{3} \pm 4\frac{2}{3}}{-1\frac{5}{9}}$ $x_1 = \frac{-4\frac{2}{3} + 4\frac{2}{3}}{-1\frac{5}{9}} \quad x_2 = \frac{-4\frac{2}{3} - 4\frac{2}{3}}{-1\frac{5}{9}}$ $x_1 = 0 \quad x_2 = 6$	$-\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 = 0 \quad / : -\frac{7}{9}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$

Aufgabe (43)

$$\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0$$

a-b-c Formel	p-q Formel
$\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0$ $x_{1/2} = \frac{+\frac{6}{49} \pm \sqrt{(-\frac{6}{49})^2 - 4 \cdot \frac{3}{49} \cdot (-2\frac{46}{49})}}{2 \cdot \frac{3}{49}}$ $x_{1/2} = \frac{+\frac{6}{49} \pm \sqrt{\frac{36}{49}}}{\frac{6}{49}}$ $x_{1/2} = \frac{\frac{6}{49} \pm \frac{6}{7}}{\frac{6}{49}}$ $x_1 = \frac{\frac{6}{49} + \frac{6}{7}}{\frac{6}{49}} \quad x_2 = \frac{\frac{6}{49} - \frac{6}{7}}{\frac{6}{49}}$ $x_1 = 8 \quad x_2 = -6$	$\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0 \quad / : \frac{3}{49}$ $x^2 - 2x - 48 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-48)}$ $x_{1/2} = 1 \pm \sqrt{49}$ $x_{1/2} = 1 \pm 7$ $x_1 = 8 \quad x_2 = -6$

Aufgabe (44)

$$\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$ $x(\frac{5}{9}x - 3\frac{1}{3}) = 0$ $\frac{5}{9}x - 3\frac{1}{3} = 0 \quad / + 3\frac{1}{3}$ $\frac{5}{9}x = 3\frac{1}{3} \quad / : \frac{5}{9}$ $x = \frac{3\frac{1}{3}}{\frac{5}{9}}$ $x_1 = 0$ $x_2 = 6$	$\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{(-3\frac{1}{3})^2 - 4 \cdot \frac{5}{9} \cdot 0}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{3\frac{1}{3} \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{3\frac{1}{3} + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{3\frac{1}{3} - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 6 \quad x_2 = 0$	$\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0 \quad / : \frac{5}{9}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{(-6)}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$

Aufgabe (45)

$$-1\frac{1}{4}x^2 - 10x - 15 = 0$$

a-b-c Formel	p-q Formel
$-1\frac{1}{4}x^2 - 10x - 15 = 0$ $x_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot (-1\frac{1}{4}) \cdot (-15)}}{2 \cdot (-1\frac{1}{4})}$ $x_{1/2} = \frac{+10 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{10 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{10 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{10 - 5}{-2\frac{1}{2}}$ $x_1 = -6 \quad x_2 = -2$	$-1\frac{1}{4}x^2 - 10x - 15 = 0 \quad / : -1\frac{1}{4}$ $x^2 + 8x + 12 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 12}$ $x_{1/2} = -4 \pm \sqrt{4}$ $x_{1/2} = -4 \pm 2$ $x_1 = -2 \quad x_2 = -6$

Aufgabe (46)

$$4x^2 - 8x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$4x^2 - 8x = 0$ $x(4x - 8) = 0$ $4x - 8 = 0 \quad / + 8$ $4x = 8 \quad / : 4$ $x = \frac{8}{4}$ $x_1 = 0$ $x_2 = 2$	$4x^2 - 8x + 0 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 0}}{2 \cdot 4}$ $x_{1/2} = \frac{+8 \pm \sqrt{64}}{8}$ $x_{1/2} = \frac{8 \pm 8}{8}$ $x_1 = \frac{8 + 8}{8} \quad x_2 = \frac{8 - 8}{8}$ $x_1 = 2 \quad x_2 = 0$	$4x^2 - 8x + 0 = 0 \quad / : 4$ $x^2 - 2x + 0 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{(-2)}{2}\right)^2 - 0}$ $x_{1/2} = 1 \pm \sqrt{1}$ $x_{1/2} = 1 \pm 1$ $x_1 = 2 \quad x_2 = 0$

Aufgabe (47)

$$-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$$

a-b-c Formel	p-q Formel
$-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$ $x_{1/2} = \frac{-2\frac{22}{49} \pm \sqrt{2\frac{22}{49}^2 - 4 \cdot \left(-\frac{24}{49}\right) \cdot 2\frac{46}{49}}}{2 \cdot \left(-\frac{24}{49}\right)}$ $x_{1/2} = \frac{-2\frac{22}{49} \pm \sqrt{11\frac{37}{49}}}{-\frac{48}{49}}$ $x_{1/2} = \frac{-2\frac{22}{49} \pm 3\frac{3}{7}}{-\frac{48}{49}}$ $x_1 = \frac{-2\frac{22}{49} + 3\frac{3}{7}}{-\frac{48}{49}} \quad x_2 = \frac{-2\frac{22}{49} - 3\frac{3}{7}}{-\frac{48}{49}}$ $x_1 = -1 \quad x_2 = 6$	$-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0 \quad / : -\frac{24}{49}$ $x^2 - 5x - 6 = 0$ $x_{1/2} = -\frac{-5}{2} \pm \sqrt{\left(\frac{-5}{2}\right)^2 - (-6)}$ $x_{1/2} = 2\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = 2\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = 6 \quad x_2 = -1$

Aufgabe (48)

$$\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$ $x\left(\frac{8}{27}x + 2\frac{2}{3}\right) = 0$ $\frac{8}{27}x + 2\frac{2}{3} = 0 \quad / - 2\frac{2}{3}$ $\frac{8}{27}x = -2\frac{2}{3} \quad / : \frac{8}{27}$ $x = \frac{-2\frac{2}{3}}{\frac{8}{27}}$ $x_1 = 0$ $x_2 = -9$	$\frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0$ $x_{1/2} = \frac{-2\frac{2}{3} \pm \sqrt{2\frac{2}{3}^2 - 4 \cdot \frac{8}{27} \cdot 0}}{2 \cdot \frac{8}{27}}$ $x_{1/2} = \frac{-2\frac{2}{3} \pm \sqrt{7\frac{1}{9}}}{\frac{16}{27}}$ $x_{1/2} = \frac{-2\frac{2}{3} \pm 2\frac{2}{3}}{\frac{16}{27}}$ $x_1 = \frac{-2\frac{2}{3} + 2\frac{2}{3}}{\frac{16}{27}} \quad x_2 = \frac{-2\frac{2}{3} - 2\frac{2}{3}}{\frac{16}{27}}$ $x_1 = 0 \quad x_2 = -9$	$\frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0 \quad / : \frac{8}{27}$ $x^2 + 9x + 0 = 0$ $x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0}$ $x_{1/2} = -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$ $x_{1/2} = -4\frac{1}{2} \pm 4\frac{1}{2}$ $x_1 = 0 \quad x_2 = -9$

Aufgabe (49)

$$\frac{20}{81}x^2 + 2\frac{2}{9}x = 0$$

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{20}{81}x^2 + 2\frac{2}{9}x = 0$ $x\left(\frac{20}{81}x + 2\frac{2}{9}\right) = 0$ $\frac{20}{81}x + 2\frac{2}{9} = 0 \quad / - 2\frac{2}{9}$ $\frac{20}{81}x = -2\frac{2}{9} \quad / : \frac{20}{81}$ $x = \frac{-2\frac{2}{9}}{\frac{20}{81}}$ $x_1 = 0$ $x_2 = -9$	$\frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0$ $x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{2\frac{2}{9}^2 - 4 \cdot \frac{20}{81} \cdot 0}}{2 \cdot \frac{20}{81}}$ $x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{4\frac{76}{81}}}{\frac{40}{81}}$ $x_{1/2} = \frac{-2\frac{2}{9} \pm 2\frac{2}{9}}{\frac{40}{81}}$ $x_1 = \frac{-2\frac{2}{9} + 2\frac{2}{9}}{\frac{40}{81}} \quad x_2 = \frac{-2\frac{2}{9} - 2\frac{2}{9}}{\frac{40}{81}}$ $x_1 = 0 \quad x_2 = -9$	$\frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0 \quad / : \frac{20}{81}$ $x^2 + 9x + 0 = 0$ $x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0}$ $x_{1/2} = -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$ $x_{1/2} = -4\frac{1}{2} \pm 4\frac{1}{2}$ $x_1 = 0 \quad x_2 = -9$

Aufgabe (50)

$$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$$

a-b-c Formel	p-q Formel
$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$ $x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{10\frac{2}{25}^2 - 4 \cdot 1\frac{11}{25} \cdot 8\frac{16}{25}}}{2 \cdot 1\frac{11}{25}}$ $x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{51\frac{21}{25}}}{2\frac{22}{25}}$ $x_{1/2} = \frac{-10\frac{2}{25} \pm 7\frac{1}{5}}{2\frac{22}{25}}$ $x_1 = \frac{-10\frac{2}{25} + 7\frac{1}{5}}{2\frac{22}{25}} \quad x_2 = \frac{-10\frac{2}{25} - 7\frac{1}{5}}{2\frac{22}{25}}$ $x_1 = -1 \quad x_2 = -6$	$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0 \quad / : 1\frac{11}{25}$ $x^2 + 7x + 6 = 0$ $x_{1/2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 6}$ $x_{1/2} = -3\frac{1}{2} \pm \sqrt{6\frac{1}{4}}$ $x_{1/2} = -3\frac{1}{2} \pm 2\frac{1}{2}$ $x_1 = -1 \quad x_2 = -6$

4 Kubische Gleichungen $ax^3 + bx^2 + cx + d = 0$

4.1 Aufgaben

- | | |
|--|--|
| (1) $-2x^3 = 0$ | (17) $-\frac{27}{28}x^3 - \frac{27}{28}x^2 + 5\frac{11}{14}x = 0$ |
| (2) $-\frac{1}{4}x^3 + \frac{2}{3}x^2 = 0$ | (18) $x^3 + 3x^2 - 4 = 0$ |
| (3) $x^3 - 3x^2 = 0$ | (19) $-5\frac{1}{16}x^3 + 10\frac{1}{8}x^2 = 0$ |
| (4) $\frac{1}{2}x^3 + 4 = 0$ | (20) $\frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$ |
| (5) $-\frac{1}{6}x^3 + 2x = 0$ | (21) $-2x^3 + 12x^2 - 18x = 0$ |
| (6) $\frac{1}{2}x^3 - 3x^2 + 5x = 0$ | (22) $40\frac{1}{2}x^3 + 81x^2 + 40\frac{1}{2}x = 0$ |
| (7) $-x^3 + 3x + 2 = 0$ | (23) $54x^3 - 270x^2 + 432x - 216 = 0$ |
| (8) $-x^3 + 3x^2 - 4 = 0$ | (24) $1\frac{19}{35}x^3 - 10\frac{4}{5}x^2 + 18\frac{18}{35}x = 0$ |
| (9) $4x^3 + 5x^2 - 6x = 0$ | (25) $-2x^3 + 6x^2 = 0$ |
| (10) $-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$ | (26) $-2x^3 + 6x^2 = 0$ |
| (11) $x^3 - 4x^2 + 3x = 0$ | (27) $5\frac{2}{5}x^3 + 27x^2 + 32\frac{2}{5}x = 0$ |
| (12) $-\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$ | (28) $\frac{1}{3}x^3 - x^2 - 1\frac{1}{3}x = 0$ |
| (13) $\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$ | (29) $-0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$ |
| (14) $-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$ | (30) $-\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$ |
| (15) $-6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$ | (31) $-13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$ |
| (16) $\frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$ | |

4.2 Lösungen

Aufgabe (1)

$$\begin{aligned}
 -2x^3 &= 0 \\
 x^3 &= 0 \Rightarrow x = 0 \\
 x_1 &= 0; \quad \underline{\text{3-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned}
 -\frac{1}{4}x^3 + \frac{2}{3}x^2 &= 0 \\
 x^2\left(-\frac{1}{4}x + \frac{2}{3}\right) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\
 -\frac{1}{4}x + \frac{2}{3} &= 0 \quad / -\frac{2}{3} \\
 -\frac{1}{4}x &= -\frac{2}{3} \quad / : \left(-\frac{1}{4}\right) \\
 x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\
 x &= 2\frac{2}{3} \\
 x_1 &= 0; \quad \underline{\text{2-fache Nullstelle}} \\
 x_2 &= 2\frac{2}{3}; \quad \underline{\text{1-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (3)

$$\begin{aligned}
 x^3 - 3x^2 &= 0 \\
 x^2(x - 3) &= 0 \Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\
 x - 3 &= 0 \quad / + 3 \\
 x &= 3 \\
 x_1 &= 0; \quad \underline{\text{2-fache Nullstelle}} \\
 x_2 &= 3; \quad \underline{\text{1-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (4)

$$\begin{aligned}
 \frac{1}{2}x^3 + 4 &= 0 \\
 \frac{1}{2}x^3 + 4 &= 0 \quad \frac{1}{2}x^3 + 4 = 0 \quad / - 4 \\
 \frac{1}{2}x^3 &= -4 \quad / : \frac{1}{2} \\
 x^3 &= \frac{-4}{\frac{1}{2}} \\
 x &= \sqrt[3]{-8} \\
 x &= -2 \\
 \text{Polynomdivision: } &(-2) \\
 \begin{array}{r}
 \left(\frac{1}{2}x^3 \right) : (x + 2) = \frac{1}{2}x^2 - x + 2 \\
 -\left(\frac{1}{2}x^3 + x^2\right) \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} -x^2 \\
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} -(-x^2 - 2x) \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} 2x \\
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} -(2x) \\
 \hline
 \phantom{-\left(\frac{1}{2}x^3 + x^2\right)} \phantom{-(2x)} 0
 \end{array} \\
 \frac{1}{2}x^2 - x + 2 &= 0 \\
 x_{1/2} &= \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{2} \cdot 2}}{2 \cdot \frac{1}{2}} \\
 x_{1/2} &= \frac{+1 \pm \sqrt{-3}}{1} \\
 \text{Diskriminante negativ keine Loesung} \\
 x_1 &= -2; \quad \underline{\text{1-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (5)

$$\begin{aligned}
 &-\frac{1}{6}x^3 + 2x = 0 \\
 &x\left(-\frac{1}{6}x^2 + 2\right) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0 \\
 &-\frac{1}{6}x^2 + 2 = 0 \quad / -2 \\
 &-\frac{1}{6}x^2 = -2 \quad / : \left(-\frac{1}{6}\right) \\
 &x^2 = \frac{-2}{-\frac{1}{6}} \\
 &x = \pm\sqrt{12} \\
 &x_1 = 3,464 \quad x_2 = -3,464 \\
 &x_1 = -3,464; \quad \underline{1\text{-fache Nullstelle}} \\
 &x_2 = 0; \quad \underline{1\text{-fache Nullstelle}} \\
 &x_3 = 3,464; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (6)

$$\begin{aligned}
 &\frac{1}{2}x^3 - 3x^2 + 5x = 0 \\
 &x\left(\frac{1}{2}x^2 - 3x + 5\right) = 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0 \\
 &\frac{1}{2}x^2 - 3x + 5 = 0 \\
 &x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}} \\
 &x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1} \\
 &\text{Diskriminante negativ keine Loesung} \\
 &x_1 = 0; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (7)

$$\begin{aligned}
 &-x^3 + 3x + 2 = 0 \\
 &-x^3 + 3x + 2 = 0 \\
 &\text{Nullstelle für Polynomdivision erraten: } -1 \\
 &\begin{array}{r}
 (-x^3 \quad -x^2) \quad +3x \quad +2 \quad) : (x+1) = -x^2 + x + 2 \\
 \underline{-(-x^3 \quad -x^2)} \\
 x^2 \quad +3x \quad +2 \\
 \underline{-(x^2 \quad +x)} \\
 2x \quad +2 \\
 \underline{-(2x \quad +2)} \\
 0
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 &-x^2 + x + 2 = 0 \\
 &x_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 &x_{1/2} = \frac{-1 \pm \sqrt{9}}{-2} \\
 &x_{1/2} = \frac{-1 \pm 3}{-2} \\
 &x_1 = \frac{-1 + 3}{-2} \quad x_2 = \frac{-1 - 3}{-2} \\
 &x_1 = -1 \quad x_2 = 2 \\
 &x_1 = -1; \quad \underline{2\text{-fache Nullstelle}} \\
 &x_2 = 2; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (8)

$$-x^3 + 3x^2 - 4 = 0$$

$$-x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-x^3 + 3x^2 - 4) : (x + 1) = -x^2 + 4x - 4 \\ -(-x^3 - x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 + 4x) \\ \hline -4x - 4 \\ -(-4x - 4) \\ \hline 0 \end{array}$$

$$-x^2 + 4x - 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{-2}$$

$$x_{1/2} = \frac{-4 \pm 0}{-2}$$

$$x_1 = \frac{-4 + 0}{-2} \quad x_2 = \frac{-4 - 0}{-2}$$

$$x_1 = 2 \quad x_2 = 2$$

$$x_1 = -1; \quad \text{1-fache Nullstelle}$$

$$x_2 = 2; \quad \text{2-fache Nullstelle}$$

Aufgabe (9)

$$4x^3 + 5x^2 - 6x = 0$$

$$x(4x^2 + 5x - 6) = 0 \Rightarrow x = 0 \quad \wedge \quad 4x^2 + 5x - 6 = 0$$

$$4x^2 + 5x - 6 = 0$$

$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$

$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$

$$x_{1/2} = \frac{-5 \pm 11}{8}$$

$$x_1 = \frac{-5 + 11}{8} \quad x_2 = \frac{-5 - 11}{8}$$

$$x_1 = \frac{3}{4} \quad x_2 = -2$$

$$x_1 = -2; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{1-fache Nullstelle}$$

$$x_3 = \frac{3}{4}; \quad \text{1-fache Nullstelle}$$

Aufgabe (10)

$$-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

$$-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6) : (x + 2) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3 \\ -(-\frac{1}{2}x^3 - x^2) \\ \hline \frac{1}{2}x^2 + 4x + 6 \\ -(\frac{1}{2}x^2 + x) \\ \hline 3x + 6 \\ -(3x + 6) \\ \hline 0 \end{array}$$

$$-\frac{1}{2}x^2 + \frac{1}{2}x + 3 = 0$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{2}^2 - 4 \cdot (-\frac{1}{2}) \cdot 3}}{2 \cdot (-\frac{1}{2})}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{6\frac{1}{4}}}{-1}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm 2\frac{1}{2}}{-1}$$

$$x_1 = \frac{-\frac{1}{2} + 2\frac{1}{2}}{-1} \quad x_2 = \frac{-\frac{1}{2} - 2\frac{1}{2}}{-1}$$

$$x_1 = -2 \quad x_2 = 3$$

$$\underline{x_1 = -2; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (11)

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0 \Rightarrow x = 0 \quad \wedge \quad x^2 - 4x + 3 = 0$$

$$1x^2 - 4x + 3 = 0$$

$$x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+4 \pm \sqrt{4}}{2}$$

$$x_{1/2} = \frac{4 \pm 2}{2}$$

$$x_1 = \frac{4 + 2}{2} \quad x_2 = \frac{4 - 2}{2}$$

$$x_1 = 3 \quad x_2 = 1$$

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 3; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (12)

$$-\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

$$-\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

NumerischeSuche :

$$\underline{x_1 = -4; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = -1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 3; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (13)

$$\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

$$\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

Nullstelle für Polynomdivision erraten: -3

$$\begin{array}{r} (\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5}) : (x + 3) = \frac{1}{10}x^2 - 0x - 1\frac{3}{5} \\ -(\frac{1}{10}x^3 + \frac{3}{10}x^2) \\ \hline -0x^2 - 1\frac{3}{5}x - 4\frac{4}{5} \\ -(-0x^2 - 0x) \\ \hline -1\frac{3}{5}x - 4\frac{4}{5} \\ -(-1\frac{3}{5}x - 4\frac{4}{5}) \\ \hline 0 \end{array}$$

$$\frac{1}{10}x^2 - 0x - 1\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+0 \pm \sqrt{(0)^2 - 4 \cdot \frac{1}{10} \cdot (-1\frac{3}{5})}}{2 \cdot \frac{1}{10}}$$

$$x_{1/2} = \frac{+0 \pm \sqrt{\frac{16}{25}}}{\frac{1}{5}}$$

$$x_{1/2} = \frac{0 \pm \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = \frac{0 + \frac{4}{5}}{\frac{1}{5}} \quad x_2 = \frac{0 - \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = 4 \quad x_2 = -4$$

$$x_1 = -4; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = -3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 4; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (14)

$$-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

$$-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5}) : (x + 2) = -5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} \\ \underline{-(-5\frac{2}{5}x^3 - 10\frac{4}{5}x^2)} \\ -27x^2 - 75\frac{3}{5}x - 43\frac{1}{5} \\ \underline{-(-27x^2 - 54x)} \\ -21\frac{3}{5}x - 43\frac{1}{5} \\ \underline{-(-21\frac{3}{5}x - 43\frac{1}{5})} \\ 0 \end{array}$$

$$-5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+27 \pm \sqrt{(-27)^2 - 4 \cdot (-5\frac{2}{5}) \cdot (-21\frac{3}{5})}}{2 \cdot (-5\frac{2}{5})}$$

$$x_{1/2} = \frac{+27 \pm \sqrt{262\frac{11}{25}}}{-10\frac{4}{5}}$$

$$x_{1/2} = \frac{27 \pm 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = \frac{27 + 16\frac{1}{5}}{-10\frac{4}{5}} \quad x_2 = \frac{27 - 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = -4 \quad x_2 = -1$$

$$x_1 = -4; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = -2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = -1; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (15)

$$-6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$$

$$x^2(-6\frac{3}{4}x - 13\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \wedge \quad -6\frac{3}{4}x - 13\frac{1}{2} = 0$$

$$-6\frac{3}{4}x - 13\frac{1}{2} = 0 \quad / + 13\frac{1}{2}$$

$$-6\frac{3}{4}x = 13\frac{1}{2} \quad / : (-6\frac{3}{4})$$

$$x = \frac{13\frac{1}{2}}{-6\frac{3}{4}}$$

$$x = -2$$

$$x_1 = -2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (16)

$$\frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

$$\frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

Numerische Suche :

$$\underline{x_1 = -3; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = -2; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (17)

$$-\frac{27}{28}x^3 - \frac{27}{28}x^2 + 5\frac{11}{14}x = 0$$

$$x(-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14}) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{\left(-\frac{27}{28}\right)^2 - 4 \cdot \left(-\frac{27}{28}\right) \cdot 5\frac{11}{14}}}{2 \cdot \left(-\frac{27}{28}\right)}$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{23,246}}{-1\frac{13}{14}}$$

$$x_{1/2} = \frac{\frac{27}{28} \pm 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = \frac{\frac{27}{28} + 4\frac{23}{28}}{-1\frac{13}{14}} \quad x_2 = \frac{\frac{27}{28} - 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = -3 \quad x_2 = 2$$

$$\underline{x_1 = -3; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (18)

$$x^3 + 3x^2 - 4 = 0$$

$$x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^3 + 3x^2 \quad -4) : (x-1) = x^2 + 4x + 4 \\ -(x^3 - x^2) \\ \hline 4x^2 \quad -4 \\ -(4x^2 - 4x) \\ \hline 4x \quad -4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$1x^2 + 4x + 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{2}$$

$$x_{1/2} = \frac{-4 \pm 0}{2}$$

$$x_1 = \frac{-4 + 0}{2} \quad x_2 = \frac{-4 - 0}{2}$$

$$x_1 = -2 \quad x_2 = -2$$

$$\underline{x_1 = -2; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 1; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (19)

$$\begin{aligned}
-5\frac{1}{16}x^3 + 10\frac{1}{8}x^2 &= 0 \\
x^2(-5\frac{1}{16}x + 10\frac{1}{8}) &= 0 \Rightarrow x = 0 \quad \wedge \quad -5\frac{1}{16}x + 10\frac{1}{8} = 0 \\
-5\frac{1}{16}x + 10\frac{1}{8} &= 0 \quad / -10\frac{1}{8} \\
-5\frac{1}{16}x &= -10\frac{1}{8} \quad / : (-5\frac{1}{16}) \\
x &= \frac{-10\frac{1}{8}}{-5\frac{1}{16}} \\
x &= 2 \\
x_1 &= 0; \quad \underline{2\text{-fache Nullstelle}} \\
x_2 &= 2; \quad \underline{1\text{-fache Nullstelle}}
\end{aligned}$$

Aufgabe (20)

$$\frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

$$\frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

Numerische Suche :

$$x_1 = -3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 4; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (21)

$$-2x^3 + 12x^2 - 18x = 0$$

$$x(-2x^2 + 12x - 18) = 0 \Rightarrow x = 0 \quad \wedge \quad -2x^2 + 12x - 18 = 0$$

$$-2x^2 + 12x - 18 = 0$$

$$x_{1/2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot (-2) \cdot (-18)}}{2 \cdot (-2)}$$

$$x_{1/2} = \frac{-12 \pm \sqrt{0}}{-4}$$

$$x_{1/2} = \frac{-12 \pm 0}{-4}$$

$$x_1 = \frac{-12 + 0}{-4} \quad x_2 = \frac{-12 - 0}{-4}$$

$$x_1 = 3 \quad x_2 = 3$$

$$x_1 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (22)

$$40\frac{1}{2}x^3 + 81x^2 + 40\frac{1}{2}x = 0$$

$$x(40\frac{1}{2}x^2 + 81x + 40\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \wedge \quad 40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} = 0$$

$$40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} = 0$$

$$x_{1/2} = \frac{-81 \pm \sqrt{81^2 - 4 \cdot 40\frac{1}{2} \cdot 40\frac{1}{2}}}{2 \cdot 40\frac{1}{2}}$$

$$x_{1/2} = \frac{-81 \pm \sqrt{0}}{81}$$

$$x_{1/2} = \frac{-81 \pm 0}{81}$$

$$x_1 = \frac{-81 + 0}{81} \quad x_2 = \frac{-81 - 0}{81}$$

$$x_1 = -1 \quad x_2 = -1$$

$$x_1 = -1; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (23)

$$54x^3 - 270x^2 + 432x - 216 = 0$$

$$54x^3 - 270x^2 + 432x - 216 = 0$$

Nullstelle für Polynomdivision erraten: 1

$$\begin{array}{r} (54x^3 - 270x^2 + 432x - 216) : (x - 1) = 54x^2 - 216x + 216 \\ -(54x^3 - 54x^2) \\ \hline -216x^2 + 432x - 216 \\ -(-216x^2 + 216x) \\ \hline 216x - 216 \\ -(216x - 216) \\ \hline 0 \end{array}$$

$$54x^2 - 216x + 216 = 0$$

$$x_{1/2} = \frac{+216 \pm \sqrt{(-216)^2 - 4 \cdot 54 \cdot 216}}{2 \cdot 54}$$

$$x_{1/2} = \frac{+216 \pm \sqrt{0}}{216 \pm 0}$$

$$x_1 = \frac{216 + 0}{108} \quad x_2 = \frac{216 - 0}{108}$$

$$x_1 = 2 \quad x_2 = 2$$

$$x_1 = 1; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 2; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (24)

$$1\frac{19}{35}x^3 - 10\frac{4}{5}x^2 + 18\frac{18}{35}x = 0$$

$$x(1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35}) = 0 \Rightarrow x = 0 \quad \wedge \quad 1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{(-10\frac{4}{5})^2 - 4 \cdot 1\frac{19}{35} \cdot 18\frac{18}{35}}}{2 \cdot 1\frac{19}{35}}$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{2,38}}{3\frac{3}{35}}$$

$$x_{1/2} = \frac{10\frac{4}{5} \pm 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = \frac{10\frac{4}{5} + 1\frac{19}{35}}{3\frac{3}{35}} \quad x_2 = \frac{10\frac{4}{5} - 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = 4 \quad x_2 = 3$$

$$x_1 = 0; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_3 = 4; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (25)

$$-2x^3 + 6x^2 = 0$$

$$x^2(-2x + 6) = 0 \Rightarrow x = 0 \quad \wedge \quad -2x + 6 = 0$$

$$-2x + 6 = 0 \quad / -6$$

$$-2x = -6 \quad / : (-2)$$

$$x = \frac{-6}{-2}$$

$$x = 3$$

$$x_1 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (26)

$$\begin{aligned}
-2x^3 + 6x^2 &= 0 \\
x^2(-2x + 6) &= 0 \Rightarrow x = 0 \quad \wedge \quad -2x + 6 = 0 \\
-2x + 6 &= 0 \quad / -6 \\
-2x &= -6 \quad / : (-2) \\
x &= \frac{-6}{-2} \\
x &= 3 \\
x_1 &= 0; \quad \underline{\text{2-fache Nullstelle}} \\
x_2 &= 3; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (27)

$$\begin{aligned}
5\frac{2}{5}x^3 + 27x^2 + 32\frac{2}{5}x &= 0 \\
x(5\frac{2}{5}x^2 + 27x + 32\frac{2}{5}) &= 0 \Rightarrow x = 0 \quad \wedge \quad 5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} = 0
\end{aligned}$$

$$\begin{aligned}
5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} &= 0 \\
x_{1/2} &= \frac{-27 \pm \sqrt{27^2 - 4 \cdot 5\frac{2}{5} \cdot 32\frac{2}{5}}}{2 \cdot 5\frac{2}{5}} \\
x_{1/2} &= \frac{-27 \pm \sqrt{29\frac{4}{25}}}{10\frac{4}{5}} \\
x_{1/2} &= \frac{-27 \pm 5\frac{2}{5}}{10\frac{4}{5}} \\
x_1 &= \frac{-27 + 5\frac{2}{5}}{10\frac{4}{5}} \quad x_2 = \frac{-27 - 5\frac{2}{5}}{10\frac{4}{5}} \\
x_1 &= -2 \quad x_2 = -3 \\
x_1 &= -3; \quad \underline{\text{1-fache Nullstelle}} \\
x_2 &= -2; \quad \underline{\text{1-fache Nullstelle}} \\
x_3 &= 0; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (28)

$$\begin{aligned}
\frac{1}{3}x^3 - x^2 - 1\frac{1}{3}x &= 0 \\
x(\frac{1}{3}x^2 - x - 1\frac{1}{3}) &= 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{3}x^2 - x - 1\frac{1}{3} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{1}{3}x^2 - x - 1\frac{1}{3} &= 0 \\
x_{1/2} &= \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{3} \cdot (-1\frac{1}{3})}}{2 \cdot \frac{1}{3}} \\
x_{1/2} &= \frac{+1 \pm \sqrt{2\frac{7}{9}}}{\frac{2}{3}} \\
x_{1/2} &= \frac{1 \pm 1\frac{2}{3}}{\frac{2}{3}} \\
x_1 &= \frac{1 + 1\frac{2}{3}}{\frac{2}{3}} \quad x_2 = \frac{1 - 1\frac{2}{3}}{\frac{2}{3}} \\
x_1 &= 4 \quad x_2 = -1 \\
x_1 &= -1; \quad \underline{\text{1-fache Nullstelle}} \\
x_2 &= 0; \quad \underline{\text{1-fache Nullstelle}} \\
x_3 &= 4; \quad \underline{\text{1-fache Nullstelle}}
\end{aligned}$$

Aufgabe (29)

$$-0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

$$-0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

NumerischeSuche :

$x_1 = -4, 019;$ 1-fache Nullstelle

$x_2 = -1, 997;$ 1-fache Nullstelle

$x_3 = 4, 005;$ 1-fache Nullstelle

Aufgabe (30)

$$-\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

$$-\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

NumerischeSuche :

$x_1 = -3;$ 1-fache Nullstelle

$x_2 = -1;$ 1-fache Nullstelle

$x_3 = 2;$ 1-fache Nullstelle

Aufgabe (31)

$$-13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

$$-13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54) : (x + 1) = -13\frac{1}{2}x^2 - 54x - 54 \\ -(-13\frac{1}{2}x^3 \quad -13\frac{1}{2}x^2) \\ \hline \quad \quad -54x^2 \quad -108x \quad -54 \\ \quad \quad -(-54x^2 \quad -54x) \\ \hline \quad \quad \quad \quad -54x \quad -54 \\ \quad \quad \quad \quad -(-54x \quad -54) \\ \hline \quad \quad \quad \quad \quad \quad \quad \quad 0 \end{array}$$

$$-13\frac{1}{2}x^2 - 54x - 54 = 0$$

$$x_{1/2} = \frac{+54 \pm \sqrt{(-54)^2 - 4 \cdot (-13\frac{1}{2}) \cdot (-54)}}{2 \cdot (-13\frac{1}{2})}$$

$$x_{1/2} = \frac{+54 \pm \sqrt{0}}{-27}$$

$$x_{1/2} = \frac{54 \pm 0}{-27}$$

$$x_1 = \frac{54 + 0}{-27} \quad x_2 = \frac{54 - 0}{-27}$$

$$x_1 = -2 \quad x_2 = -2$$

$x_1 = -2;$ 2-fache Nullstelle

$x_2 = -1;$ 1-fache Nullstelle

5 Funktionen 4. Grades $ax^4 + bx^3 + cx^2 + dx + e = 0$

5.1 Aufgaben

(1) $x^4 - 4x^3 + 2x^2 + 4 = 0$

(2) $x^4 - 4x^3 - 16x - 16 = 0$

(3) $x^4 - 18x^2 + 81 = 0$

(4) $-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$

(5) $-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$

(6) $x^4 - 3x^3 = 0$

(7) $x^4 + x^3 - 9x^2 + 11x - 4 = 0$

(8) $-\frac{1}{6}x^4 + 2x^2 = 0$

(9) $-\frac{1}{6}x^4 + 2x^2 = 0$

(10) $\frac{1}{2}x^4 - 3x^3 + 5x^2 = 0$

(11) $-x^4 + 3x^2 + 2x = 0$

(12) $-x^4 + 3x^3 - 4x = 0$

(13) $4x^4 + 5x^3 - 6x^2 = 0$

(14) $x^4 - 2x^2 + 1 = 0$

(15) $x^4 + 2x^2 + 1 = 0$

(16) $x^4 - 13x^2 + 36 = 0$

(17) $x^4 + x^3 - 7x^2 - x + 7 = 0$

5.2 Lösungen

Aufgabe (1)

$$x^4 - 4x^3 + 2x^2 + 4 = 0$$

$$x^4 - 4x^3 + 2x^2 + 4$$

NumerischeSuche :

$$\underline{x_1 = 1,508; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3,276; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (2)

$$x^4 - 4x^3 - 16x - 16 = 0$$

$$x^4 - 4x^3 - 16x - 16$$

NumerischeSuche :

$$\underline{x_1 = -0,828; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 4,828; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (3)

$$x^4 - 18x^2 + 81 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 18u + 81 = 0$$

$$u_{1/2} = \frac{+18 \pm \sqrt{(-18)^2 - 4 \cdot 1 \cdot 81}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+18 \pm \sqrt{0}}{2}$$

$$u_{1/2} = \frac{18 \pm 0}{2}$$

$$u_1 = \frac{18+0}{2} \quad u_2 = \frac{18-0}{2}$$

$$u_1 = 9 \quad u_2 = 9$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$\underline{x_1 = -3; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 2\text{-fache Nullstelle}}$$

Aufgabe (4)

$$-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$$

$$x^3\left(-\frac{1}{4}x + \frac{2}{3}\right) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0$$

$$-\frac{1}{4}x + \frac{2}{3} = 0 \quad / -\frac{2}{3}$$

$$-\frac{1}{4}x = -\frac{2}{3} \quad / : \left(-\frac{1}{4}\right)$$

$$x = \frac{-\frac{2}{3}}{-\frac{1}{4}}$$

$$x = 2\frac{2}{3}$$

$$x = 2\frac{2}{3}$$

$$\begin{aligned} x_1 &= 0; && \text{3-fache Nullstelle} \\ x_2 &= 2\frac{2}{3}; && \text{1-fache Nullstelle} \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} -\frac{1}{4}x^4 + \frac{2}{3}x^3 &= 0 \\ x^3(-\frac{1}{4}x + \frac{2}{3}) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\ -\frac{1}{4}x + \frac{2}{3} &= 0 \quad / -\frac{2}{3} \\ -\frac{1}{4}x &= -\frac{2}{3} \quad / : (-\frac{1}{4}) \\ x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\ x &= 2\frac{2}{3} \end{aligned}$$

$$\begin{aligned} x_1 &= 0; && \text{3-fache Nullstelle} \\ x_2 &= 2\frac{2}{3}; && \text{1-fache Nullstelle} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} x^4 - 3x^3 &= 0 \\ x^3(x - 3) &= 0 \Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\ x - 3 &= 0 \quad / + 3 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} x_1 &= 0; && \text{3-fache Nullstelle} \\ x_2 &= 3; && \text{1-fache Nullstelle} \end{aligned}$$

Aufgabe (7)

$$x^4 + x^3 - 9x^2 + 11x - 4 = 0$$

$$x^4 + x^3 - 9x^2 + 11x - 4$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^4 + x^3 - 9x^2 + 11x - 4) : (x - 1) = x^3 + 2x^2 - 7x + 4 \\ -(x^4 - x^3) \\ \hline 2x^3 - 9x^2 + 11x - 4 \\ -(2x^3 - 2x^2) \\ \hline -7x^2 + 11x - 4 \\ -(-7x^2 + 7x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$x^3 + 2x^2 - 7x + 4 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^3 + 2x^2 - 7x + 4) : (x - 1) = x^2 + 3x - 4 \\ -(x^3 - x^2) \\ \hline 3x^2 - 7x + 4 \\ -(3x^2 - 3x) \\ \hline -4x + 4 \\ -(-4x + 4) \\ \hline 0 \end{array}$$

$$1x^2 + 3x - 4 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{25}}{2}$$

$$x_{1/2} = \frac{-3 \pm 5}{2}$$

$$x_1 = \frac{-3+5}{2} \quad x_2 = \frac{-3-5}{2}$$

$$x_1 = 1 \quad x_2 = -4$$

$$x_1 = -4; \quad \text{1-fache Nullstelle}$$

$$x_2 = 1; \quad \text{3-fache Nullstelle}$$

Aufgabe (8)

$$-\frac{1}{6}x^4 + 2x^2 = 0$$

$$x^2(-\frac{1}{6}x^2 + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0$$

$$-\frac{1}{6}x^2 + 2 = 0 \quad / -2$$

$$-\frac{1}{6}x^2 = -2 \quad / : (-\frac{1}{6})$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,464 \quad x_2 = -3,464$$

$$x_1 = -3,464; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{2-fache Nullstelle}$$

$$x_3 = 3,464; \quad \text{1-fache Nullstelle}$$

Aufgabe (9)

$$-\frac{1}{6}x^4 + 2x^2 = 0$$

$$x^2(-\frac{1}{6}x^2 + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0$$

$$-\frac{1}{6}x^2 + 2 = 0 \quad / -2$$

$$-\frac{1}{6}x^2 = -2 \quad / : (-\frac{1}{6})$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,464 \quad x_2 = -3,464$$

$$x_1 = -3,464; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{2-fache Nullstelle}$$

$$x_3 = 3,464; \quad \text{1-fache Nullstelle}$$

Aufgabe (10)

$$\frac{1}{2}x^4 - 3x^3 + 5x^2 = 0$$

$$x^2(\frac{1}{2}x^2 - 3x + 5) = 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0$$

$$\frac{1}{2}x^2 - 3x + 5 = 0$$

$$x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1}$$

Diskriminante negativ keine Loesung

$$x_1 = 0; \quad \text{2-fache Nullstelle}$$

Aufgabe (11)

$$-x^4 + 3x^2 + 2x = 0$$

$$x(-x^3 + 3x + 2) = 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x + 2 = 0$$

$$-x^3 + 3x + 2 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-x^3 \quad +3x \quad +2) : (x+1) = -x^2 + x + 2 \\ -(-x^3 \quad -x^2) \\ \hline x^2 \quad +3x \quad +2 \\ -(x^2 \quad +x) \\ \hline 2x \quad +2 \\ -(2x \quad +2) \\ \hline 0 \end{array}$$

$$\begin{aligned}
 -x^2 + x + 2 &= 0 \\
 x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
 x_{1/2} &= \frac{-1 \pm 3}{-2} \\
 x_1 &= \frac{-1 + 3}{-2} & x_2 &= \frac{-1 - 3}{-2} \\
 x_1 &= -1 & x_2 &= 2 \\
 \underline{x_1 = -1; \quad 2\text{-fache Nullstelle}} \\
 \underline{x_2 = 0; \quad 1\text{-fache Nullstelle}} \\
 \underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (12)

$$\begin{aligned}
 -x^4 + 3x^3 - 4x &= 0 \\
 x(-x^3 + 3x^2 - 4) = 0 &\Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x^2 - 4 = 0 \\
 -x^3 + 3x^2 - 4 &= 0 \\
 \text{Nullstelle für Polynomdivision erraten: } -1 & \\
 \begin{array}{r}
 (-x^3 + 3x^2 - 4) : (x + 1) = -x^2 + 4x - 4 \\
 \underline{-(-x^3 - x^2)} \\
 4x^2 - 4 \\
 \underline{-(4x^2 + 4x)} \\
 -4x - 4 \\
 \underline{-(-4x - 4)} \\
 0
 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 -x^2 + 4x - 4 &= 0 \\
 x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-4 \pm \sqrt{0}}{-2} \\
 x_{1/2} &= \frac{-4 \pm 0}{-2} \\
 x_1 &= \frac{-4 + 0}{-2} & x_2 &= \frac{-4 - 0}{-2} \\
 x_1 &= 2 & x_2 &= 2 \\
 \underline{x_1 = -1; \quad 1\text{-fache Nullstelle}} \\
 \underline{x_2 = 0; \quad 1\text{-fache Nullstelle}} \\
 \underline{x_3 = 2; \quad 2\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (13)

$$\begin{aligned}
 4x^4 + 5x^3 - 6x^2 &= 0 \\
 x^2(4x^2 + 5x - 6) = 0 &\Rightarrow x = 0 \quad \wedge \quad 4x^2 + 5x - 6 = 0 \\
 4x^2 + 5x - 6 &= 0 \\
 x_{1/2} &= \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4} \\
 x_{1/2} &= \frac{-5 \pm \sqrt{121}}{8} \\
 x_{1/2} &= \frac{-5 \pm 11}{8} \\
 x_1 &= \frac{-5 + 11}{8} & x_2 &= \frac{-5 - 11}{8} \\
 x_1 &= \frac{3}{4} & x_2 &= -2
 \end{aligned}$$

$$\begin{array}{l} x_1 = -2; \quad 1\text{-fache Nullstelle} \\ x_2 = 0; \quad 2\text{-fache Nullstelle} \\ x_3 = \frac{3}{4}; \quad 1\text{-fache Nullstelle} \end{array}$$

Aufgabe (14)

$$x^4 - 2x^2 + 1 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 2u + 1 = 0$$

$$u_{1/2} = \frac{+2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+2 \pm \sqrt{0}}{2}$$

$$u_{1/2} = \frac{2 \pm 0}{2}$$

$$u_1 = \frac{2+0}{2} \quad u_2 = \frac{2-0}{2}$$

$$u_1 = 1 \quad u_2 = 1$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x_1 = 1 \quad x_2 = -1$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x_1 = 1 \quad x_2 = -1$$

$$x_1 = -1; \quad 2\text{-fache Nullstelle}$$

$$x_2 = 1; \quad 2\text{-fache Nullstelle}$$

Aufgabe (15)

$$x^4 + 2x^2 + 1 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 + 2u + 1 = 0$$

$$u_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$u_{1/2} = \frac{-2 \pm \sqrt{0}}{2}$$

$$u_{1/2} = \frac{-2 \pm 0}{2}$$

$$u_1 = \frac{-2+0}{2} \quad u_2 = \frac{-2-0}{2}$$

$$u_1 = -1 \quad u_2 = -1$$

$$x^2 = -1 \quad x = \pm\sqrt{-1}$$

Diskriminante negativ keine Loesung

$$x^2 = -1 \quad x = \pm\sqrt{-1}$$

Diskriminante negativ keine Loesung

Aufgabe (16)

$$x^4 - 13x^2 + 36 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 13u + 36 = 0$$

$$u_{1/2} = \frac{+13 \pm \sqrt{(-13)^2 - 4 \cdot 1 \cdot 36}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+13 \pm \sqrt{25}}{2}$$

$$u_{1/2} = \frac{13 \pm 5}{2}$$

$$u_1 = \frac{13+5}{2} \quad u_2 = \frac{13-5}{2}$$

$$u_1 = 9 \quad u_2 = 4$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 4$$

$$x = \pm\sqrt{4}$$

$$x_1 = 2 \quad x_2 = -2$$

$$x_1 = -3; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

$$x_3 = 2; \quad \text{1-fache Nullstelle}$$

$$x_4 = 3; \quad \text{1-fache Nullstelle}$$

Aufgabe (17)

$$x^4 + x^3 - 7x^2 - x + 7 = 0$$

$$x^4 + x^3 - 7x^2 - x + 7$$

NumerischeSuche :

$$x_1 = -2,974; \quad \text{1-fache Nullstelle}$$

$$x_2 = -1,081; \quad \text{1-fache Nullstelle}$$

$$x_3 = 1,131; \quad \text{1-fache Nullstelle}$$

$$x_4 = 1,925; \quad \text{1-fache Nullstelle}$$

6 Funktionen höheren Grades

6.1 Aufgaben

(1) $-2x^5 = 0$

(2) $-\frac{1}{4}x^5 + \frac{2}{3}x^4 = 0$

(3) $x^5 - 3x^4 = 0$

(4) $x^5 - 10x^3 + 9x = 0$

(5) $\frac{1}{2}x^5 + 2x^2 = 0$

(6) $-\frac{1}{6}x^5 + 2x^3 = 0$

(7) $\frac{1}{2}x^5 - 3x^4 + 5x^3 = 0$

(8) $-x^5 + 3x^3 + 2x^2 = 0$

(9) $-x^5 + 3x^4 - 4x^2 = 0$

(10) $4x^5 + 5x^4 - 6x^3 = 0$

(11) $4x^5 + 5x^4 - 6x^3 + 1 = 0$

(12) $x^6 - 3x^4 + 3x^2 - 1 = 0$

(13) $x^6 - 12x^4 + 48x^2 - 64 = 0$

6.2 Lösungen

Aufgabe (1)

$$\begin{aligned} -2x^5 &= 0 \\ x^5 &= 0 \Rightarrow x = 0 \\ x_1 &= 0; \quad \underline{\text{5-fache Nullstelle}} \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} -\frac{1}{4}x^5 + \frac{2}{3}x^4 &= 0 \\ x^4\left(-\frac{1}{4}x + \frac{2}{3}\right) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\ -\frac{1}{4}x + \frac{2}{3} &= 0 \quad / -\frac{2}{3} \\ -\frac{1}{4}x &= -\frac{2}{3} \quad / : \left(-\frac{1}{4}\right) \\ x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\ x &= 2\frac{2}{3} \\ x_1 &= 0; \quad \underline{\text{4-fache Nullstelle}} \\ x_2 &= 2\frac{2}{3}; \quad \underline{\text{1-fache Nullstelle}} \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} x^5 - 3x^4 &= 0 \\ x^4(x - 3) &= 0 \Rightarrow x = 0 \quad \wedge \quad x - 3 = 0 \\ x - 3 &= 0 \quad / + 3 \\ x &= 3 \\ x_1 &= 0; \quad \underline{\text{4-fache Nullstelle}} \\ x_2 &= 3; \quad \underline{\text{1-fache Nullstelle}} \end{aligned}$$

Aufgabe (4)

$$\begin{aligned} x^5 - 10x^3 + 9x &= 0 \\ x(x^4 - 10x^2 + 9) &= 0 \Rightarrow x = 0 \quad \wedge \quad x^4 - 10x^2 + 9 = 0 \\ u = x^2 \quad u^2 &= x^4 \\ 1u^2 - 10u + 9 &= 0 \\ u_{1/2} &= \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1} \\ u_{1/2} &= \frac{+10 \pm \sqrt{64}}{2} \\ u_{1/2} &= \frac{10 \pm 8}{2} \\ u_1 &= \frac{10 + 8}{2} \quad u_2 = \frac{10 - 8}{2} \\ u_1 &= 9 \quad u_2 = 1 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} \\ x_1 &= 3 \quad x_2 = -3 \\ x^2 &= 1 \\ x &= \pm\sqrt{1} \\ x_1 &= 1 \quad x_2 = -1 \\ x_1 &= -3; \quad \underline{\text{1-fache Nullstelle}} \\ x_2 &= -1; \quad \underline{\text{1-fache Nullstelle}} \\ x_3 &= 0; \quad \underline{\text{1-fache Nullstelle}} \\ x_4 &= 1; \quad \underline{\text{1-fache Nullstelle}} \\ x_5 &= 3; \quad \underline{\text{1-fache Nullstelle}} \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} \frac{1}{2}x^5 + 2x^2 &= 0 \\ x^2(\frac{1}{2}x^3 + 2) &= 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^3 + 2 = 0 \\ \frac{1}{2}x^3 + 2 &= 0 \quad / -2 \\ \frac{1}{2}x^3 &= -2 \quad / : \frac{1}{2} \end{aligned}$$

$$\begin{aligned} x^3 &= \frac{-2}{\frac{1}{2}} \\ x &= \sqrt[3]{-4} \\ x &= -1,587 \end{aligned}$$

Polynomdivision: $(-1, 587)$

$$\begin{array}{r} (\frac{1}{2}x^3 \quad +0,794x^2) \quad +2 \quad) : (x + 1,587) = \frac{1}{2}x^2 - 0,794x + 1,26 \\ -(\frac{1}{2}x^3 \quad +0,794x^2) \\ \hline \quad -0,794x^2 \quad \quad +2 \\ \quad -(-0,794x^2 \quad -1,26x) \quad +2 \\ \hline \quad \quad 1,26x \quad +2 \\ \quad \quad -(1,26x \quad +2) \\ \hline \quad \quad \quad 0 \end{array}$$

$$\begin{aligned} \frac{1}{2}x^2 - 0,794x + 1,26 &= 0 \\ x_{1/2} &= \frac{+0,794 \pm \sqrt{(-0,794)^2 - 4 \cdot \frac{1}{2} \cdot 1,26}}{2 \cdot \frac{1}{2}} \end{aligned}$$

$$\begin{aligned} x_{1/2} &= \frac{+0,794 \pm \sqrt{-1,89}}{1} \\ \text{Diskriminante negativ keine Loesung} \\ x_1 &= -1,587; \quad \underline{1\text{-fache Nullstelle}} \\ x_2 &= 0; \quad \underline{2\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} -\frac{1}{6}x^5 + 2x^3 &= 0 \\ x^3(-\frac{1}{6}x^2 + 2) &= 0 \Rightarrow x = 0 \quad \wedge \quad -\frac{1}{6}x^2 + 2 = 0 \\ -\frac{1}{6}x^2 + 2 &= 0 \quad / -2 \\ -\frac{1}{6}x^2 &= -2 \quad / : (-\frac{1}{6}) \end{aligned}$$

$$\begin{aligned} x^2 &= \frac{-2}{-\frac{1}{6}} \\ x &= \pm\sqrt{12} \\ x_1 &= 3,464 \quad x_2 = -3,464 \\ x_1 &= -3,464; \quad \underline{1\text{-fache Nullstelle}} \\ x_2 &= 0; \quad \underline{3\text{-fache Nullstelle}} \\ x_3 &= 3,464; \quad \underline{1\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} \frac{1}{2}x^5 - 3x^4 + 5x^3 &= 0 \\ x^3(\frac{1}{2}x^2 - 3x + 5) &= 0 \Rightarrow x = 0 \quad \wedge \quad \frac{1}{2}x^2 - 3x + 5 = 0 \\ \frac{1}{2}x^2 - 3x + 5 &= 0 \end{aligned}$$

$$\begin{aligned} x_{1/2} &= \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}} \\ x_{1/2} &= \frac{+3 \pm \sqrt{-1}}{1} \end{aligned}$$

$$\begin{aligned} \text{Diskriminante negativ keine Loesung} \\ x_1 &= 0; \quad \underline{3\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (8)

$$\begin{aligned}
 -x^5 + 3x^3 + 2x^2 &= 0 \\
 x^2(-x^3 + 3x + 2) &= 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x + 2 = 0 \\
 -x^3 + 3x + 2 &= 0
 \end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x \quad +2) : (x+1) = -x^2 + x + 2 \\
 \underline{-(-x^3 \quad -x^2)} \\
 x^2 \quad +3x \quad +2 \\
 \underline{-(x^2 \quad +x)} \\
 2x \quad +2 \\
 \underline{-(2x \quad +2)} \\
 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + x + 2 &= 0 \\
 x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
 x_{1/2} &= \frac{-1 \pm 3}{-2} \\
 x_1 &= \frac{-1 + 3}{-2} \quad x_2 = \frac{-1 - 3}{-2} \\
 x_1 &= -1 \quad x_2 = 2 \\
 x_1 &= -1; \quad \underline{2\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{2\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{1\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (9)

$$\begin{aligned}
 -x^5 + 3x^4 - 4x^2 &= 0 \\
 x^2(-x^3 + 3x^2 - 4) &= 0 \Rightarrow x = 0 \quad \wedge \quad -x^3 + 3x^2 - 4 = 0 \\
 -x^3 + 3x^2 - 4 &= 0
 \end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-x^3 \quad +3x^2 \quad -4) : (x+1) = -x^2 + 4x - 4 \\
 \underline{-(-x^3 \quad -x^2)} \\
 4x^2 \quad -4 \\
 \underline{-(4x^2 \quad +4x)} \\
 -4x \quad -4 \\
 \underline{-(-4x \quad -4)} \\
 0
 \end{array}$$

$$\begin{aligned}
 -x^2 + 4x - 4 &= 0 \\
 x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)} \\
 x_{1/2} &= \frac{-4 \pm \sqrt{0}}{-2} \\
 x_{1/2} &= \frac{-4 \pm 0}{-2} \\
 x_1 &= \frac{-4 + 0}{-2} \quad x_2 = \frac{-4 - 0}{-2} \\
 x_1 &= 2 \quad x_2 = 2 \\
 x_1 &= -1; \quad \underline{1\text{-fache Nullstelle}} \\
 x_2 &= 0; \quad \underline{2\text{-fache Nullstelle}} \\
 x_3 &= 2; \quad \underline{2\text{-fache Nullstelle}}
 \end{aligned}$$

Aufgabe (10)

$$\begin{aligned}
 4x^5 + 5x^4 - 6x^3 &= 0 \\
 x^3(4x^2 + 5x - 6) &= 0 \Rightarrow x = 0 \quad \wedge \quad 4x^2 + 5x - 6 = 0
 \end{aligned}$$

$$4x^2 + 5x - 6 = 0$$

$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$

$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$

$$x_{1/2} = \frac{-5 \pm 11}{8}$$

$$x_1 = \frac{-5 + 11}{8} \quad x_2 = \frac{-5 - 11}{8}$$

$$x_1 = \frac{3}{4} \quad x_2 = -2$$

$$x_1 = -2; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{3\text{-fache Nullstelle}}$$

$$x_3 = \frac{3}{4}; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (11)

$$4x^5 + 5x^4 - 6x^3 + 1 = 0$$

NumerischeSuche :

$$\underline{x_1 = -2,011; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (12)

$$x^6 - 3x^4 + 3x^2 - 1 = 0$$

NumerischeSuche :

$$\underline{x_1 = -1; \quad 3\text{-fache Nullstelle}}$$

$$\underline{x_2 = 1; \quad 3\text{-fache Nullstelle}}$$

Aufgabe (13)

$$x^6 - 12x^4 + 48x^2 - 64 = 0$$

NumerischeSuche :

$$\underline{x_1 = -2; \quad 3\text{-fache Nullstelle}}$$

$$\underline{x_2 = 2; \quad 3\text{-fache Nullstelle}}$$