Geometrie Dreiecke
Aufgaben und Lösungen

http://www.fersch.de

21. September 2010

Inhaltsverzeichnis

1 Dreiecke

1.1 Aufgaben

Gegeben:
Seite-Seite-Seite:$a-b-c$
Seite-Winkel-Seite:$a-b-\gamma, a-c-\beta, b-c-\alpha$
Seite-Seite-Winkel:$a-b-\alpha, a-b-\beta, a-c-\alpha, a-c-\gamma, b-c-\beta, b-c-\gamma$
Winkel-Winkel-Seite:$c-\beta-\gamma, a-\alpha-\beta, a-\alpha-\gamma, a-\beta-\gamma, b-\alpha-\beta, b-\alpha-\gamma, b-\beta-\gamma, c-\alpha-\beta, c-\alpha-\gamma$
Gesucht:Alle Seiten und Winkel
Die Zeichnung ist nur bei der PDF-Ausgabe möglich.
Um eigene Aufgaben zu lösen, klicken Sie hier: Neue Rechnung

(1) $a=4$ $b=4$ $c=4$
(2) $a=3$ $b=4$ $c=5$
(3) $a=3$ $b=5$ $c=4$
(4) $a=5$ $b=4$ $c=3$
(5) $a=4$ $b=3$ $\alpha=90$
(6) $a=8$ $c=5$ $\alpha=90$
(7) $b=3$ $c=5$ $\alpha=90$
(8) $a=3$ $b=4$ $\beta=90$
(9) $a=3$ $c=5$ $\beta=90$
(10) $b=8$ $c=5$ $\beta=90$
(11) $a=3$ $b=4$ $\gamma=90$
(12) $a=3$ $c=5$ $\gamma=90$
(13) $b=3$ $c=5$ $\gamma=90$
(14) $a=4$ $\alpha=90$ $\beta=70$
(15) $b=5$ $\alpha=90$ $\beta=30$
(16) $c=5$ $\alpha=90$ $\gamma=40$
(17) $a=3$ $\alpha=20$ $\beta=90$
(18) $c=5$ $\alpha=30$ $\beta=90$
(19) $b=8$ $\beta=90$ $\gamma=45$
(20) $a=3$ $\alpha=20$ $\gamma=90$
(21) $c=5$ $\alpha=35$ $\gamma=90$
(22) $b=3$ $\beta=65$ $\gamma=90$
(23) $a=6$ $\alpha=90$ $\beta=30$
(24) $a=5$ $\alpha=90$ $\gamma=30$
(25) $b=3$ $c=5$ $\alpha=90$
(26) $a=3$ $b=4$ $\beta=90$
(27) $a=3$ $c=5$ $\beta=90$
(28) $a=8$ $b=4$ $c=5$
(29) $a=3$ $b=7$ $c=4$
(30) $a=7$ $b=4$ $c=5$
(31) $a=6$ $b=2$ $c=5$
(32) $a=6$ $b=5$ $\gamma=25$
(33) $b=5$ $c=10$ $\alpha=155$
(34) $b=7$ $c=5$ $\alpha=30$
(35) $a=6$ $c=5$ $\beta=40$
(36) $a=6$ $b=5$ $\gamma=120$
(37) $a=6$ $b=5$ $\alpha=50$
(38) $a=6$ $b=7$ $\beta=60$
(39) $a=6$ $c=3\frac{1}{2}$ $\alpha=50$
(40) $a=2\frac{1}{2}$ $c=4\frac{1}{2}$ $\beta=60$
(41) $b=4$ $c=3\frac{1}{2}$ $\beta=40$
(42) $b=3\frac{1}{2}$ $c=4\frac{1}{2}$ $\gamma=70$
(43) $a=6$ $\alpha=30$ $\beta=50$
(44) $a=6$ $\alpha=30$ $\gamma=50$
(45) $b=7$ $\alpha=30$ $\beta=50$
(46) $b=7$ $\beta=50$ $\gamma=80$
(47) $c=7$ $\alpha=30$ $\gamma=70$
(48) $c=6$ $\beta=50$ $\gamma=40$

1.2 Lösungen

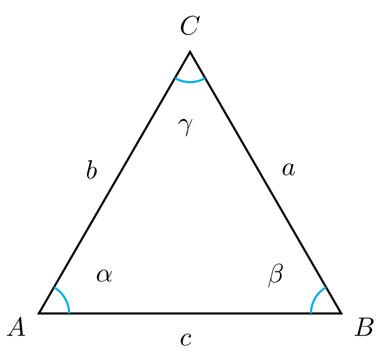
Aufgabe (1)

$Seite - Seite - Seite$
$a = 4 \quad b = 4 \quad c = 4$
Gleichseitiges Dreieck
$\alpha = 60° \quad \beta = 60° \quad \gamma = 60°$

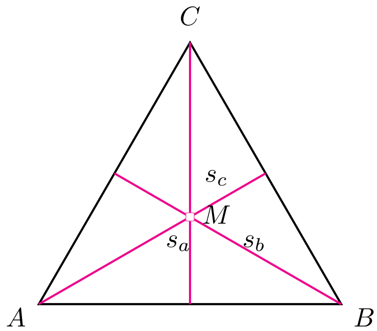
Höhe h_c
$h_c = \frac{1}{2} \cdot a \cdot \sqrt{3}$
$h_c = \frac{1}{2} \cdot 4 \cdot \sqrt{3}$
$h_c = 3,464$
$h_a = h_b = h_c = 3,464$
$s_a = s_b = s_c = 3,464$
$wha = whb = whc = 3,464$

Fläche A
A= $\frac{1}{4} \cdot a^2 \cdot \sqrt{3}$
$A = \frac{1}{3} \cdot 4^2 \cdot \sqrt{3}$
$A = 6,928$

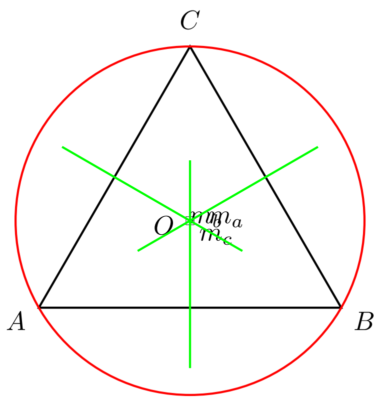
Umfang:$U = a + b + c$
$U = 4 + 4 + 4$
$U = 12$
Umkreisradius:$2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 60°}$
$r_u = 2,309$
Innkreisradius:$r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,928}{12}$
$r_i = 1,155$



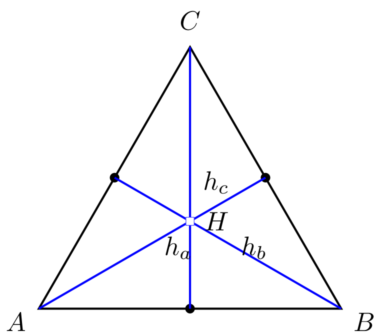
Seitenhalbierende-Schwerpunkt



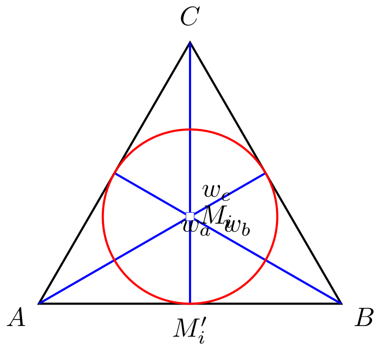
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_c
M_i
w_a
w_b
A
B
M_i'

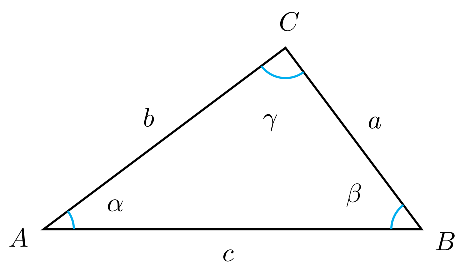
Aufgabe (2)

$Seite - Seite - Seite$
$a = 3 \quad b = 4 \quad c = 5$

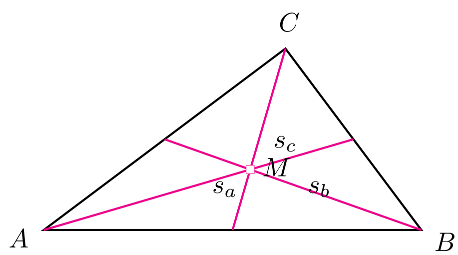
Pythagoras $c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$ Rechtwinkeliges Dreieck
Kathete $a = 3$ $b = 4$ Hypothenuse $c = 5$ $\gamma = 90°$

Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,87$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,87° - 90°$
$\beta = 53,13°$
Umfang: $U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,13°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,87°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

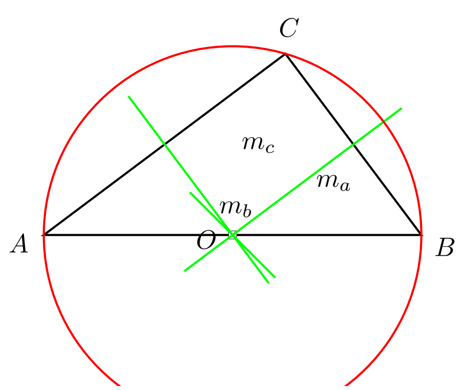
$wha = \frac{5 \cdot \sin 53,13}{\sin 108,435}$
$wha = 4,216$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,435}$
$whb = 3,354$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,87}{\sin 108,435}$
$whc = 1,897$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,272$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,606$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,915$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,87^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



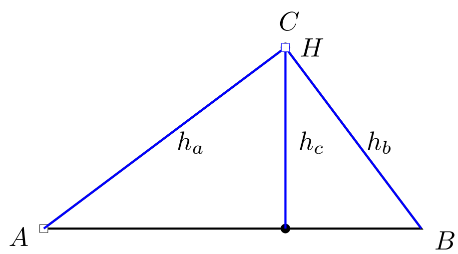
Seitenhalbierende-Schwerpunkt



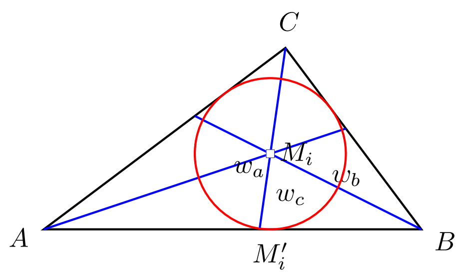
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



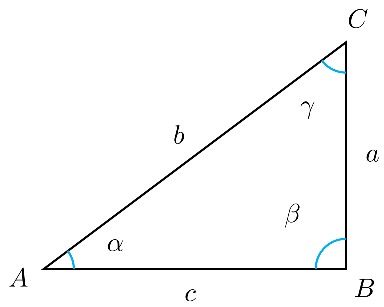
Aufgabe (3)

$Seite - Seite - Seite$
$a = 3 \quad b = 5 \quad c = 4$

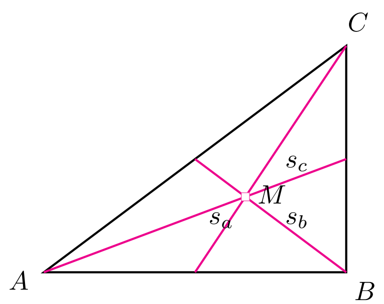
Pythagoras $b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 4^2}$
$b = 5$ Rechtwinkeliges Dreieck
Kathete $a = 3$ Hypothenuse $b = 5$ Kathete $c = 4$ $\beta = 90°$

Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,87$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 36,87° - 90°$
$\gamma = 53,13°$
Umfang:$U = a + b + c$
$U = 3 + 5 + 4$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4 \cdot \sin 90°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 53,13°$
$h_b = 2\frac{2}{5}$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 36,87°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

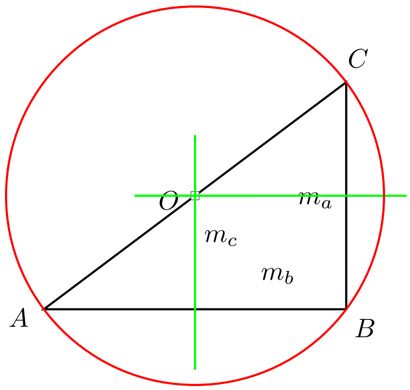
$wha = \frac{4 \cdot \sin 90}{\sin 71,565}$
$wha = 4,216$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 53,13}{\sin 81,87}$
$whb = 2,424$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 36,87}{\sin 71,565}$
$whc = 1,897$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 4^2) - 3^2}$
$s_a = 4,272$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_b = 2\frac{1}{2}$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_c = 3,279$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,87^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



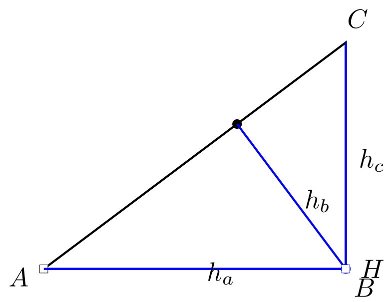
Seitenhalbierende-Schwerpunkt



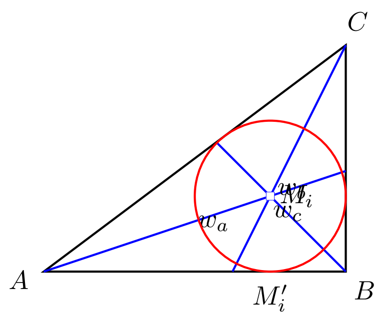
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



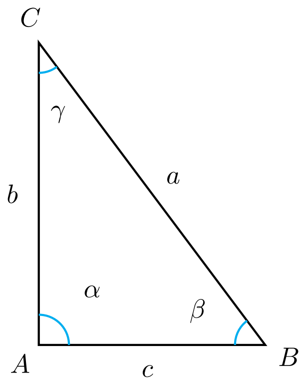
Aufgabe (4)

$Seite - Seite - Seite$
$a = 5 \quad b = 4 \quad c = 3$

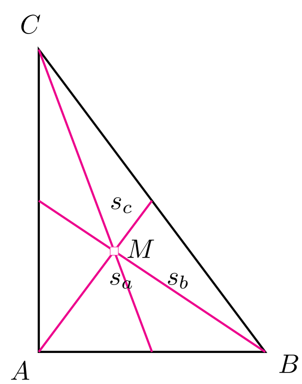
Pythagoras $a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{4^2 + 3^2}$
$a = 5$ Rechtwinkeliges Dreieck
Hypothenuse $a = 5$ Kathete $b = 4$ Kathete $c = 3$ $\alpha = 90°$

Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{4}{5}$
$\beta = 53,13$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 53,13°$
$\gamma = 36,87°$
Umfang:$U = a + b + c$
$U = 5 + 4 + 3$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3 \cdot \sin 53,13°$
$h_a = 2\frac{2}{5}$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 2\frac{2}{5}$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 36,87°$
$h_b = 3$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 90°$
$h_c = 4$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

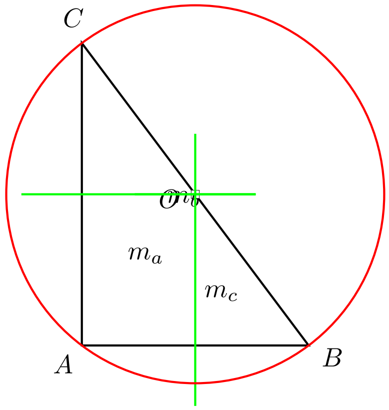
$wha = \frac{3 \cdot \sin 53,13}{\sin 81,87}$
$wha = 2,424$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 36,87}{\sin 116,565}$
$whb = 3,354$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 90}{\sin 81,87}$
$whc = 5,051$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 5^2}$
$s_a = 2\frac{1}{2}$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2 + 3^2) - 4^2}$
$s_b = 3,606$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2 + 4^2) - 3^2}$
$s_c = 4,062$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 90^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



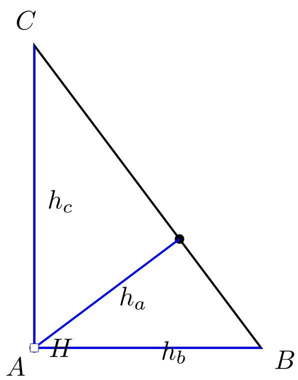
Seitenhalbierende-Schwerpunkt



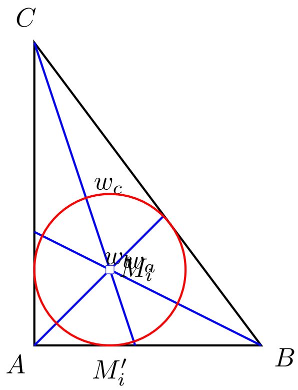
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

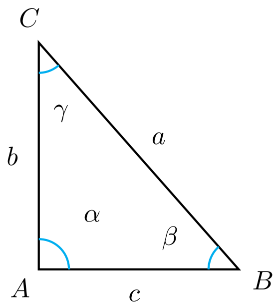

C
w_c
w_b w_a
M_i
A
M_i'
B

Aufgabe (5)

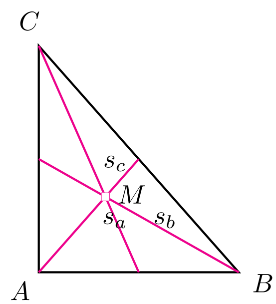
$Seite - Seite - Winkel$
$a = 4 \quad b = 3 \quad \alpha = 90°$

Pythagoras$a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,646$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{4}$
$\beta = 48,59$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 48,59°$
$\gamma = 41,41°$
Umfang:$U = a + b + c$
$U = 4 + 3 + 2,646$
$U = 9,646$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,646 \cdot \sin 48,59°$
$h_a = 1,984$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 1,984$
$A = 3,969$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 41,41°$
$h_b = 2,646$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,646 \cdot \sin 48,59}{\sin 86,41}$

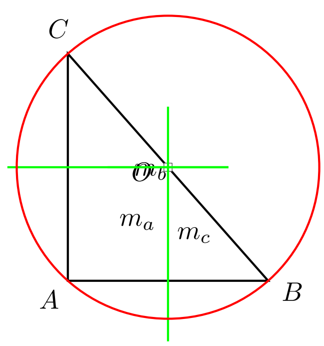
$wha = 1,988$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 41,41}{\sin 114,295}$
$whb = 2,903$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 86,41}$
$whc = 4,008$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 2,646^2) - 4^2}$
$s_a = 2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 2,646^2) - 3^2}$
$s_b = 3,041$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3^2) - 2,646^2}$
$s_c = 3,202$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 90^\circ}$
$r_u = 2$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,969}{9,646}$
$r_i = 0,823$



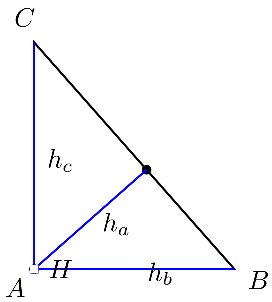
Seitenhalbierende-Schwerpunkt



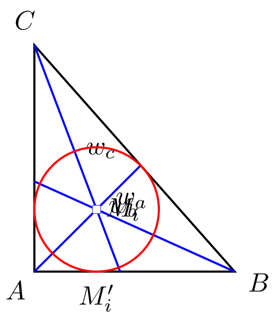
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

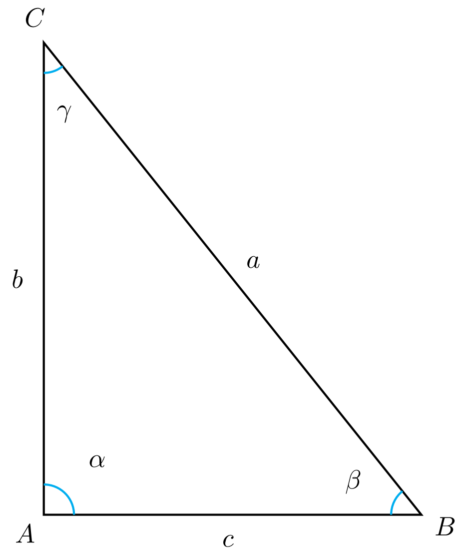


Aufgabe (6)

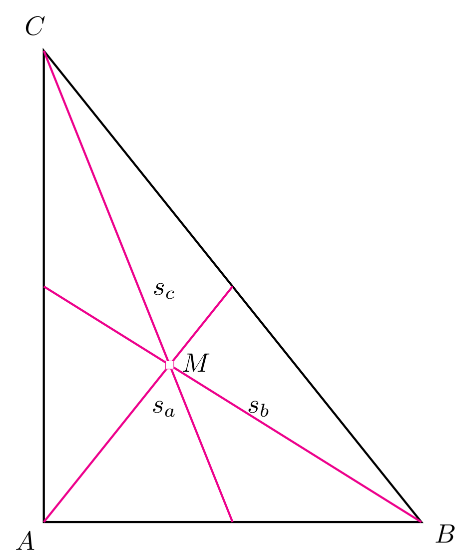
$Seite - Seite - Winkel$
$a = 8 \quad c = 5 \quad \alpha = 90°$

Pythagoras$a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{8^2 - 5^2}$
$b = 6,245$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{6,245}{8}$
$\beta = 51,318$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 51,318°$
$\gamma = 38,682°$
Umfang:$U = a + b + c$
$U = 8 + 6,245 + 5$
$U = 19,245$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 51,318°$
$h_a = 3,903$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 8 \cdot 3,903$
$A = 15,612$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 8 \cdot \sin 38,682°$
$h_b = 5$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 6,245 \cdot \sin 90°$
$h_c = 6,245$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 51,318}{\sin 83,682}$

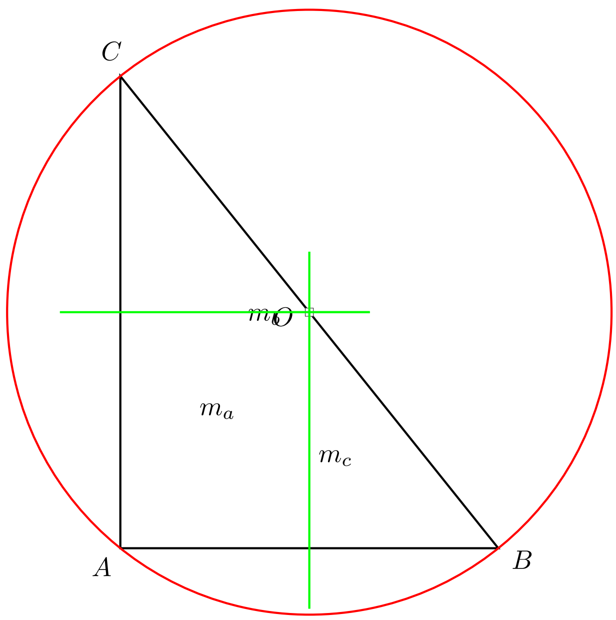
$wha = 3,927$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{8 \cdot \sin 38,682}{\sin 115,659}$
$whb = 5,547$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{6,245 \cdot \sin 90}{\sin 83,682}$
$whc = 8,049$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(6,245^2 + 5^2) - 8^2}$
$s_a = 4$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 6,245^2}$
$s_b = 5,895$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(8^2 + 6,245^2) - 5^2}$
$s_c = 6,461$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{8}{2 \cdot \sin 90^\circ}$
$r_u = 4$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15,612}{19,245}$
$r_i = 1,622$



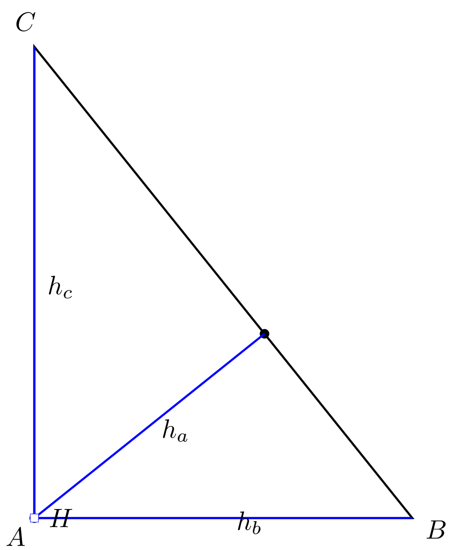
Seitenhalbierende-Schwerpunkt



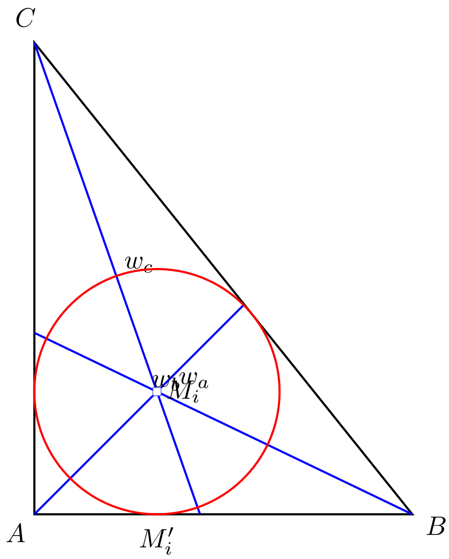
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

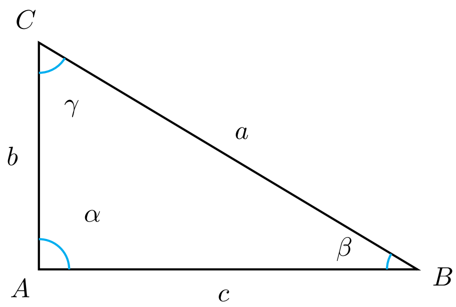

C
w_c
w_b w_a
M_i
A
M_i'
B

Aufgabe (7)

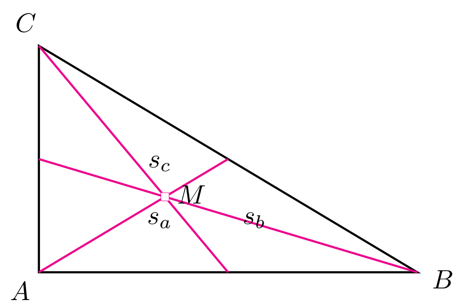
$Seite - Winkel - Seite$
$b = 3 \quad c = 5 \quad \alpha = 90°$

Pythagoras$a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{3^2 + 5^2}$
$a = 5,831$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{5,831}$
$\beta = 30,964$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30,964°$
$\gamma = 59,036°$
Umfang:$U = a + b + c$
$U = 5,831 + 3 + 5$
$U = 13,831$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 30,964°$
$h_a = 2,572$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,831 \cdot 2,572$
$A = 7\frac{1}{2}$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,831 \cdot \sin 59,036°$
$h_b = 5$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 30,964}{\sin 104,036}$

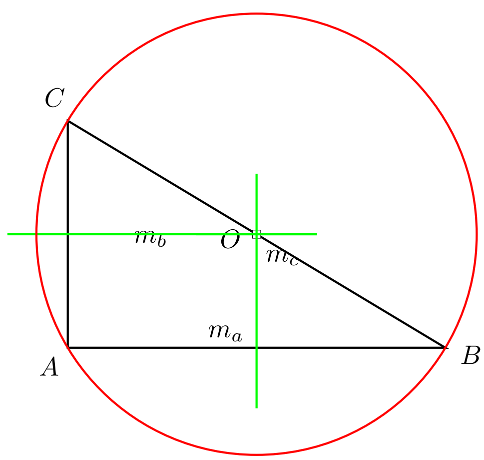
$wha = 2,652$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,831 \cdot \sin 59,036}{\sin 105,482}$
$whb = 5,188$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 104,036}$
$whc = 6,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,831^2}$
$s_a = 2,915$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,831^2 + 5^2) - 3^2}$
$s_b = 5,22$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,831^2 + 3^2) - 5^2}$
$s_c = 4,387$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,831}{2 \cdot \sin 90^\circ}$
$r_u = 2,915$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,831}$
$r_i = 1,085$



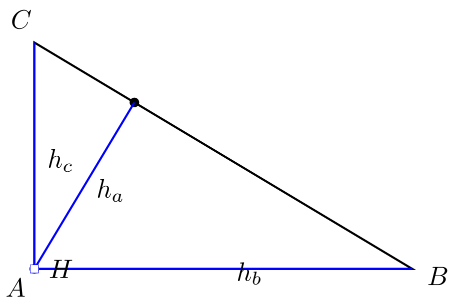
Seitenhalbierende-Schwerpunkt



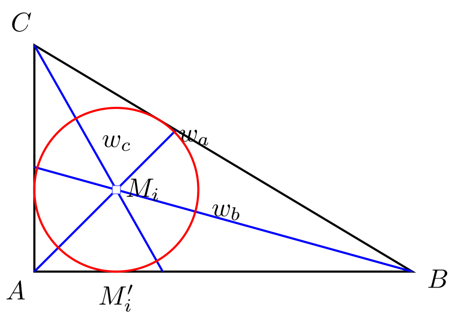
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

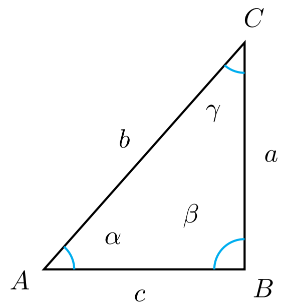


Aufgabe (8)

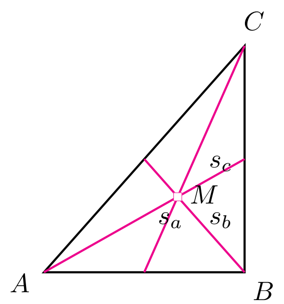
$Seite - Seite - Winkel$
$a = 3 \quad b = 4 \quad \beta = 90°$

Pythagoras$b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,646$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{4}$
$\alpha = 48,59$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 48,59° - 90°$
$\gamma = 41,41°$
Umfang:$U = a + b + c$
$U = 3 + 4 + 2,646$
$U = 9,646$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,646 \cdot \sin 90°$
$h_a = 2,646$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 2,646$
$A = 3,969$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 41,41°$
$h_b = 1,984$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 48,59°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,646 \cdot \sin 90}{\sin 65,705}$

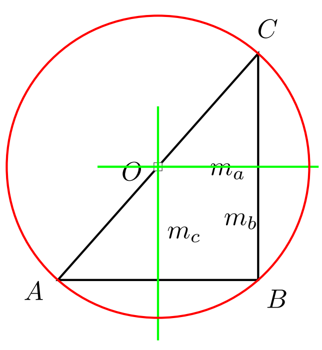
$wha = 2,903$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 41,41}{\sin 93,59}$
$whb = 1,988$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 48,59}{\sin 65,705}$
$whc = 2,469$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 2,646^2) - 3^2}$
$s_a = 3,041$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 2,646^2) - 4^2}$
$s_b = 2$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2,646^2}$
$s_c = 2,915$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 48,59^\circ}$
$r_u = 2$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,969}{9,646}$
$r_i = 0,823$



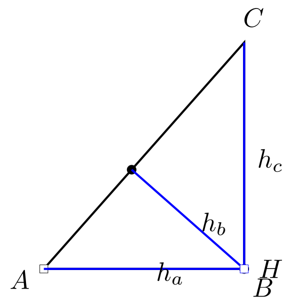
Seitenhalbierende-Schwerpunkt



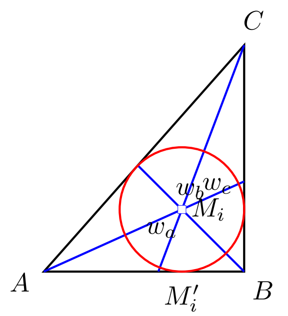
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

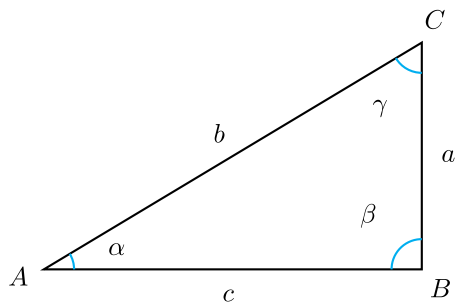


Aufgabe (9)

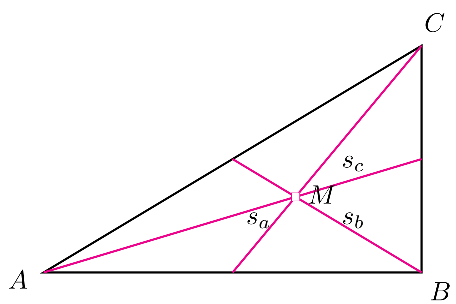
$Seite - Winkel - Seite$
$a = 3 \quad c = 5 \quad \beta = 90°$

Pythagoras$b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 5^2}$
$b = 5,831$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5,831}$
$\alpha = 30,964$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30,964° - 90°$
$\gamma = 59,036°$
Umfang:$U = a + b + c$
$U = 3 + 5,831 + 5$
$U = 13,831$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 5$
$A = 7\frac{1}{2}$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 59,036°$
$h_b = 2,572$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,831 \cdot \sin 30,964°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 74,518}$

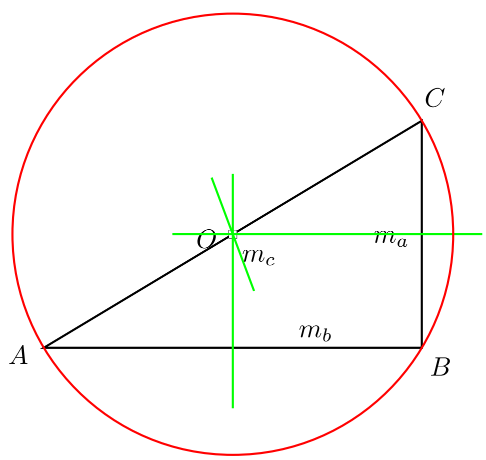
$wha = 5,188$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 59,036}{\sin 75,964}$
$whb = 2,652$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,831 \cdot \sin 30,964}{\sin 74,518}$
$whc = 1,602$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,831^2 + 5^2) - 3^2}$
$s_a = 5,22$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,831^2}$
$s_b = 2,915$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 5,831^2) - 5^2}$
$s_c = 3,606$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 30,964^\circ}$
$r_u = 2,915$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,831}$
$r_i = 1,085$



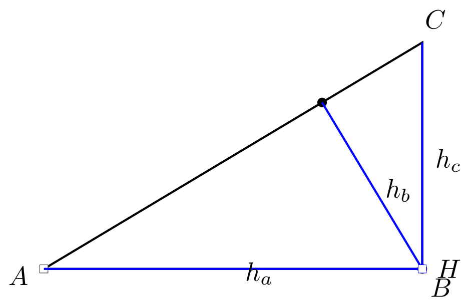
Seitenhalbierende-Schwerpunkt



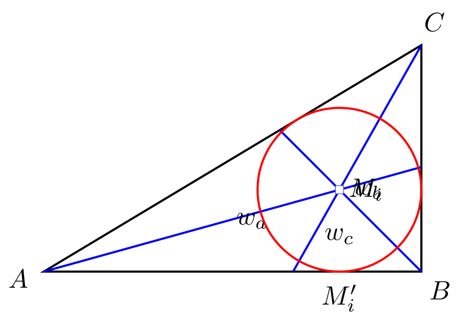
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

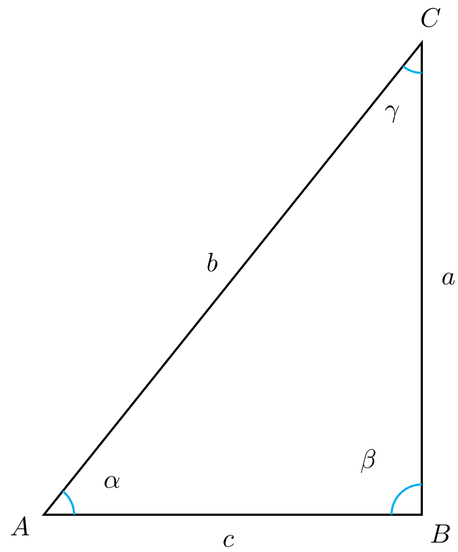


Aufgabe (10)

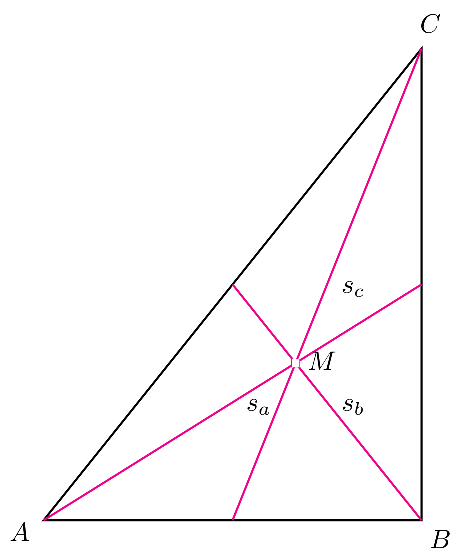
$Seite - Seite - Winkel$
$b = 8 \quad c = 5 \quad \beta = 90°$

Pythagoras$b^2 = a^2 + c^2 \quad / - c^2$
$a^2 = b^2 - c^2$
$a = \sqrt{b^2 - c^2}$
$a = \sqrt{8^2 - 5^2}$
$a = 6,245$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{6,245}{8}$
$\alpha = 51,318$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 51,318° - 90°$
$\gamma = 38,682°$
Umfang:$U = a + b + c$
$U = 6,245 + 8 + 5$
$U = 19,245$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6,245 \cdot 5$
$A = 15,612$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6,245 \cdot \sin 38,682°$
$h_b = 3,903$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8 \cdot \sin 51,318°$
$h_c = 6,245$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 64,341}$

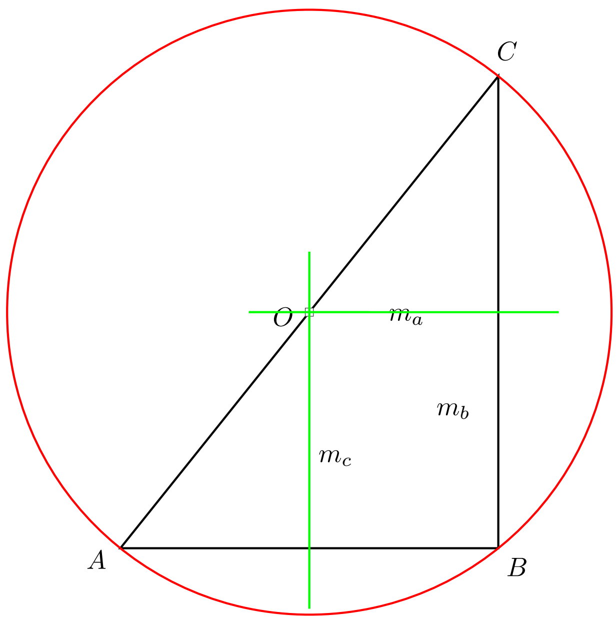
$wha = 5,547$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6,245 \cdot \sin 38,682}{\sin 96,318}$
$whb = 3,927$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8 \cdot \sin 51,318}{\sin 64,341}$
$whc = 5,408$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 6,245^2}$
$s_a = 5,895$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6,245^2 + 5^2) - 8^2}$
$s_b = 4$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6,245^2 + 8^2) - 5^2}$
$s_c = 5,958$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6,245}{2 \cdot \sin 51,318^\circ}$
$r_u = 4$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15,612}{19,245}$
$r_i = 1,622$



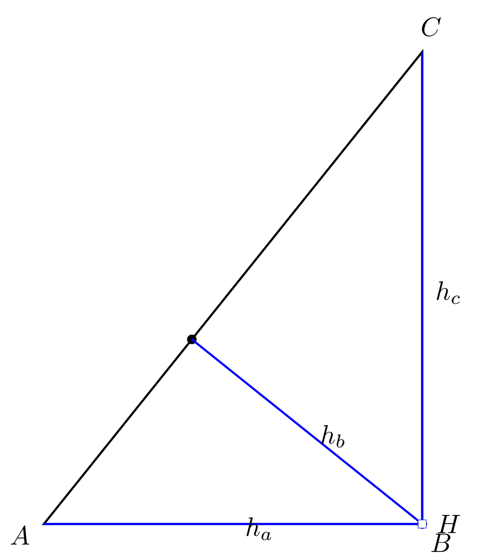
Seitenhalbierende-Schwerpunkt



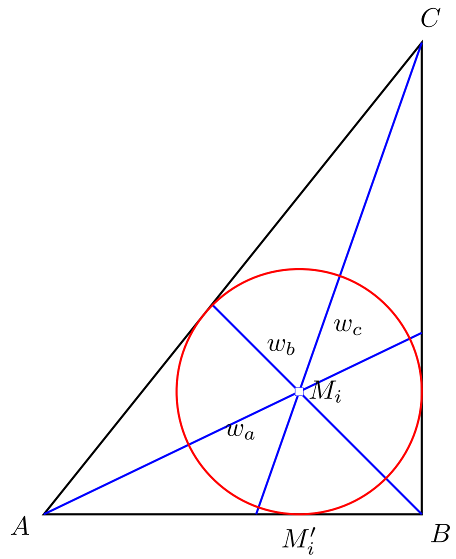
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

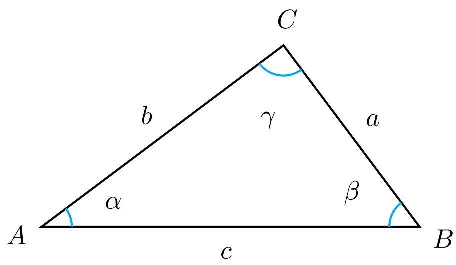

C
w_b
w_c
M_i
w_a
A
M_i'
B

Aufgabe (11)

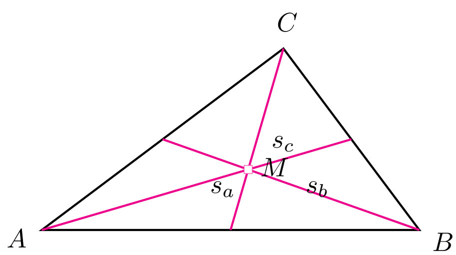
$Seite - Winkel - Seite$
$a = 3 \quad b = 4 \quad \gamma = 90°$

Pythagoras$c^2 = a^2 + b^2$
$c = \sqrt{a^2 + b^2}$
$c = \sqrt{3^2 + 4^2}$
$c = 5$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,87$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,87° - 90°$
$\beta = 53,13°$
Umfang:$U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,13°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,87°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 53,13}{\sin 108,435}$

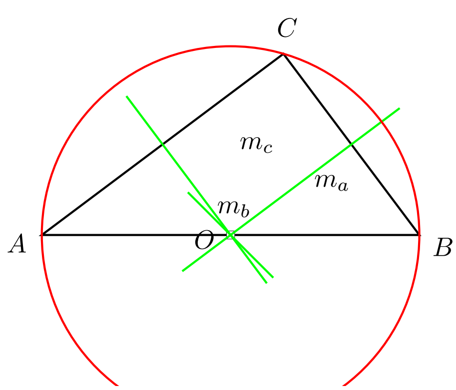
$wha = 4,216$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,435}$
$whb = 3,354$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,87}{\sin 108,435}$
$whc = 1,897$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,272$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,606$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,915$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,87^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



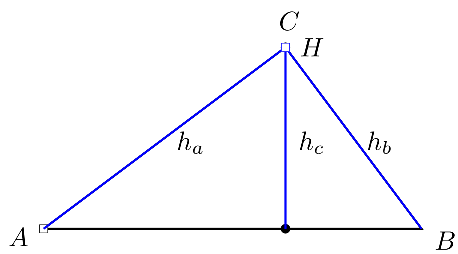
Seitenhalbierende-Schwerpunkt



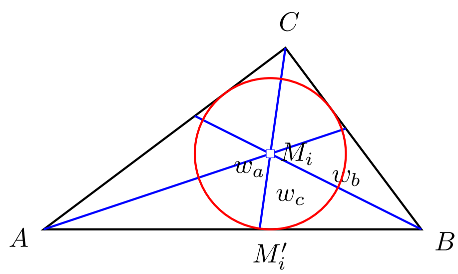
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

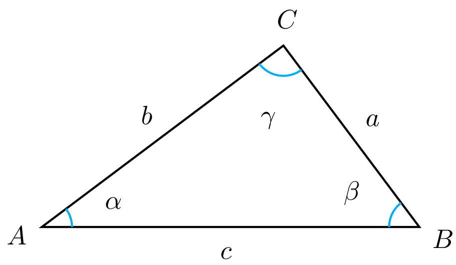


Aufgabe (12)

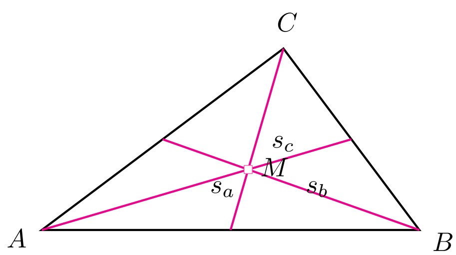
$Seite - Seite - Winkel$
$a = 3 \quad c = 5 \quad \gamma = 90°$

Pythagoras$c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{5^2 - 3^2}$
$b = 4$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{3}{5}$
$\alpha = 36,87$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 36,87° - 90°$
$\beta = 53,13°$
Umfang:$U = a + b + c$
$U = 3 + 4 + 5$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 53,13°$
$h_a = 4$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 4$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 36,87°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

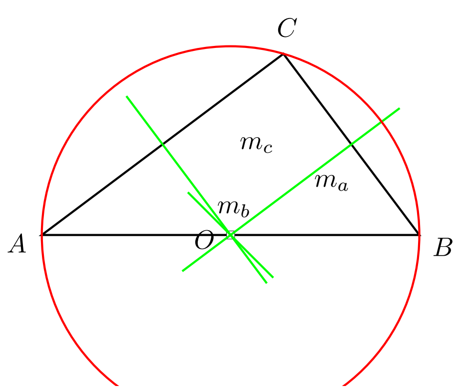
$wha = \frac{5 \cdot \sin 53,13}{\sin 108,435}$
$wha = 4,216$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 63,435}$
$whb = 3,354$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 36,87}{\sin 108,435}$
$whc = 1,897$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_a = 4,272$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_b = 3,606$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 5^2}$
$s_c = 2,915$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 36,87^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6}{12}$
$r_i = 1$



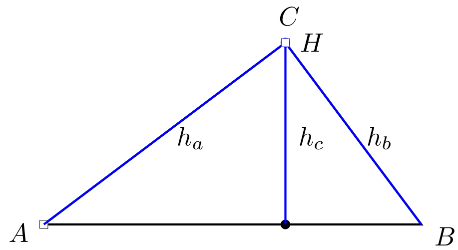
Seitenhalbierende-Schwerpunkt



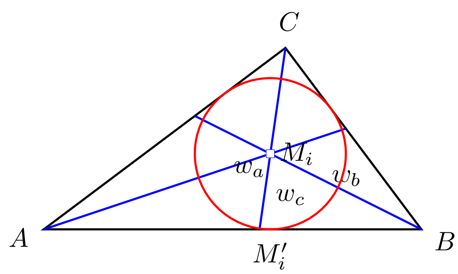
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

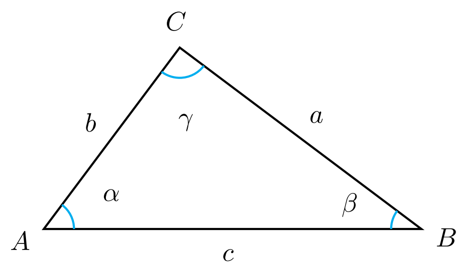


Aufgabe (13)

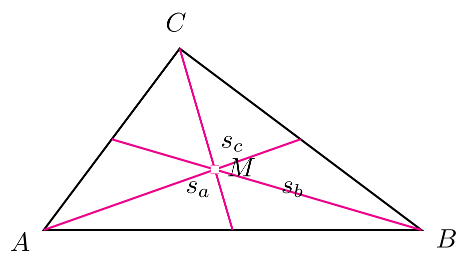
$Seite - Seite - Winkel$
$b = 3 \quad c = 5 \quad \gamma = 90°$

Pythagoras$c^2 = a^2 + b^2 \quad / - b^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2 - b^2}$
$a = \sqrt{5^2 - 3^2}$
$a = 4$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{4}{5}$
$\alpha = 53,13$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 53,13° - 90°$
$\beta = 36,87°$
Umfang:$U = a + b + c$
$U = 4 + 3 + 5$
$U = 12$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 36,87°$
$h_a = 3$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 3$
$A = 6$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 90°$
$h_b = 4$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 53,13°$
$h_c = 2\frac{2}{5}$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

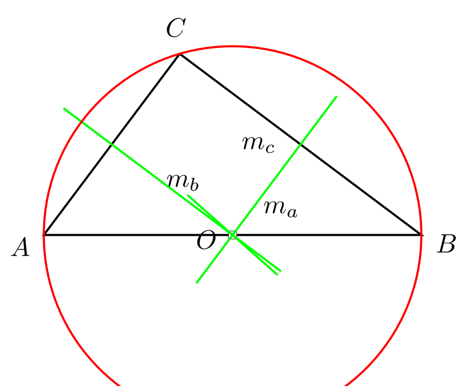
$wha = \dfrac{5 \cdot \sin 36,87}{\sin 116,565}$
$wha = 3,354$
Winkelhalbierende β
$\delta = 180 - \dfrac{\beta}{2} - \gamma$
Sinus-Satz: $\dfrac{whb}{\sin\gamma} = \dfrac{a}{\sin\delta}$
$\dfrac{whb}{\sin\gamma} = \dfrac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \dfrac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \dfrac{4 \cdot \sin 90}{\sin 71,565}$
$whb = 4,216$
Winkelhalbierende γ
$\delta = 180 - \alpha - \dfrac{\gamma}{2}$
Sinus-Satz: $\dfrac{whc}{\sin\alpha} = \dfrac{b}{\sin\delta}$
$\dfrac{whc}{\sin\alpha} = \dfrac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \dfrac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \dfrac{3 \cdot \sin 53,13}{\sin 116,565}$
$whc = 3,578$
Seitenhalbierende:
$s_a = \dfrac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \dfrac{1}{2}\sqrt{2(3^2 + 5^2) - 4^2}$
$s_a = 3,606$
Seitenhalbierende: $s_b = \dfrac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \dfrac{1}{2}\sqrt{2(4^2 + 5^2) - 3^2}$
$s_b = 4,272$
Seitenhalbierende: $s_c = \dfrac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \dfrac{1}{2}\sqrt{2(4^2 + 3^2) - 5^2}$
$s_c = 3,202$
Umkreisradius: $2 \cdot r_u = \dfrac{a}{\sin\alpha}$
$r_u = \dfrac{a}{2 \cdot \sin\alpha}$
$r_u = \dfrac{4}{2 \cdot \sin 53,13^\circ}$
$r_u = 2\dfrac{1}{2}$
Innkreisradius: $r_i = \dfrac{2 \cdot A}{U}$
$r_i = \dfrac{2 \cdot 6}{12}$
$r_i = 1$



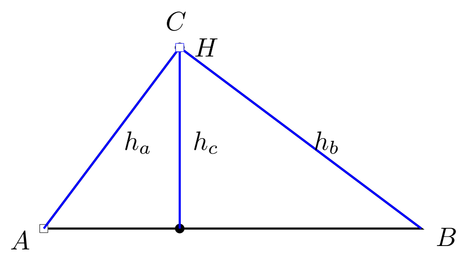
Seitenhalbierende-Schwerpunkt



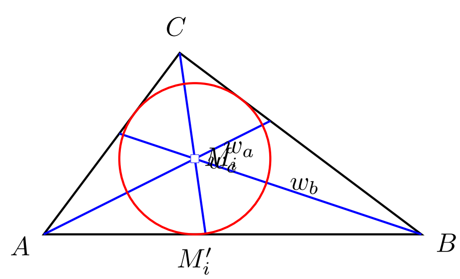
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

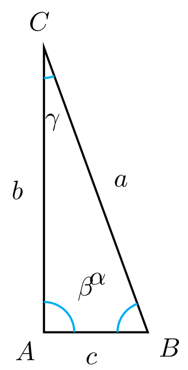


Aufgabe (14)

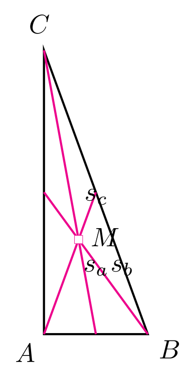
$Winkel - Winkel - Seite$
$a = 4 \quad \alpha = 90° \quad \beta = 70°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 70°$
$\gamma = 20°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$b = a \cdot \sin\beta$
$b = 4 \cdot \sin 70$
$b = 3,759$
Pythagoras$a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{4^2 - 3,759^2}$
$c = 1,368$
Umfang:$U = a + b + c$
$U = 4 + 3,759 + 1,368$
$U = 9,127$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 1,368 \cdot \sin 70°$
$h_a = 1,286$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4 \cdot 1,286$
$A = 2,571$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4 \cdot \sin 20°$
$h_b = 1,368$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3,759 \cdot \sin 90°$
$h_c = 3,759$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

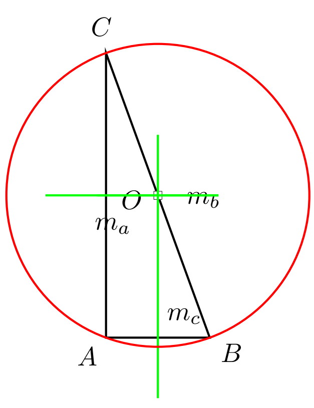
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{1,368 \cdot \sin 70}{\sin 65}$
$wha = 1,418$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4 \cdot \sin 20}{\sin 125}$
$whb = 1,67$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3,759 \cdot \sin 90}{\sin 65}$
$whc = 4,414$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,759^2 + 1,368^2) - 4^2}$
$s_a = 2$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4^2 + 1,368^2) - 3,759^2}$
$s_b = 2,325$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4^2 + 3,759^2) - 1,368^2}$
$s_c = 3,396$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4}{2 \cdot \sin 90^\circ}$
$r_u = 2$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 2,571}{9,127}$
$r_i = 0,563$



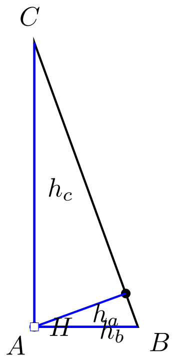
Seitenhalbierende-Schwerpunkt



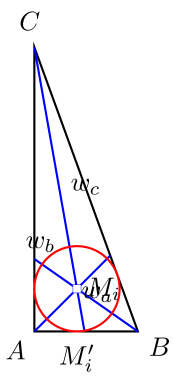
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (15)

$Winkel - Winkel - Seite$
$b = 5 \quad \alpha = 90° \quad \beta = 30°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30°$
$\gamma = 60°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$a \cdot \sin\beta = b \quad / : sin\beta$
$a = \frac{b}{\sin\beta}$
$a = \frac{5}{\sin 30}$
$a = 10$
Pythagoras$a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{10^2 - 5^2}$
$c = 8,66$
Umfang:$U = a + b + c$
$U = 10 + 5 + 8,66$
$U = 23,66$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,66 \cdot \sin 30°$
$h_a = 4,33$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 10 \cdot 4,33$
$A = 21,651$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 10 \cdot \sin 60°$
$h_b = 8,66$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 90°$
$h_c = 5$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{8,66 \cdot \sin 30}{\sin 105}$

$wha = 4,483$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{10 \cdot \sin 60}{\sin 105}$

$whb = 8,966$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{5 \cdot \sin 90}{\sin 105}$

$whc = 10,353$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(5^2 + 8,66^2) - 10^2}$

$s_a = 5$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(10^2 + 8,66^2) - 5^2}$

$s_b = 9,014$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(10^2 + 5^2) - 8,66^2}$

$s_c = 7\frac{1}{2}$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

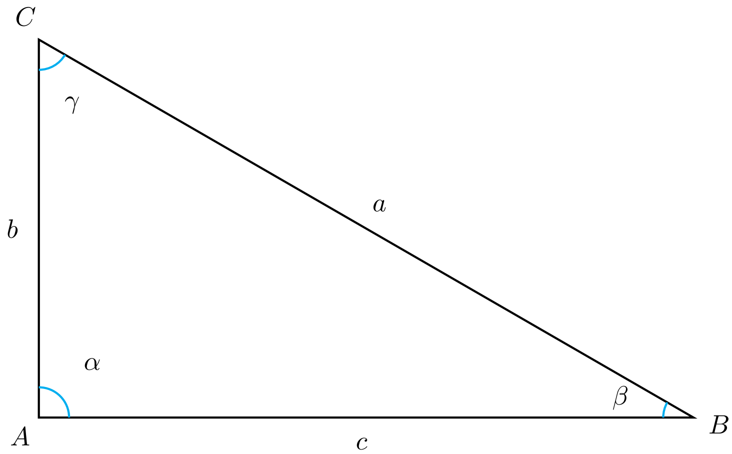
$r_u = \frac{10}{2 \cdot \sin 90^\circ}$

$r_u = 5$

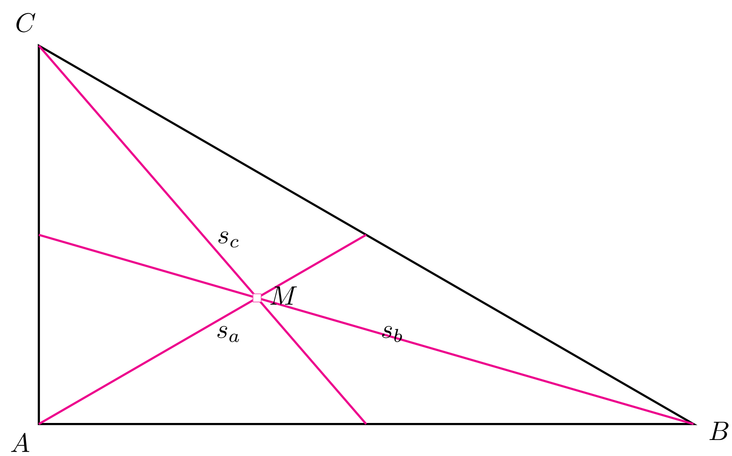
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 21,651}{23,66}$

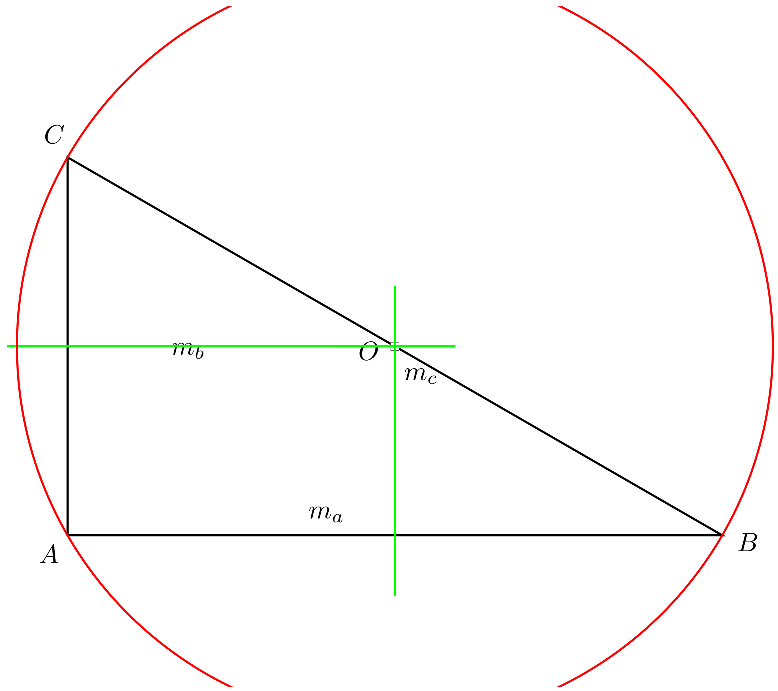
$r_i = 1,83$



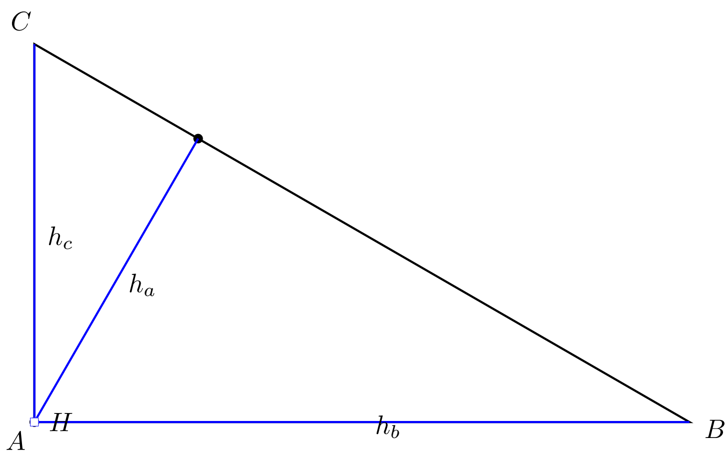
Seitenhalbierende-Schwerpunkt



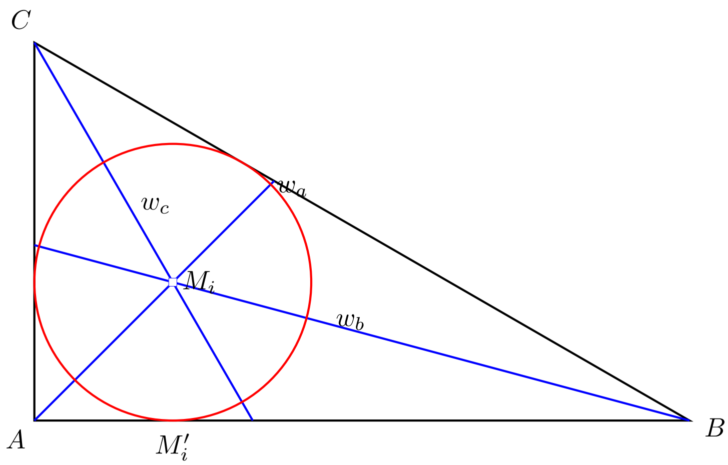
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_c
w_a
M_i
w_b
A
M_i'
B

Aufgabe (16)

$Winkel - Winkel - Seite$
$c = 5 \quad \gamma = 40° \quad \alpha = 90°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 90° - 40°$
$\beta = 50°$
Cosinus: $\cos\beta = \frac{c}{a}$
$\cos\beta = \frac{c}{a} \quad / \cdot a$
$a \cdot \cos\beta = c \quad / : cos\beta$
$a = \frac{c}{\cos\beta}$
$a = \frac{5}{\cos 50}$
$a = 7,779$
Pythagoras$a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{7,779^2 - 5^2}$
$b = 5,959$
Umfang:$U = a + b + c$
$U = 7,779 + 5,959 + 5$
$U = 18,737$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 50°$
$h_a = 3,83$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7,779 \cdot 3,83$
$A = 14,897$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7,779 \cdot \sin 40°$
$h_b = 5$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,959 \cdot \sin 90°$
$h_c = 5,959$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{5 \cdot \sin 50}{\sin 85}$

$wha = 3,845$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{7,779 \cdot \sin 40}{\sin 115}$

$whb = 5,517$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{5,959 \cdot \sin 90}{\sin 85}$

$whc = 7,808$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(5,959^2 + 5^2) - 7,779^2}$

$s_a = 3,889$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(7,779^2 + 5^2) - 5,959^2}$

$s_b = 5,82$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(7,779^2 + 5,959^2) - 5^2}$

$s_c = 6,255$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

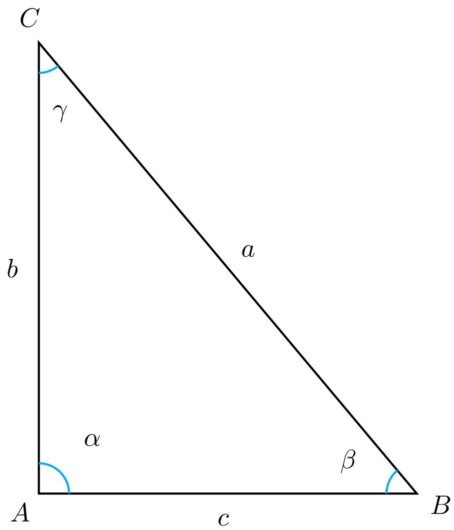
$r_u = \frac{7,779}{2 \cdot \sin 90^\circ}$

$r_u = 3,889$

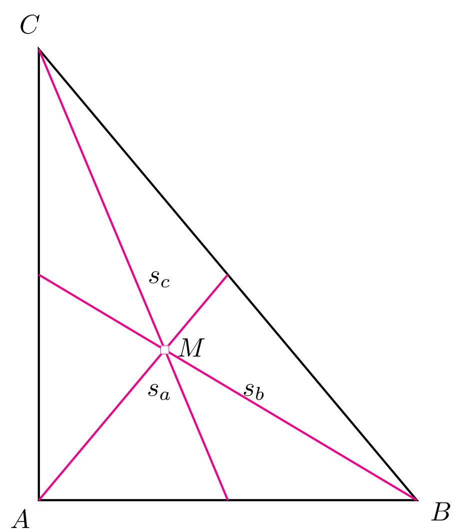
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 14,897}{18,737}$

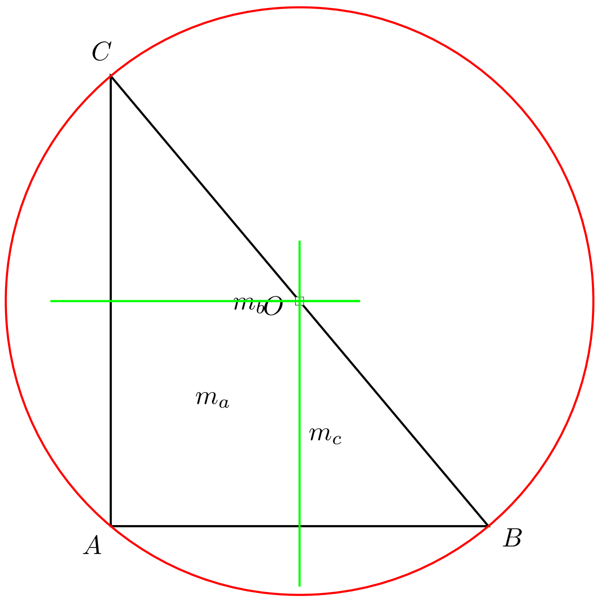
$r_i = 1,59$



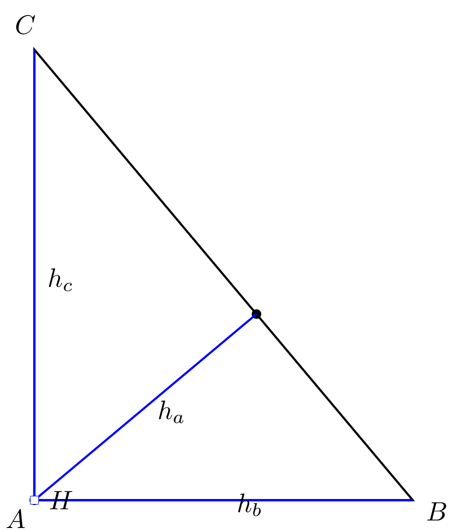
Seitenhalbierende-Schwerpunkt



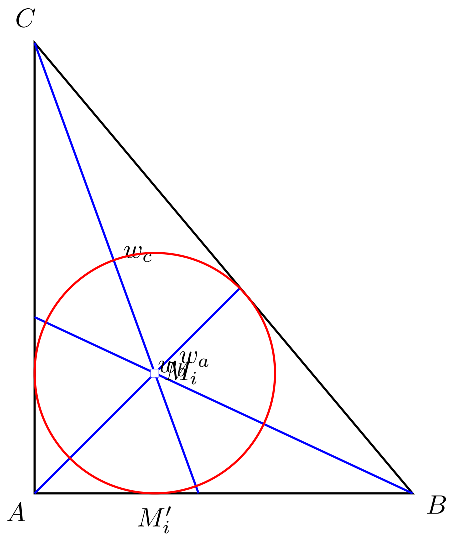
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_c
w_a
M_i
A
M_i'
B

Aufgabe (17)

$Winkel - Winkel - Seite$
$a = 3 \quad \alpha = 20^\circ \quad \beta = 90^\circ$

Winkelsumme:$\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180^\circ - \alpha - \beta$
$\gamma = 180^\circ - 20^\circ - 90^\circ$
$\gamma = 70^\circ$
Cosinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{a}{b} \quad / \cdot b$
$b \cdot \sin\alpha = a \quad / : sin\alpha$
$b = \frac{a}{\sin\alpha}$
$b = \frac{3}{\sin 20}$
$b = 8,771$
Pythagoras$b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{8,771^2 - 3^2}$
$c = 8,242$
Umfang:$U = a + b + c$
$U = 3 + 8,771 + 8,242$
$U = 20,014$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,242 \cdot \sin 90^\circ$
$h_a = 8,242$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 8,242$
$A = 12,364$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 70^\circ$
$h_b = 2,819$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8,771 \cdot \sin 20^\circ$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{8,242 \cdot \sin 90}{\sin 80}$

$wha = 8,37$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{3 \cdot \sin 70}{\sin 65}$

$whb = 3,111$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{8,771 \cdot \sin 20}{\sin 80}$

$whc = 1,042$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(8,771^2 + 8,242^2) - 3^2}$

$s_a = 8,378$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(3^2 + 8,242^2) - 8,771^2}$

$s_b = 4,386$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(3^2 + 8,771^2) - 8,242^2}$

$s_c = 4,872$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

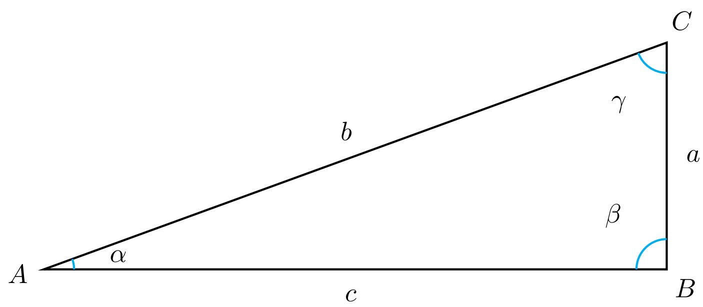
$r_u = \frac{3}{2 \cdot \sin 20^\circ}$

$r_u = 4,386$

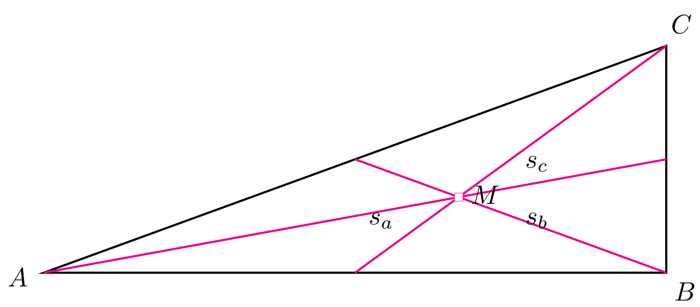
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 12,364}{20,014}$

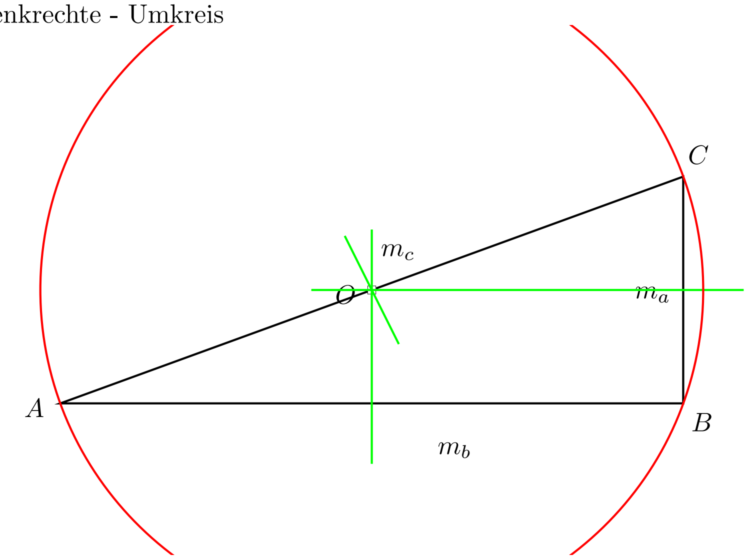
$r_i = 1,236$



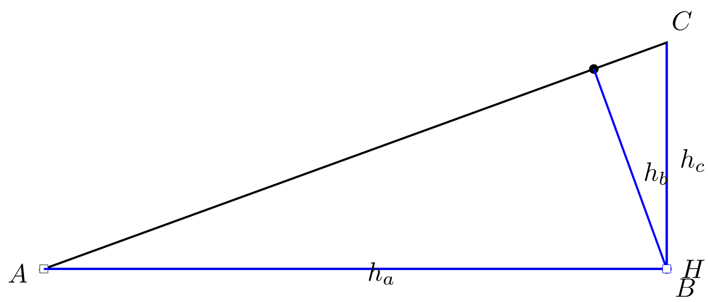
Seitenhalbierende-Schwerpunkt



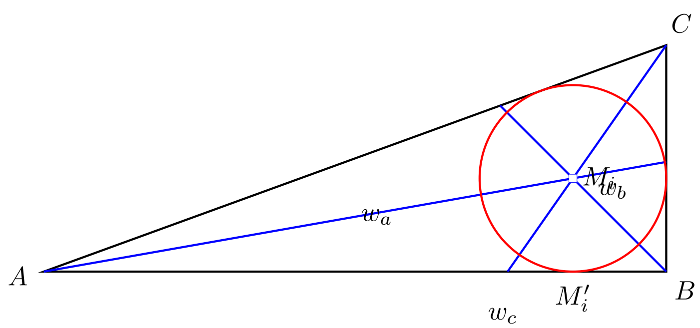
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

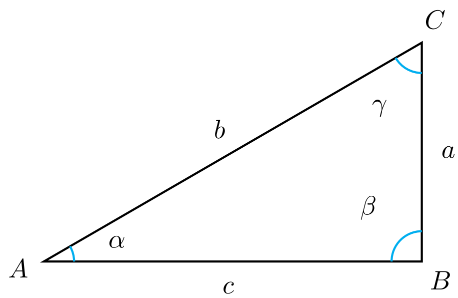


Aufgabe (18)

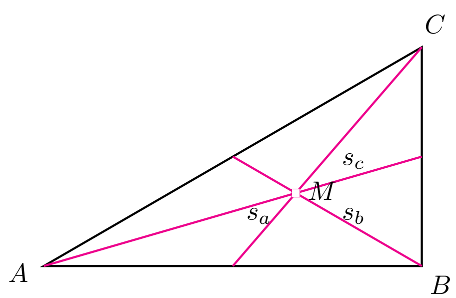
$Winkel - Seite - Winkel$
$c = 5 \quad \alpha = 30° \quad \beta = 90°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 90°$
$\gamma = 60°$
Sinus: $\cos\alpha = \frac{c}{b}$
$\cos\alpha = \frac{c}{b} \quad / \cdot b$
$b \cdot \cos\alpha = c \quad / : cos\alpha$
$b = \frac{c}{\cos\alpha}$
$b = \frac{5}{\cos 30}$
$b = 5,774$
Pythagoras$b^2 = a^2 + c^2 \quad / - c^2$
$a^2 = b^2 - c^2$
$a = \sqrt{b^2 - c^2}$
$a = \sqrt{5,774^2 - 5^2}$
$a = 2,887$
Umfang:$U = a + b + c$
$U = 2,887 + 5,774 + 5$
$U = 13,66$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,887 \cdot 5$
$A = 7,217$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,887 \cdot \sin 60°$
$h_b = 2\frac{1}{2}$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,774 \cdot \sin 30°$
$h_c = 2,887$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

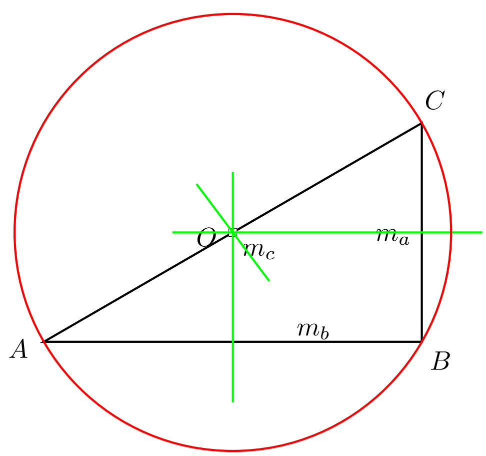
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 75}$
$wha = 5,176$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,887 \cdot \sin 60}{\sin 75}$
$whb = 2,588$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,774 \cdot \sin 30}{\sin 75}$
$whc = 1,494$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,774^2 + 5^2) - 2,887^2}$
$s_a = 5,204$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,887^2 + 5^2) - 5,774^2}$
$s_b = 2,887$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,887^2 + 5,774^2) - 5^2}$
$s_c = 3,536$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,887}{2 \cdot \sin 30^\circ}$
$r_u = 2,887$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,217}{13,66}$
$r_i = 1,057$



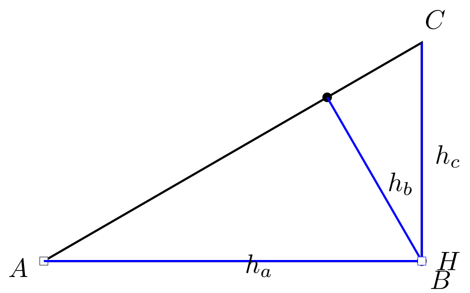
Seitenhalbierende-Schwerpunkt



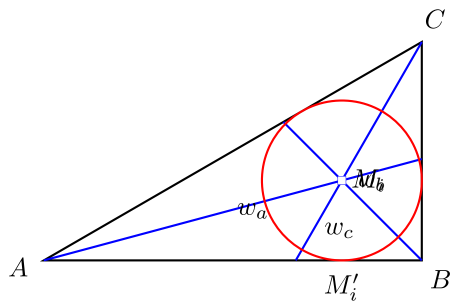
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

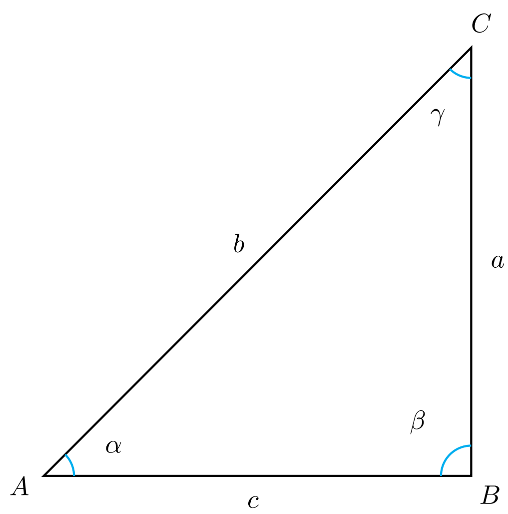


Aufgabe (19)

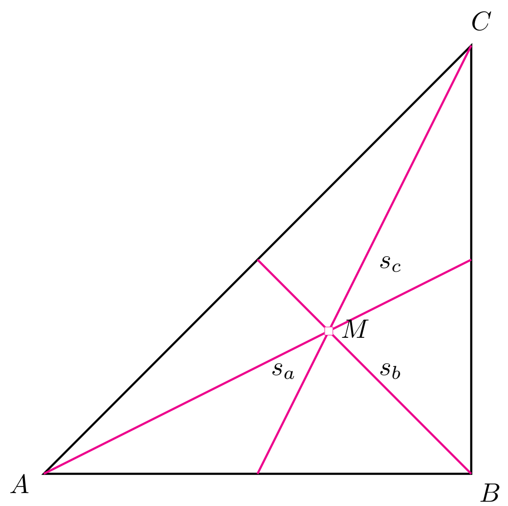
$Winkel - Winkel - Seite$
$b = 8 \quad \gamma = 45° \quad \beta = 90°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 90° - 45°$
$\alpha = 45°$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{a}{b} \quad / \cdot b \; a = b \cdot \sin\alpha$
$a = 8 \cdot \sin 45$
$a = 5,657$
Pythagoras$b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{8^2 - 5,657^2}$
$c = 5,657$
Umfang:$U = a + b + c$
$U = 5,657 + 8 + 5,657$
$U = 19,314$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,657 \cdot \sin 90°$
$h_a = 5,657$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,657 \cdot 5,657$
$A = 16$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,657 \cdot \sin 45°$
$h_b = 4$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8 \cdot \sin 45°$
$h_c = 5,657$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

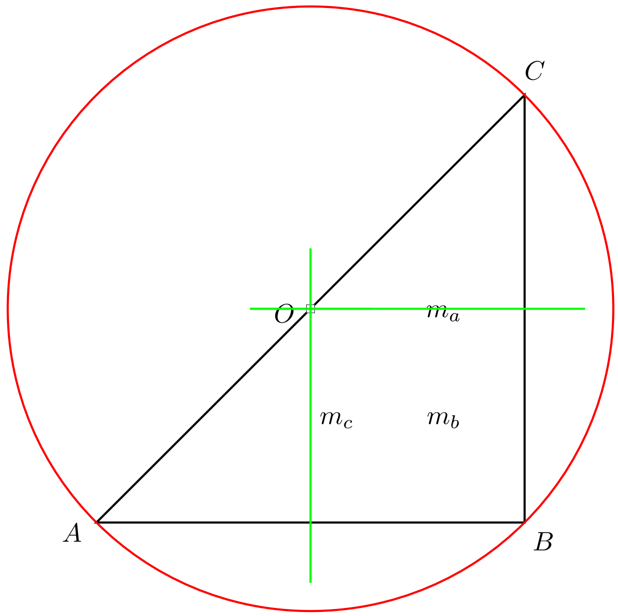
$wha = \frac{5,657 \cdot \sin 90}{\sin 67\frac{1}{2}}$
$wha = 6,123$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,657 \cdot \sin 45}{\sin 90}$
$whb = 4$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8 \cdot \sin 45}{\sin 67\frac{1}{2}}$
$whc = 4,33$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(8^2 + 5,657^2) - 5,657^2}$
$s_a = 6,325$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,657^2 + 5,657^2) - 8^2}$
$s_b = 4$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,657^2 + 8^2) - 5,657^2}$
$s_c = 5,657$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,657}{2 \cdot \sin 45^\circ}$
$r_u = 4$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 16}{19,314}$
$r_i = 1,657$



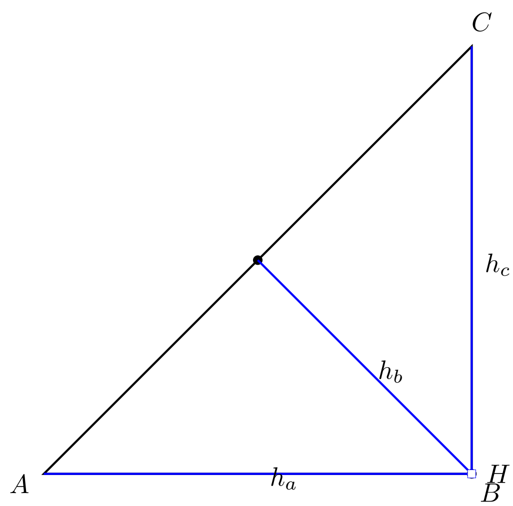
Seitenhalbierende-Schwerpunkt



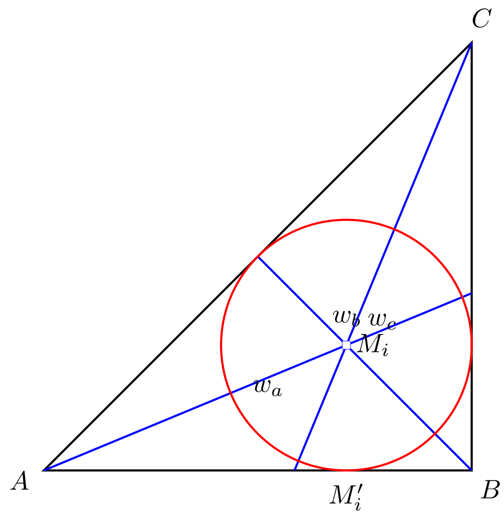
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

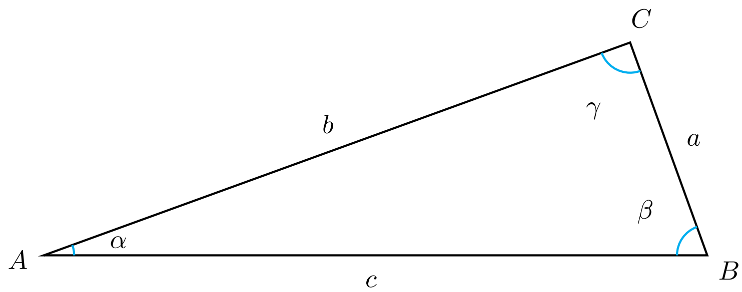

C
w_b w_c
M_i
w_a
A
M_i'
B

Aufgabe (20)

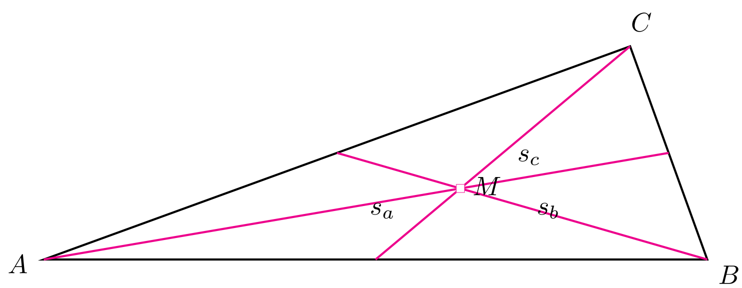
$Winkel - Winkel - Seite$
$a = 3 \quad \alpha = 20° \quad \gamma = 90°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 20° - 90°$
$\beta = 70°$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{a}{c} \quad / \cdot c$
$c \cdot \sin\alpha = a \quad / : sin\alpha$
$c = \frac{a}{\sin\alpha}$
$c = \frac{3}{\sin 20}$
$c = 8,771$
Pythagoras$c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{8,771^2 - 3^2}$
$b = 8,242$
Umfang:$U = a + b + c$
$U = 3 + 8,242 + 8,771$
$U = 20,014$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,771 \cdot \sin 70°$
$h_a = 8,242$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 8,242$
$A = 12,364$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 90°$
$h_b = 3$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 8,242 \cdot \sin 20°$
$h_c = 2,819$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

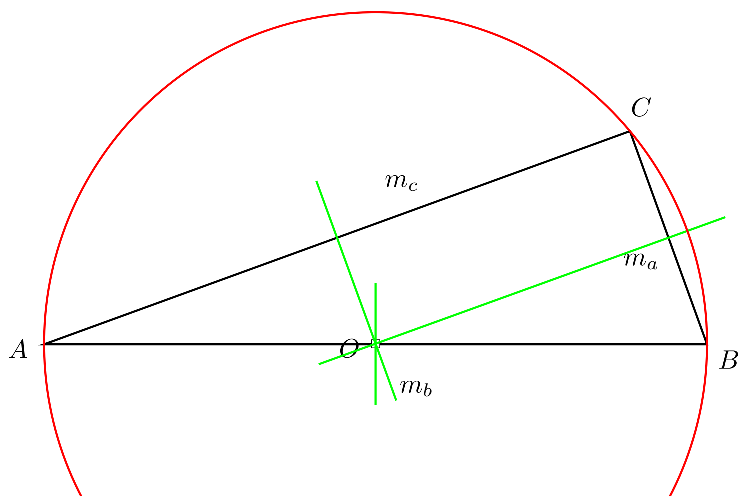
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{8,771 \cdot \sin 70}{\sin 100}$
$wha = 8,37$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 90}{\sin 55}$
$whb = 3,662$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{8,242 \cdot \sin 20}{\sin 100}$
$whc = 1,042$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(8,242^2 + 8,771^2) - 3^2}$
$s_a = 8,378$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 8,771^2) - 8,242^2}$
$s_b = 5,097$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 8,242^2) - 8,771^2}$
$s_c = 4,635$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 20^\circ}$
$r_u = 4,386$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,364}{20,014}$
$r_i = 1,236$



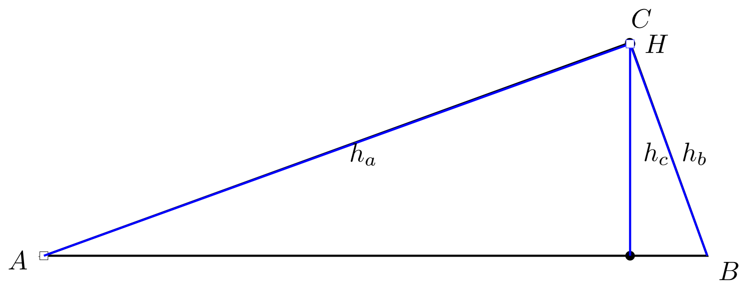
Seitenhalbierende-Schwerpunkt



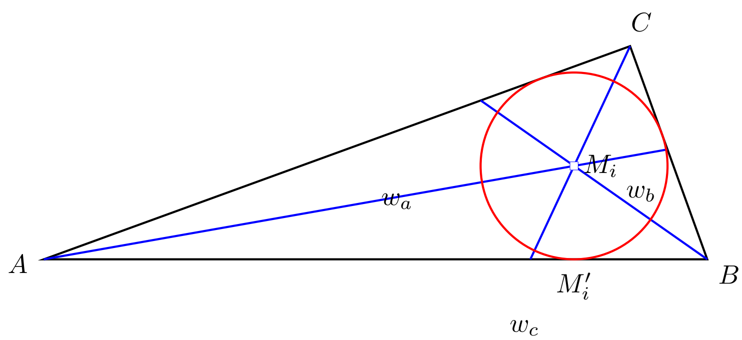
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

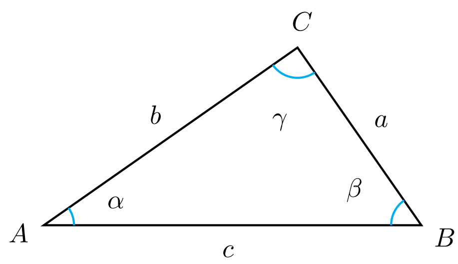


Aufgabe (21)

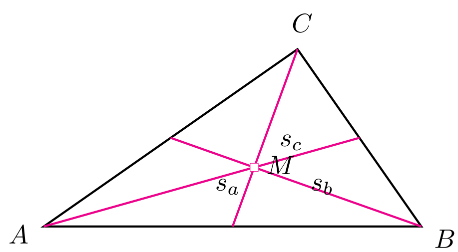
$Winkel - Winkel - Seite$
$c = 5 \quad \gamma = 90° \quad \alpha = 35°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 35° - 90°$
$\beta = 55°$
Sinus: $\sin\alpha = \frac{a}{c}$
$\sin\alpha = \frac{a}{c} \quad / \cdot c \; a = c \cdot \sin\alpha$
$a = 5 \cdot \sin 35$
$a = 2,868$
Pythagoras$c^2 = a^2 + b^2 \quad / - a^2$
$b^2 = c^2 - a^2$
$b = \sqrt{c^2 - a^2}$
$b = \sqrt{5^2 - 2,868^2}$
$b = 4,096$
Umfang:$U = a + b + c$
$U = 2,868 + 4,096 + 5$
$U = 11,964$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 55°$
$h_a = 4,096$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 2,868 \cdot 4,096$
$A = 5,873$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2,868 \cdot \sin 90°$
$h_b = 2,868$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,096 \cdot \sin 35°$
$h_c = 2,349$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

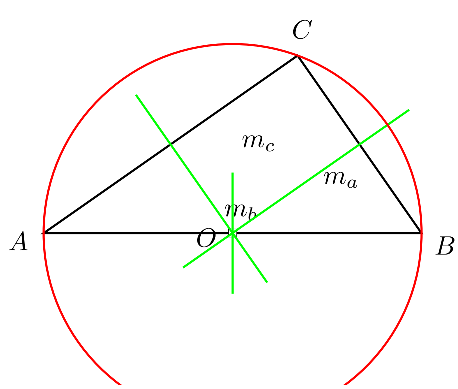
$wha = \frac{5 \cdot \sin 55}{\sin 107\frac{1}{2}}$
$wha = 4,295$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2,868 \cdot \sin 90}{\sin 62\frac{1}{2}}$
$whb = 3,233$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,096 \cdot \sin 35}{\sin 107\frac{1}{2}}$
$whc = 1,725$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,096^2 + 5^2) - 2,868^2}$
$s_a = 4,34$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2,868^2 + 5^2) - 4,096^2}$
$s_b = 3,524$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2,868^2 + 4,096^2) - 5^2}$
$s_c = 2,882$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2,868}{2 \cdot \sin 35^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 5,873}{11,964}$
$r_i = 0,982$



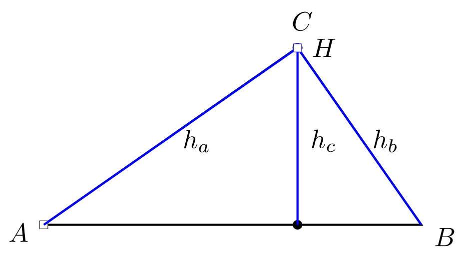
Seitenhalbierende-Schwerpunkt



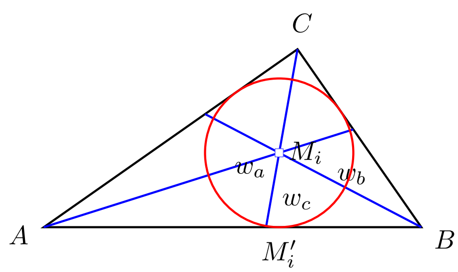
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (22)

$Winkel - Winkel - Seite$
$b = 3 \quad \gamma = 90° \quad \beta = 65°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 65° - 90°$
$\alpha = 25°$
Cosinus: $\cos\alpha = \frac{b}{c}$
$\cos\alpha = \frac{b}{c} \quad / \cdot c$
$c \cdot \cos\alpha = b \quad / : cos\alpha$
$c = \frac{b}{\cos\alpha}$
$c = \frac{3}{\cos 25}$
$c = 3,31$
Pythagoras$c^2 = a^2 + b^2 \quad / - b^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2 - b^2}$
$a = \sqrt{3,31^2 - 3^2}$
$a = 1,399$
Umfang:$U = a + b + c$
$U = 1,399 + 3 + 3,31$
$U = 7,709$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3,31 \cdot \sin 65°$
$h_a = 3$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 1,399 \cdot 3$
$A = 2,098$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 1,399 \cdot \sin 90°$
$h_b = 1,399$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 25°$
$h_c = 1,268$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{3,31 \cdot \sin 65}{\sin 102\frac{1}{2}}$

$wha = 3,073$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{1,399 \cdot \sin 90}{\sin 57\frac{1}{2}}$

$whb = 1,659$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{3 \cdot \sin 25}{\sin 102\frac{1}{2}}$

$whc = 0,606$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(3^2 + 3,31^2) - 1,399^2}$

$s_a = 3,08$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(1,399^2 + 3,31^2) - 3^2}$

$s_b = 2,051$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(1,399^2 + 3^2) - 3,31^2}$

$s_c = 1,797$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

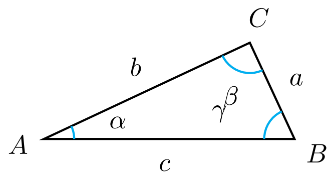
$r_u = \frac{1,399}{2 \cdot \sin 25°}$

$r_u = 1,655$

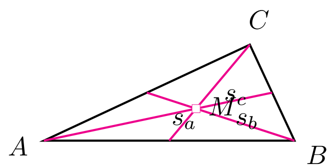
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 2,098}{7,709}$

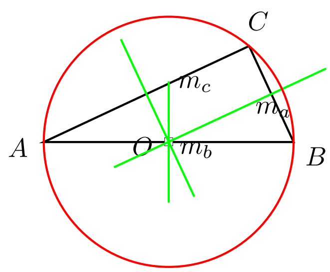
$r_i = 0,544$



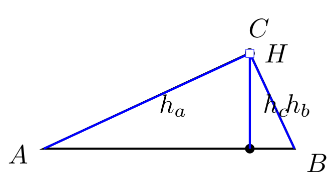
Seitenhalbierende-Schwerpunkt



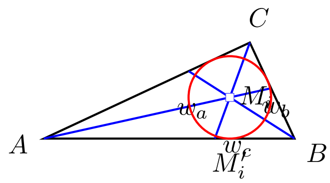
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

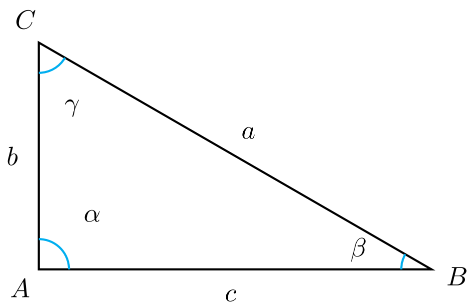

C
A
B
w_a
w_b
w_c
M_i
M_i

Aufgabe (23)

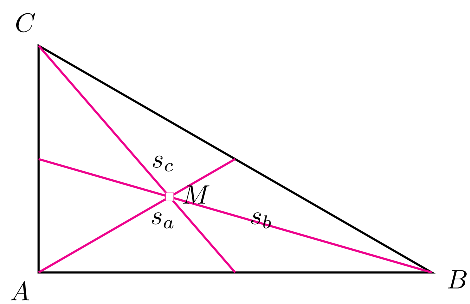
$Winkel - Winkel - Seite$
$a = 6 \quad \alpha = 90° \quad \beta = 30°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30°$
$\gamma = 60°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$b = a \cdot \sin\beta$
$b = 6 \cdot \sin 30$
$b = 3$
Pythagoras$a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{6^2 - 3^2}$
$c = 5,196$
Umfang:$U = a + b + c$
$U = 6 + 3 + 5,196$
$U = 14,196$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5,196 \cdot \sin 30°$
$h_a = 2,598$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 2,598$
$A = 7,794$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 60°$
$h_b = 5,196$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

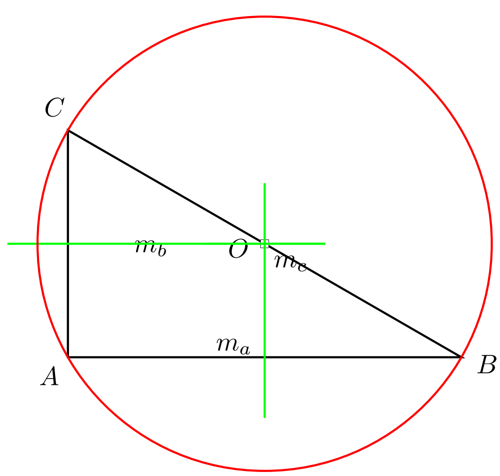
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5,196 \cdot \sin 30}{\sin 105}$
$wha = 2,69$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 60}{\sin 105}$
$whb = 5,379$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 105}$
$whc = 6,212$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5,196^2) - 6^2}$
$s_a = 3$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5,196^2) - 3^2}$
$s_b = 5,408$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 3^2) - 5,196^2}$
$s_c = 4\frac{1}{2}$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 90^\circ}$
$r_u = 3$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,794}{14,196}$
$r_i = 1,098$



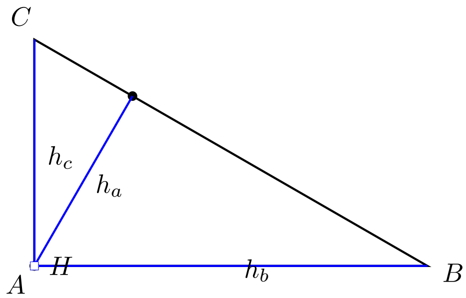
Seitenhalbierende-Schwerpunkt



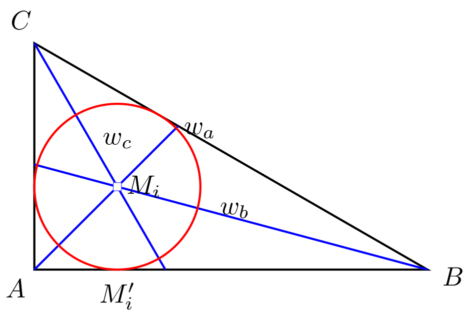
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

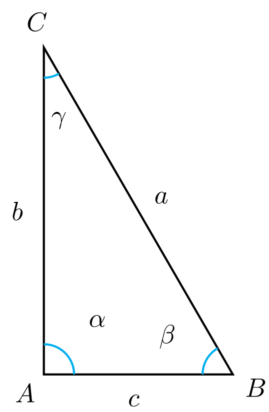


Aufgabe (24)

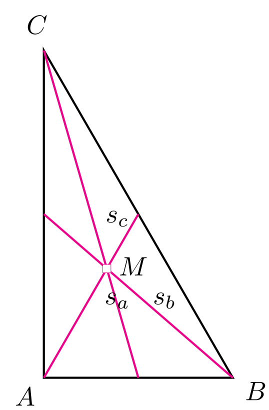
$Winkel - Winkel - Seite$
$a = 5 \quad \alpha = 90° \quad \gamma = 30°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 90° - 30°$
$\beta = 60°$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{b}{a} \quad / \cdot a$
$b = a \cdot \sin\beta$
$b = 5 \cdot \sin 60$
$b = 4,33$
Pythagoras$a^2 = b^2 + c^2 \quad / - b^2$
$c^2 = a^2 - b^2$
$c = \sqrt{a^2 - b^2}$
$c = \sqrt{5^2 - 4,33^2}$
$c = 2\frac{1}{2}$
Umfang:$U = a + b + c$
$U = 5 + 4,33 + 2\frac{1}{2}$
$U = 11,83$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2\frac{1}{2} \cdot \sin 60°$
$h_a = 2,165$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5 \cdot 2,165$
$A = 5,413$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5 \cdot \sin 30°$
$h_b = 2\frac{1}{2}$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4,33 \cdot \sin 90°$
$h_c = 4,33$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

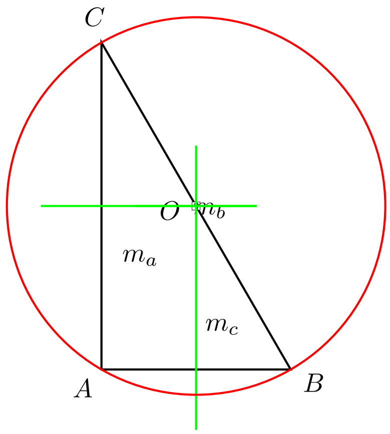
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2\frac{1}{2} \cdot \sin 60}{\sin 75}$
$wha = 2,241$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5 \cdot \sin 30}{\sin 120}$
$whb = 2,887$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4,33 \cdot \sin 90}{\sin 75}$
$whc = 5,176$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4,33^2 + 2\frac{1}{2}^2) - 5^2}$
$s_a = 2\frac{1}{2}$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5^2 + 2\frac{1}{2}^2) - 4,33^2}$
$s_b = 3,307$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5^2 + 4,33^2) - 2\frac{1}{2}^2}$
$s_c = 4,146$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5}{2 \cdot \sin 90^\circ}$
$r_u = 2\frac{1}{2}$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 5,413}{11,83}$
$r_i = 0,915$



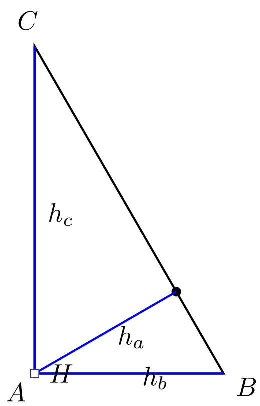
Seitenhalbierende-Schwerpunkt



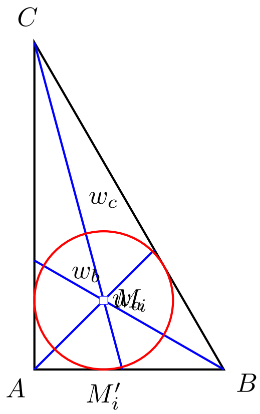
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

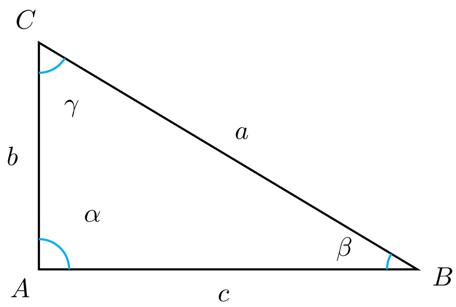

C
w_c
w_b
M_i
w_a
A
M_i'
B

Aufgabe (25)

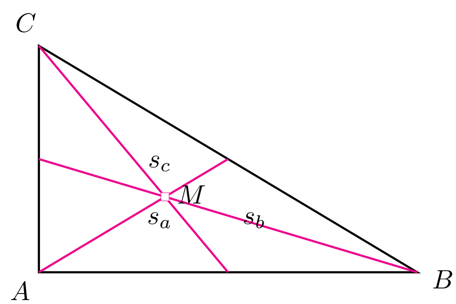
$Seite - Winkel - Seite$
$b = 3 \quad c = 5 \quad \alpha = 90°$

Pythagoras$a^2 = b^2 + c^2$
$a = \sqrt{b^2 + c^2}$
$a = \sqrt{3^2 + 5^2}$
$a = 5,831$
Sinus: $\sin\beta = \frac{b}{a}$
$\sin\beta = \frac{3}{5,831}$
$\beta = 30,964$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 90° - 30,964°$
$\gamma = 59,036°$
Umfang:$U = a + b + c$
$U = 5,831 + 3 + 5$
$U = 13,831$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 30,964°$
$h_a = 2,572$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,831 \cdot 2,572$
$A = 7\frac{1}{2}$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,831 \cdot \sin 59,036°$
$h_b = 5$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3 \cdot \sin 90°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 30,964}{\sin 104,036}$

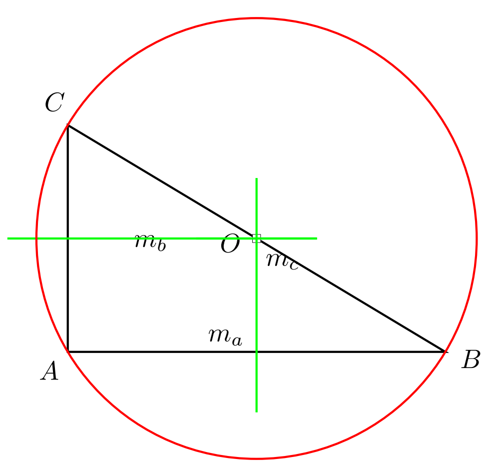
$wha = 2,652$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,831 \cdot \sin 59,036}{\sin 105,482}$
$whb = 5,188$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3 \cdot \sin 90}{\sin 104,036}$
$whc = 6,01$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,831^2}$
$s_a = 2,915$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,831^2 + 5^2) - 3^2}$
$s_b = 5,22$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,831^2 + 3^2) - 5^2}$
$s_c = 4,387$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,831}{2 \cdot \sin 90^\circ}$
$r_u = 2,915$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,831}$
$r_i = 1,085$



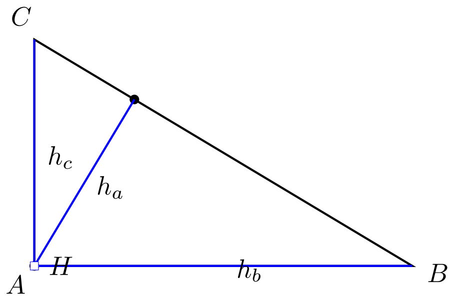
Seitenhalbierende-Schwerpunkt



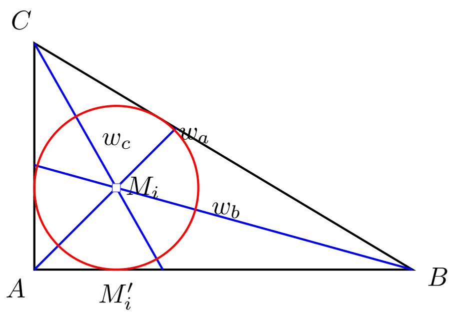
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

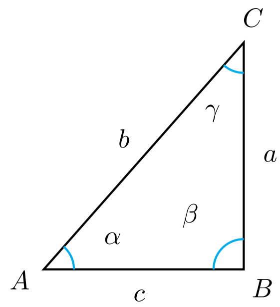


Aufgabe (26)

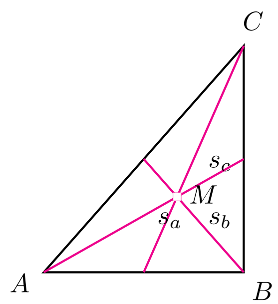
$Seite - Seite - Winkel$
$a = 3 \quad b = 4 \quad \beta = 90°$

Pythagoras$b^2 = a^2 + c^2 \quad / - a^2$
$c^2 = b^2 - a^2$
$c = \sqrt{b^2 - a^2}$
$c = \sqrt{4^2 - 3^2}$
$c = 2,646$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{4}$
$\alpha = 48,59$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 48,59° - 90°$
$\gamma = 41,41°$
Umfang:$U = a + b + c$
$U = 3 + 4 + 2,646$
$U = 9,646$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,646 \cdot \sin 90°$
$h_a = 2,646$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 2,646$
$A = 3,969$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 41,41°$
$h_b = 1,984$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 48,59°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{2,646 \cdot \sin 90}{\sin 65,705}$

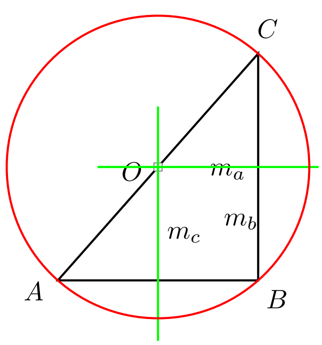
$wha = 2,903$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 41,41}{\sin 93,59}$
$whb = 1,988$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 48,59}{\sin 65,705}$
$whc = 2,469$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 2,646^2) - 3^2}$
$s_a = 3,041$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 2,646^2) - 4^2}$
$s_b = 2$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 4^2) - 2,646^2}$
$s_c = 2,915$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 48,59^\circ}$
$r_u = 2$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 3,969}{9,646}$
$r_i = 0,823$



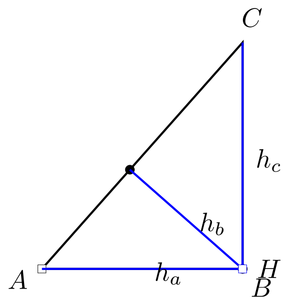
Seitenhalbierende-Schwerpunkt



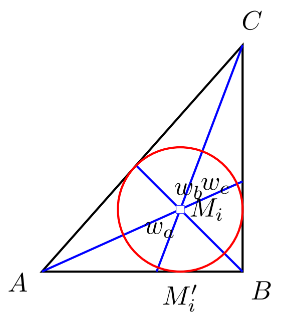
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

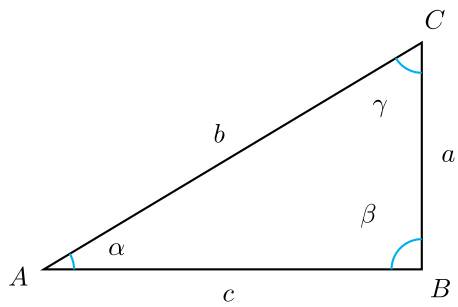


Aufgabe (27)

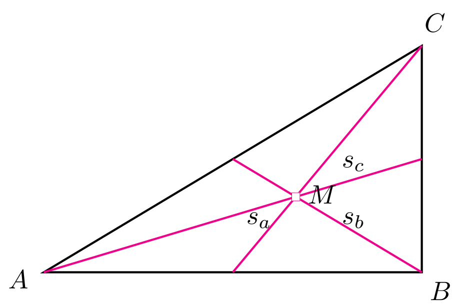
$Seite - Winkel - Seite$
$a = 3 \quad c = 5 \quad \beta = 90°$

Pythagoras$b^2 = a^2 + c^2$
$b = \sqrt{a^2 + c^2}$
$b = \sqrt{3^2 + 5^2}$
$b = 5,831$
Sinus: $\sin\alpha = \frac{a}{b}$
$\sin\alpha = \frac{3}{5,831}$
$\alpha = 30,964$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30,964° - 90°$
$\gamma = 59,036°$
Umfang:$U = a + b + c$
$U = 3 + 5,831 + 5$
$U = 13,831$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 90°$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3 \cdot 5$
$A = 7\frac{1}{2}$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3 \cdot \sin 59,036°$
$h_b = 2,572$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5,831 \cdot \sin 30,964°$
$h_c = 3$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 90}{\sin 74,518}$

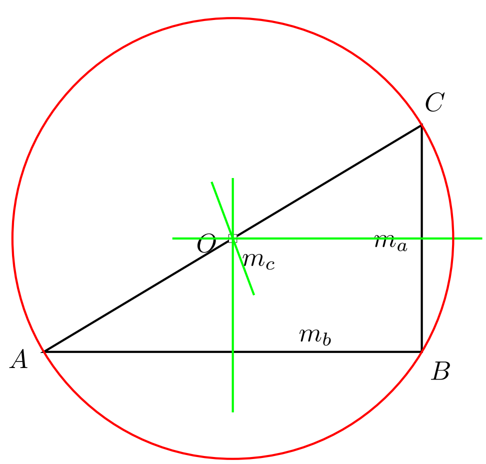
$wha = 5,188$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3 \cdot \sin 59,036}{\sin 75,964}$
$whb = 2,652$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5,831 \cdot \sin 30,964}{\sin 74,518}$
$whc = 1,602$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5,831^2 + 5^2) - 3^2}$
$s_a = 5,22$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3^2 + 5^2) - 5,831^2}$
$s_b = 2,915$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3^2 + 5,831^2) - 5^2}$
$s_c = 3,606$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3}{2 \cdot \sin 30,964^\circ}$
$r_u = 2,915$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7\frac{1}{2}}{13,831}$
$r_i = 1,085$



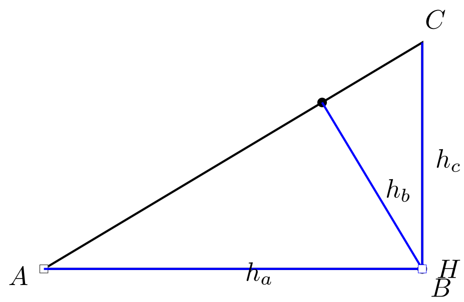
Seitenhalbierende-Schwerpunkt



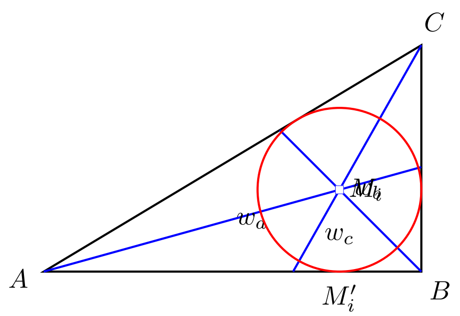
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



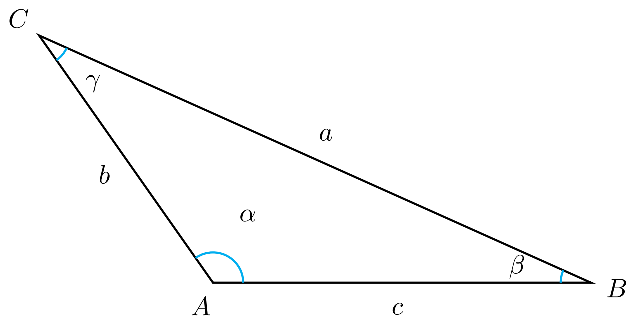
Aufgabe (28)

$Seite - Seite - Seite$
$a = 8 \quad b = 4 \quad c = 5$

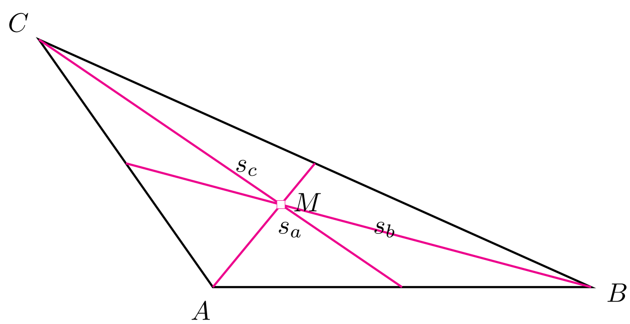
Umfang:$U = a + b + c$
$U = 8 + 4 + 5$
$U = 17$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{4^2 + 5^2 - 8^2}{2 \cdot 4 \cdot 5}$
$\cos\alpha = -\frac{23}{40}$
$\alpha = \arccos(-\frac{23}{40})$
$\alpha = 125,1°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{4 \cdot \sin 125,1°}{8}$
$\sin\beta = 0,409$
$\beta = \arcsin(0,409)$
$\beta = 24,147°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 125,1° - 24,147°$
$\gamma = 30,754°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 24,147°$
$h_a = 2,045$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 8 \cdot 2,045$
$A = 8,182$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 8 \cdot \sin 30,754°$
$h_b = 4,091$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$

$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 125,1°$
$h_c = 3,273$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \quad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 24,147}{\sin 93,303}$
$wha = 2,049$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \quad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{8 \cdot \sin 30,754}{\sin 137,173}$
$whb = 6,018$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \quad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 125,1}{\sin 93,303}$
$whc = 6,556$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 8^2}$
$s_a = 2,121$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(8^2 + 5^2) - 4^2}$
$s_b = 6,364$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(8^2 + 4^2) - 5^2}$
$s_c = 6$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{8}{2 \cdot \sin 125,1°}$
$r_u = 4,889$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

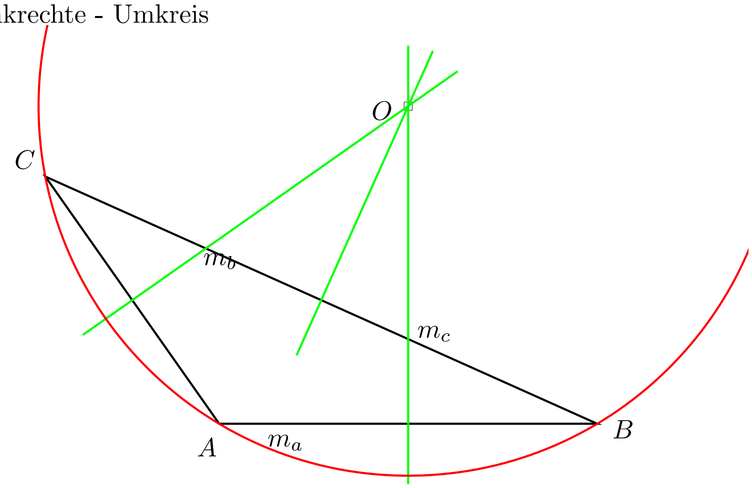
$r_i = \frac{2 \cdot 8,182}{17}$
$r_i = 0,963$



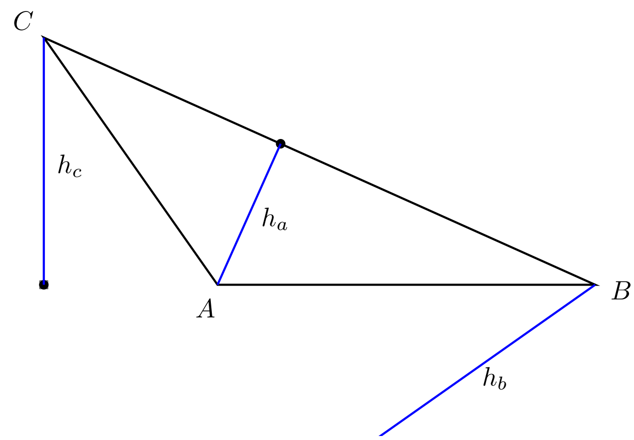
Seitenhalbierende-Schwerpunkt



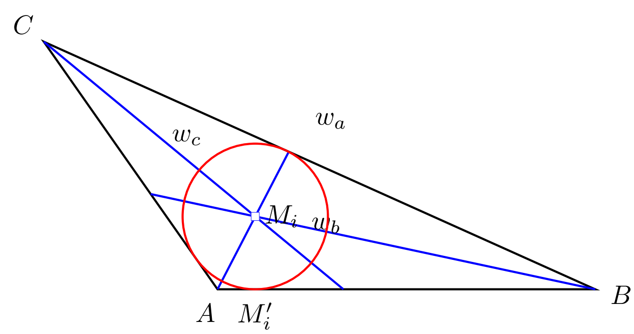
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (29)

$Seite - Seite - Seite$
$a = 3 \quad b = 7 \quad c = 4$
$b > a + c$ Berechnung nicht möglich: Dreiecksungleichung nicht erfüllt
$b > a + c$
Zeichnung nicht möglich

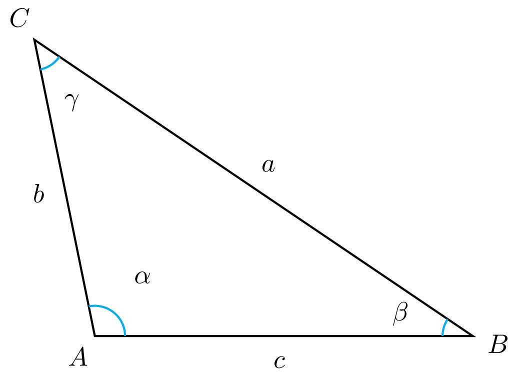
Aufgabe (30)

$Seite - Seite - Seite$
$a = 7 \quad b = 4 \quad c = 5$

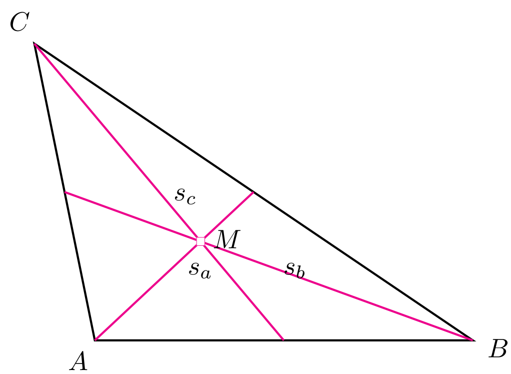
Umfang:$U = a + b + c$
$U = 7 + 4 + 5$
$U = 16$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{4^2 + 5^2 - 7^2}{2 \cdot 4 \cdot 5}$
$\cos\alpha = -\frac{1}{5}$
$\alpha = \arccos(-\frac{1}{5})$
$\alpha = 101,537°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{4 \cdot \sin 101,537°}{7}$
$\sin\beta = 0,56$
$\beta = \arcsin(0,56)$
$\beta = 34,048°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 101,537° - 34,048°$
$\gamma = 44,415°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 34,048°$
$h_a = 2,799$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7 \cdot 2,799$
$A = 9,798$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7 \cdot \sin 44,415°$
$h_b = 4,899$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$

$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 101,537°$
$h_c = 3,919$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 34,048}{\sin 95,184}$
$wha = 2,811$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7 \cdot \sin 44,415}{\sin 118,561}$
$whb = 5,578$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 101,537}{\sin 95,184}$
$whc = 6,887$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 5^2) - 7^2}$
$s_a = 2,872$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(7^2 + 5^2) - 4^2}$
$s_b = 5,745$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(7^2 + 4^2) - 5^2}$
$s_c = 5,339$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7}{2 \cdot \sin 101,537°}$
$r_u = 3,572$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

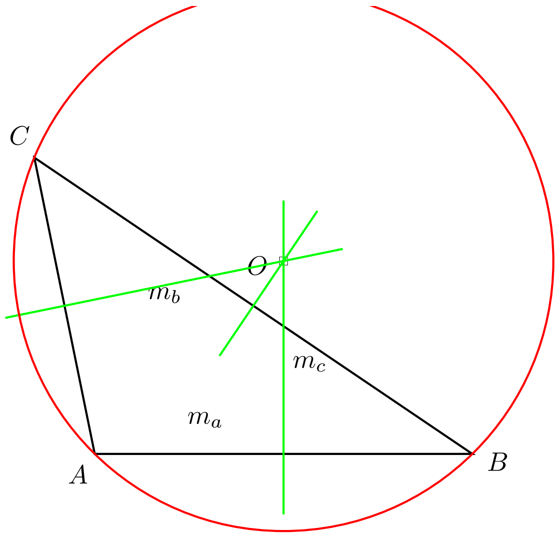
$r_i = \frac{2 \cdot 9,798}{16}$
$r_i = 1,225$



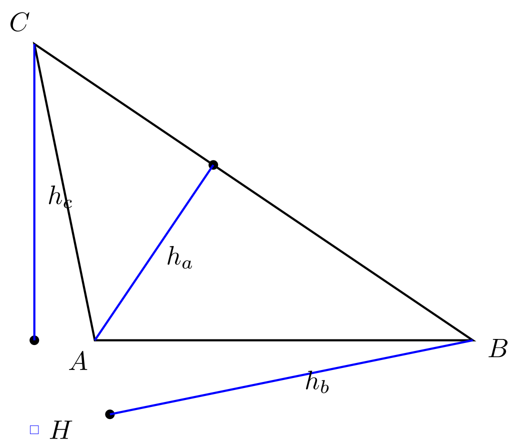
Seitenhalbierende-Schwerpunkt



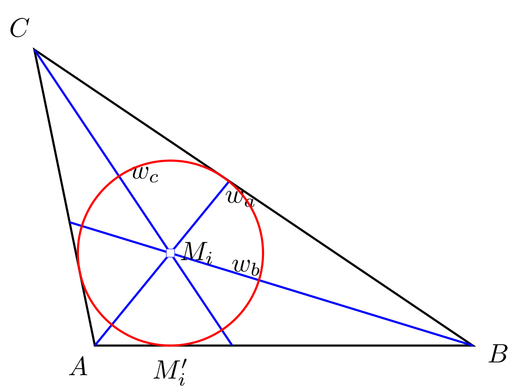
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



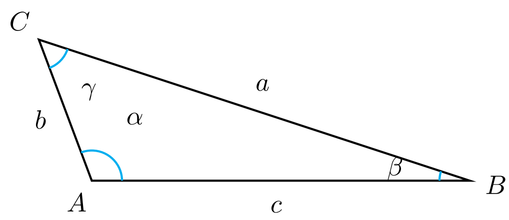
Aufgabe (31)

$Seite - Seite - Seite$
$a = 6 \quad b = 2 \quad c = 5$

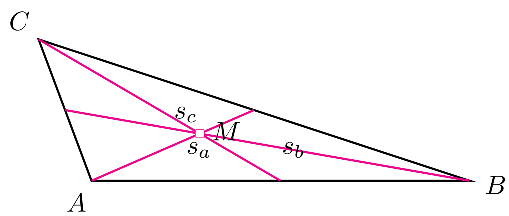
Umfang:$U = a + b + c$
$U = 6 + 2 + 5$
$U = 13$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{2^2 + 5^2 - 6^2}{2 \cdot 2 \cdot 5}$
$\cos\alpha = -\frac{7}{20}$
$\alpha = \arccos(-\frac{7}{20})$
$\alpha = 110,487°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{2 \cdot \sin 110,487°}{6}$
$\sin\beta = 0,312$
$\beta = \arcsin(0,312)$
$\beta = 18,195°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 110,487° - 18,195°$
$\gamma = 51,318°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 18,195°$
$h_a = 1,561$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 1,561$
$A = 4,684$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 51,318°$
$h_b = 4,684$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$

$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 2 \cdot \sin 110,487°$
$h_c = 1,873$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 18,195}{\sin 106,561}$
$wha = 1,629$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 51,318}{\sin 119,585}$
$whb = 5,386$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{2 \cdot \sin 110,487}{\sin 106,561}$
$whc = 5,864$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(2^2 + 5^2) - 6^2}$
$s_a = 2,345$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 2^2}$
$s_b = 5,431$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 2^2) - 5^2}$
$s_c = 4,359$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 110,487°}$
$r_u = 3,203$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

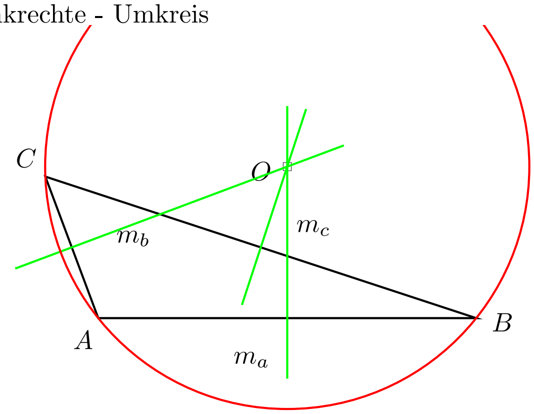
$r_i = \frac{2 \cdot 4,684}{13}$
$r_i = 0,721$



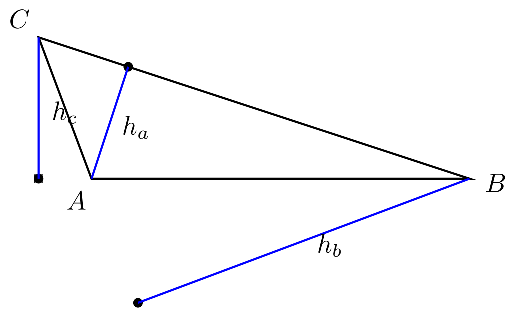
Seitenhalbierende-Schwerpunkt



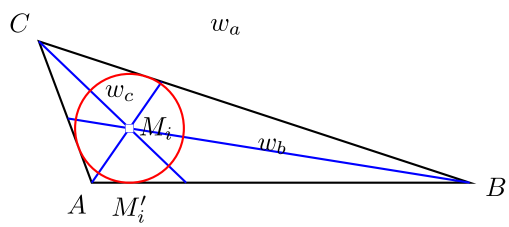
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (32)

$Seite - Winkel - Seite$
$a = 6 \quad b = 5 \quad \gamma = 25°$

Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 25°}$
$c = 2,573$
Umfang:$U = a + b + c$
$U = 6 + 5 + 2,573$
$U = 13,573$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{5^2 + 2,573^2 - 6^2}{2 \cdot 5 \cdot 2,573}$
$\cos\alpha = -0,17$
$\alpha = \arccos(-0,17)$
$\alpha = 99,797°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 99,797° - 25°$
$\beta = 55,203°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 2,573 \cdot \sin 55,203°$
$h_a = 2,113$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 2,113$
$A = 6,339$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 25°$
$h_b = 2,536$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 99,797°$
$h_c = 4,927$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{2,573 \cdot \sin 55,203}{\sin 74,898}$

$wha = 2,189$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{6 \cdot \sin 25}{\sin 127,398}$

$whb = 3,192$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{5 \cdot \sin 99,797}{\sin 74,898}$

$whc = 6,124$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(5^2 + 2,573^2) - 6^2}$

$s_a = 2,61$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(6^2 + 2,573^2) - 5^2}$

$s_b = 3,881$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 2,573^2}$

$s_c = 4,924$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

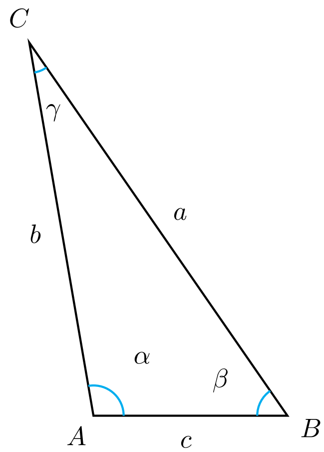
$r_u = \frac{6}{2 \cdot \sin 99,797^\circ}$

$r_u = 3,044$

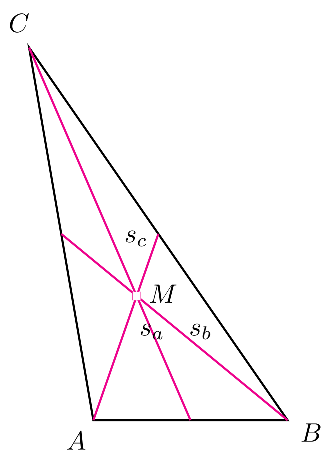
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 6,339}{13,573}$

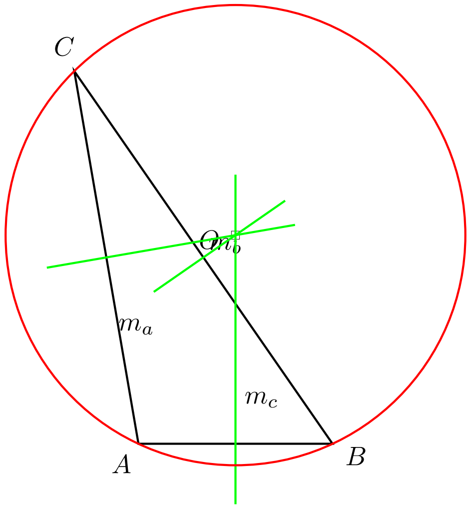
$r_i = 0,934$



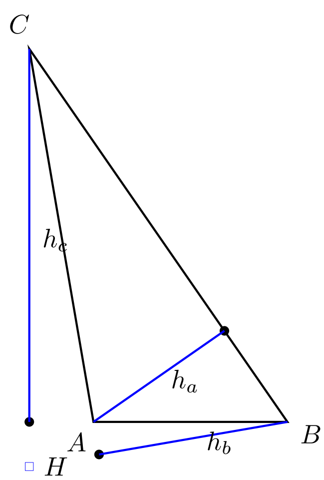
Seitenhalbierende-Schwerpunkt



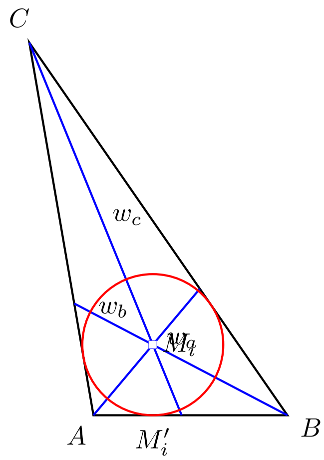
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_c
w_b
w_a
M_i
A
M_i'
B

Aufgabe (33)

$Seite - Winkel - Seite$
$b = 5 \quad c = 10 \quad \alpha = 155°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{5^2 + 10^2 - 2 \cdot 5 \cdot 10 \cdot \cos 155°}$
$a = 14,684$
Umfang:$U = a + b + c$
$U = 14,684 + 5 + 10$
$U = 29,684$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{5 \cdot \sin 155°}{14,684}$
$\sin\beta = 0,144$
$\beta = \arcsin(0,144)$
$\beta = 8,274°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 155° - 8,274°$
$\gamma = 16,726°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 10 \cdot \sin 8,274°$
$h_a = 1,439$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 14,684 \cdot 1,439$
$A = 10,565$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 14,684 \cdot \sin 16,726°$
$h_b = 4,226$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 155°$
$h_c = 2,113$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$

Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{10 \cdot \sin 8,274}{\sin 94,226}$

$wha = 1,443$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{14,684 \cdot \sin 16,726}{\sin 159,137}$

$whb = 11,867$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{5 \cdot \sin 155}{\sin 94,226}$

$whc = 6,223$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(5^2 + 10^2) - 14,684^2}$

$s_a = 2,931$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(14,684^2 + 10^2) - 5^2}$

$s_b = 12,311$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(14,684^2 + 5^2) - 10^2}$

$s_c = 10,68$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

$r_u = \frac{14,684}{2 \cdot \sin 155^\circ}$

$r_u = 17,373$

Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 10,565}{29,684}$

$r_i = 0,712$

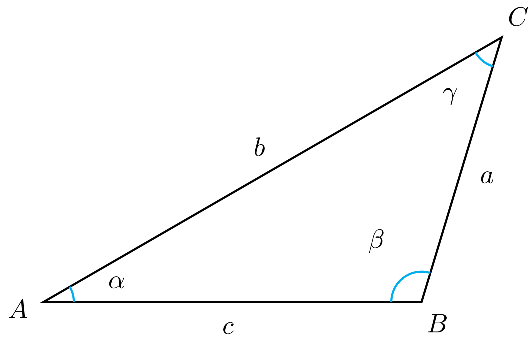
Werte zu groß - Zeichnung nicht möglich

Aufgabe (34)

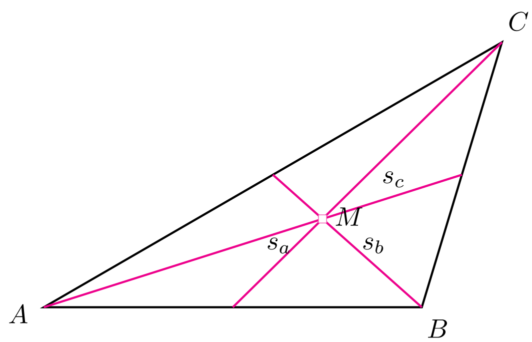
$Seite - Winkel - Seite$
$b = 7 \quad c = 5 \quad \alpha = 30°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos 30°}$
$a = 3,658$
Umfang:$U = a + b + c$
$U = 3,658 + 7 + 5$
$U = 15,658$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{7 \cdot \sin 30°}{3,658}$
$\sin\beta = 0,957$
$\beta = \arcsin(0,957)$
$\beta = 73,118°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 73,118°$
$\gamma = 76,882°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 73,118°$
$h_a = 4,785$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3,658 \cdot 4,785$
$A = 8\frac{3}{4}$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3,658 \cdot \sin 76,882°$
$h_b = 3,562$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 30°$
$h_c = 3\frac{1}{2}$
Winkelhalbierende α

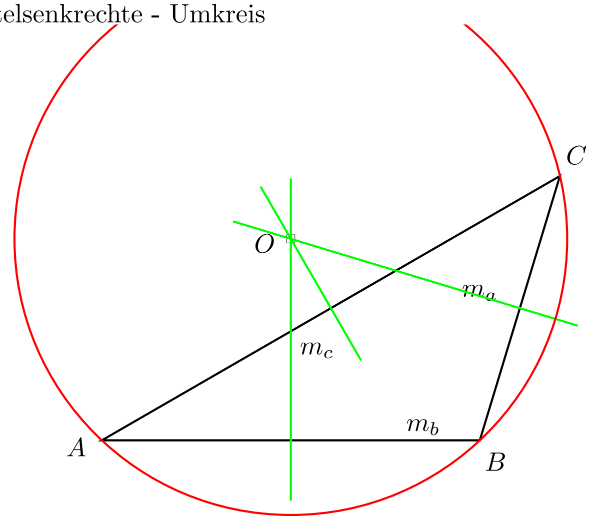
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 73,118}{\sin 91,882}$
$wha = 4,787$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3,658 \cdot \sin 76,882}{\sin 66,559}$
$whb = 3,883$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 30}{\sin 91,882}$
$whc = 1,83$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 5^2) - 3,658^2}$
$s_a = 5,801$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3,658^2 + 5^2) - 7^2}$
$s_b = 2,634$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3,658^2 + 7^2) - 5^2}$
$s_c = 4,352$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3,658}{2 \cdot \sin 30^\circ}$
$r_u = 3,658$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 8\frac{3}{4}}{15,658}$
$r_i = 1,118$



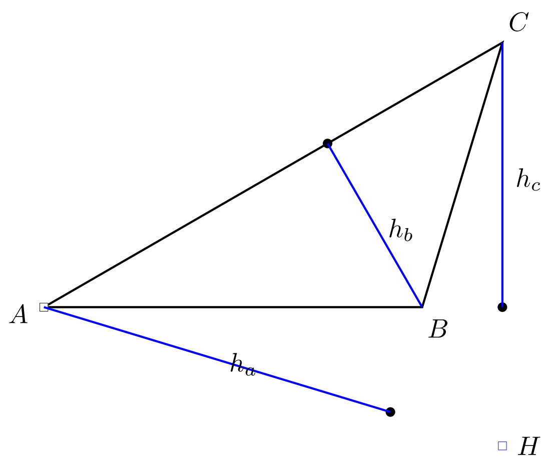
Seitenhalbierende-Schwerpunkt



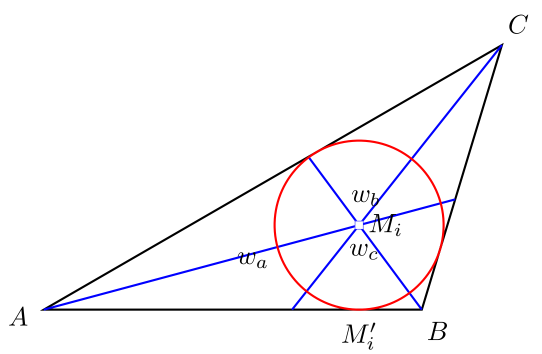
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

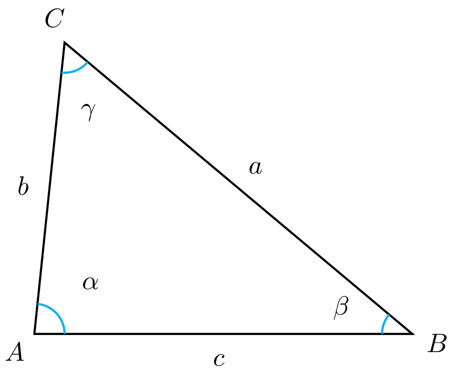


Aufgabe (35)

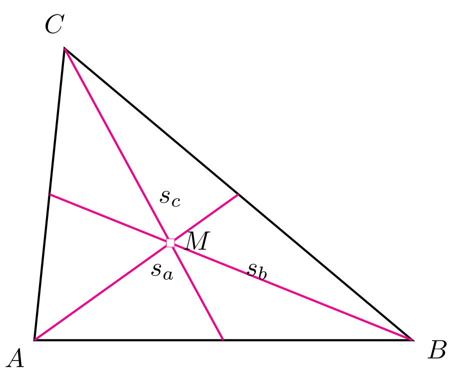
$Seite - Winkel - Seite$
$a = 6 \quad c = 5 \quad \beta = 40°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 40°}$
$b = 3,878$
Umfang:$U = a + b + c$
$U = 6 + 3,878 + 5$
$U = 14,878$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{3,878^2 + 5^2 - 6^2}{2 \cdot 3,878 \cdot 5}$
$\cos\alpha = 0,104$
$\alpha = \arccos(0,104)$
$\alpha = 84,024°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 84,024° - 40°$
$\gamma = 55,976°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 5 \cdot \sin 40°$
$h_a = 3,214$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 3,214$
$A = 9,642$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 55,976°$
$h_b = 4,973$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3,878 \cdot \sin 84,024°$
$h_c = 3,857$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

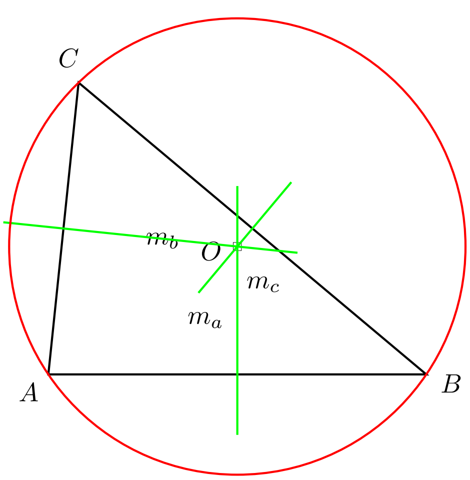
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{5 \cdot \sin 40}{\sin 97,988}$
$wha = 3,245$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 55,976}{\sin 104,024}$
$whb = 5,126$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3,878 \cdot \sin 84,024}{\sin 97,988}$
$whc = 6,026$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,878^2 + 5^2) - 6^2}$
$s_a = 3,319$
Seitenhalbierende:$s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 3,878^2}$
$s_b = 5,171$
Seitenhalbierende:$s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 3,878^2) - 5^2}$
$s_c = 4,665$
Umkreisradius:$2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 84,024^\circ}$
$r_u = 3\frac{1}{61}$
Innkreisradius:$r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 9,642}{14,878}$
$r_i = 1,296$



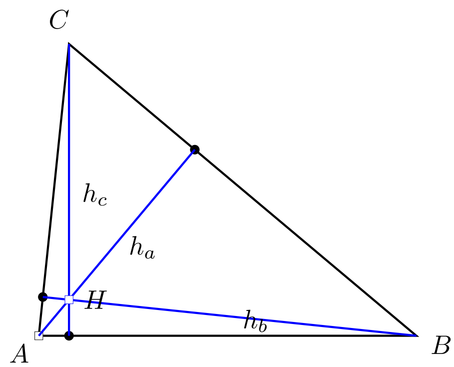
Seitenhalbierende-Schwerpunkt



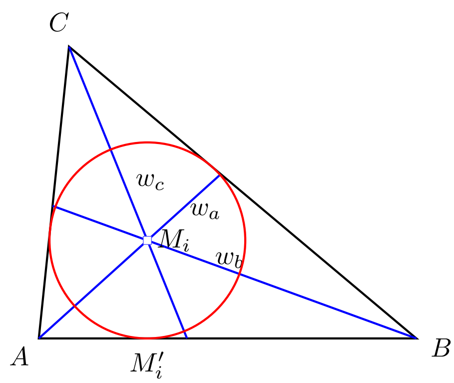
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

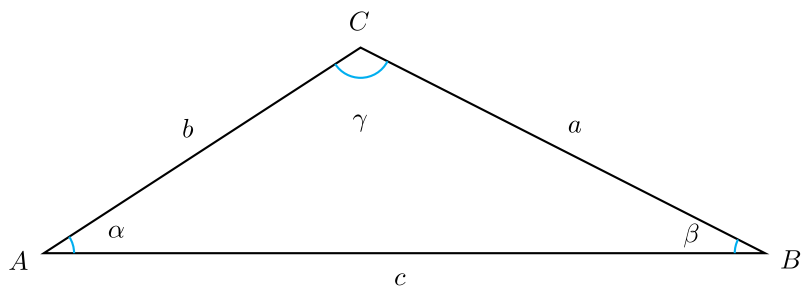


Aufgabe (36)

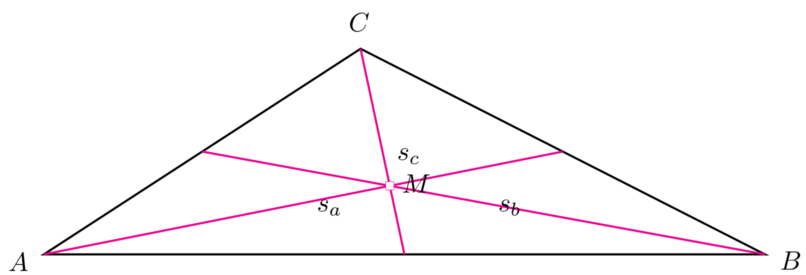
$Seite - Winkel - Seite$
$a = 6 \quad b = 5 \quad \gamma = 120°$

Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = \overline{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 120°}$
$c = 9,539$
Umfang:$U = a + b + c$
$U = 6 + 5 + 9,539$
$U = 20,539$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \frac{5^2 + 9,539^2 - 6^2}{2 \cdot 5 \cdot 9,539}$
$\cos\alpha = 0,839$
$\alpha = \arccos(0,839)$
$\alpha = 33,004°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 33,004° - 120°$
$\beta = 26,996°$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 9,539 \cdot \sin 26,996°$
$h_a = 4,33$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 4,33$
$A = 12,99$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 120°$
$h_b = 5,196$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 33,004°$
$h_c = 2,724$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

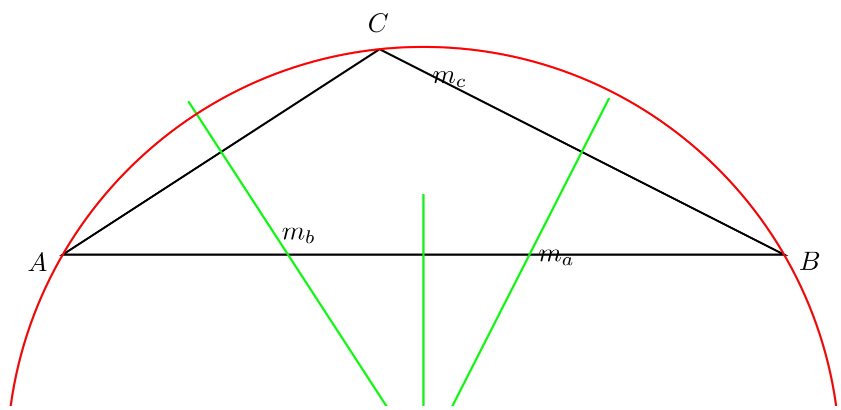
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9,539 \cdot \sin 26,996}{\sin 136,502}$
$wha = 6,291$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 120}{\sin 46,502}$
$whb = 7,163$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 33,004}{\sin 136,502}$
$whc = 4,748$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 9,539^2) - 6^2}$
$s_a = 7$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 9,539^2) - 5^2}$
$s_b = 7,566$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 9,539^2}$
$s_c = 4,924$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 33,004^\circ}$
$r_u = 5,508$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,99}{20,539}$
$r_i = 1,265$



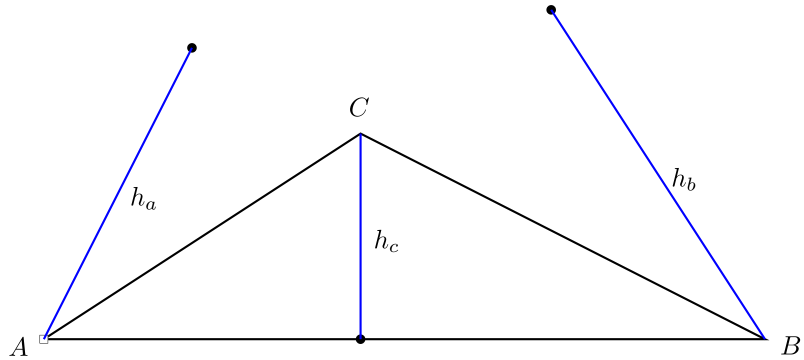
Seitenhalbierende-Schwerpunkt



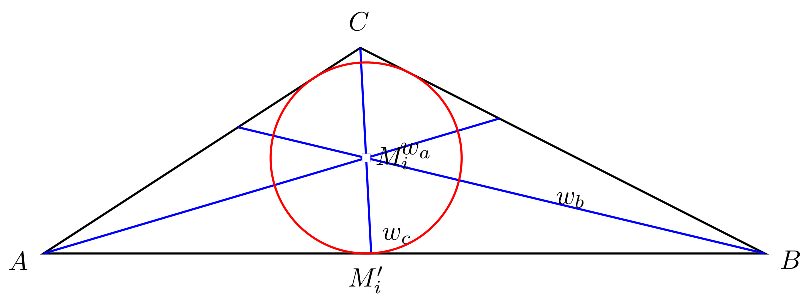
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

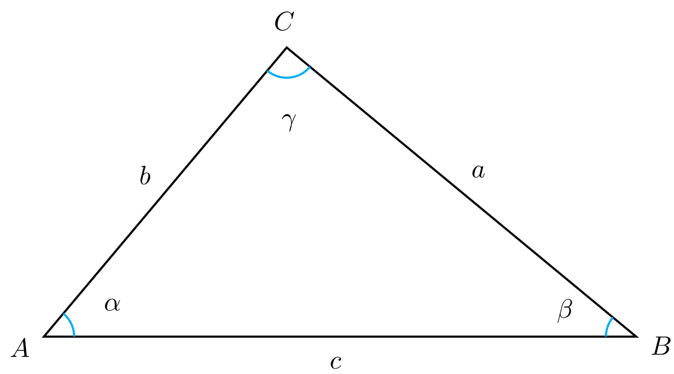


Aufgabe (37)

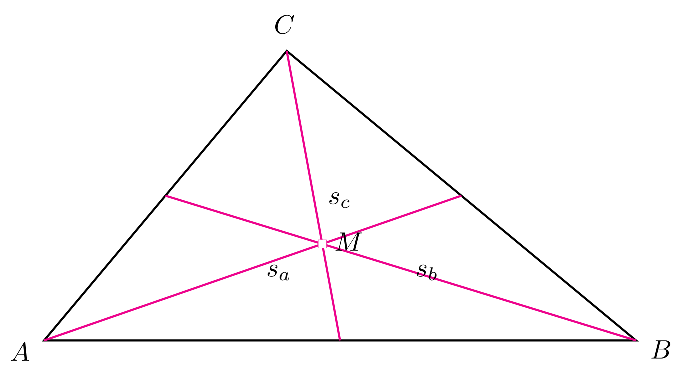
$Seite - Seite - Winkel$
$a = 6 \quad b = 5 \quad \alpha = 50^\circ$

Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : a$
$\sin\beta = \frac{b \cdot \sin\alpha}{a}$
$\sin\beta = \frac{5 \cdot \sin 50^\circ}{6}$
$\sin\beta = 0,638$
$\beta = \arcsin(0,638)$
$\beta = 39,67^\circ$
Winkelsumme: $\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180^\circ - \alpha - \beta$
$\gamma = 180^\circ - 50^\circ - 39,67^\circ$
$\gamma = 90,33^\circ$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 5^2 - 2 \cdot 6 \cdot 5 \cdot \cos 90,33^\circ}$
$c = 7,832$
Umfang: $U = a + b + c$
$U = 6 + 5 + 7,832$
$U = 18,832$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 7,832 \cdot \sin 39,67^\circ$
$h_a = 5$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 5$
$A = 15$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 90,33^\circ$
$h_b = 6$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 5 \cdot \sin 50^\circ$
$h_c = 3,83$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$

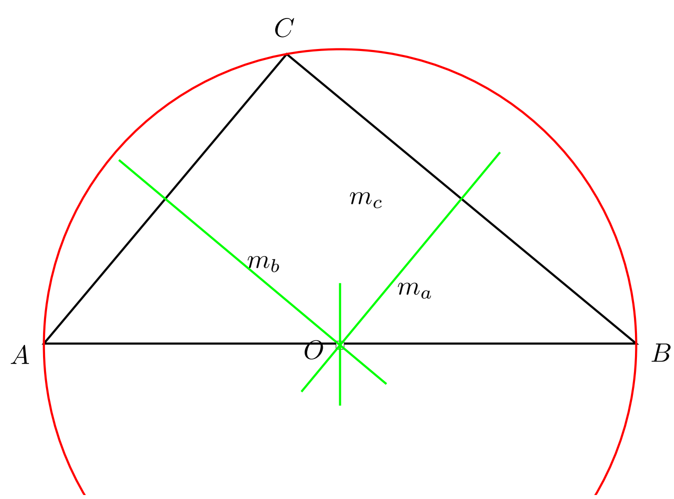
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{7,832 \cdot \sin 39,67}{\sin 115,33}$
$wha = 5,532$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 90,33}{\sin 69,835}$
$whb = 6,392$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{5 \cdot \sin 50}{\sin 115,33}$
$whc = 5,085$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(5^2 + 7,832^2) - 6^2}$
$s_a = 5,846$
Seitenhalbierende:$s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 7,832^2) - 5^2}$
$s_b = 6,513$
Seitenhalbierende:$s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 5^2) - 7,832^2}$
$s_c = 4,924$
Umkreisradius:$2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 50^\circ}$
$r_u = 3,916$
Innkreisradius:$r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 15}{18,832}$
$r_i = 1,593$



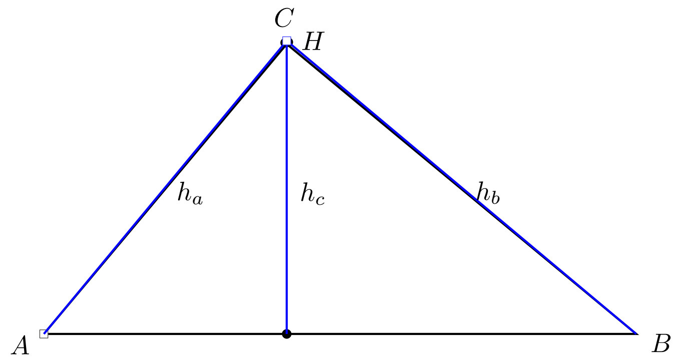
Seitenhalbierende-Schwerpunkt



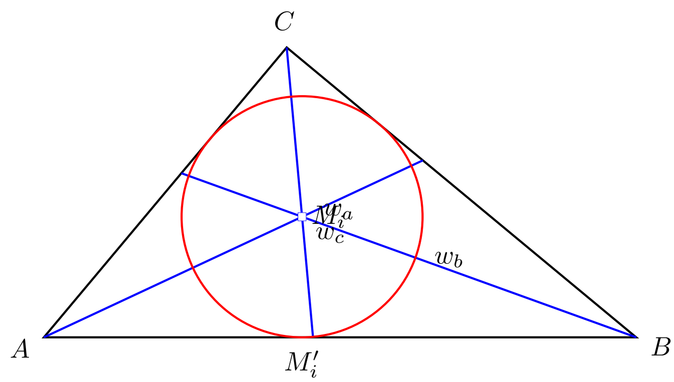
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (38)

$Seite - Seite - Winkel$
$a = 6 \quad b = 7 \quad \beta = 60°$

Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta \qquad / \cdot \sin\alpha$
$a \cdot \sin\beta = b \cdot \sin\alpha \qquad / : b$
$\sin\alpha = \frac{a \cdot \sin\beta}{b}$
$\sin\alpha = \frac{6 \cdot \sin 60°}{7}$
$\sin\alpha = 0,742$
$\alpha = \arcsin(0,742)$
$\alpha = 47,928°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 47,928° - 60°$
$\gamma = 72,072°$
Kosinus-Satz: $c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 7^2 - 2 \cdot 6 \cdot 7 \cdot \cos 72,072°}$
$c = 7,69$
Umfang: $U = a + b + c$
$U = 6 + 7 + 7,69$
$U = 20,69$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 7,69 \cdot \sin 60°$
$h_a = 6,66$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 6,66$
$A = 19,98$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 72,072°$
$h_b = 5,709$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 47,928°$
$h_c = 5,196$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$

Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{7,69 \cdot \sin 60}{\sin 96,036}$

$wha = 6,697$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{6 \cdot \sin 72,072}{\sin 77,928}$

$whb = 5,838$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{7 \cdot \sin 47,928}{\sin 96,036}$

$whc = 4,479$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(7^2 + 7,69^2) - 6^2}$

$s_a = 6,714$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(6^2 + 7,69^2) - 7^2}$

$s_b = 5,943$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(6^2 + 7^2) - 7,69^2}$

$s_c = 5\frac{1}{2}$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

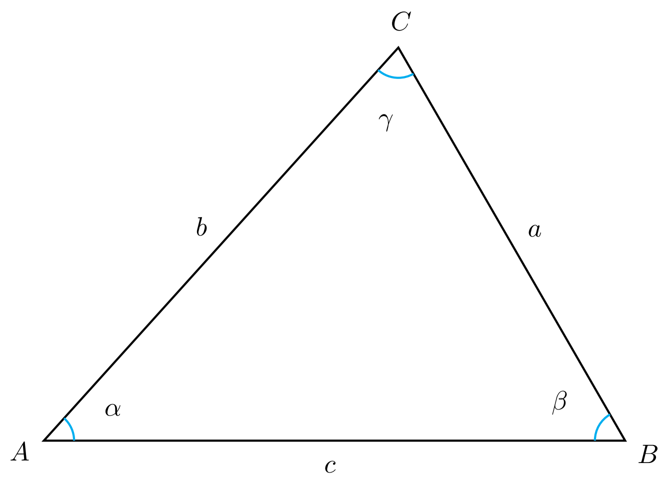
$r_u = \frac{6}{2 \cdot \sin 47,928°}$

$r_u = 4,041$

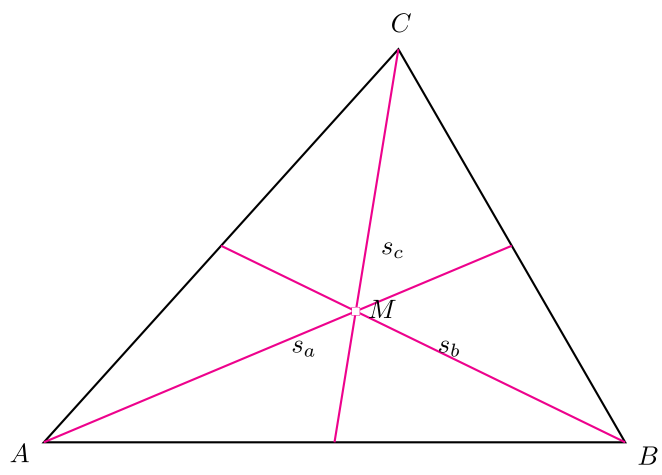
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 19,98}{20,69}$

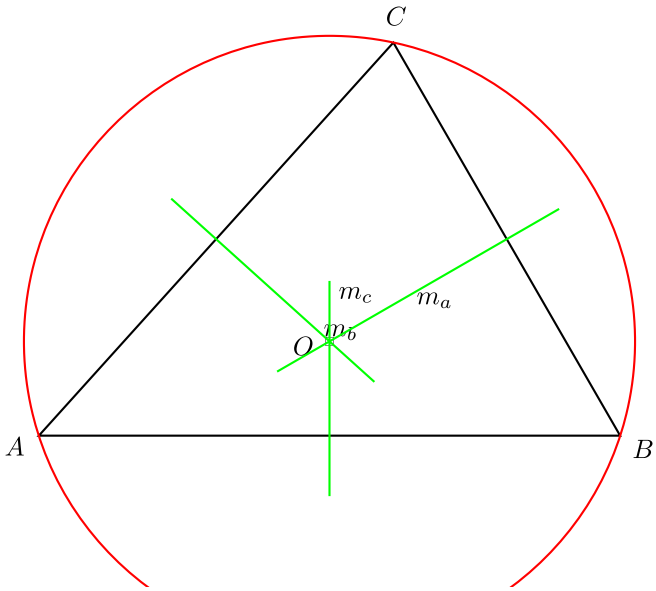
$r_i = 1,931$



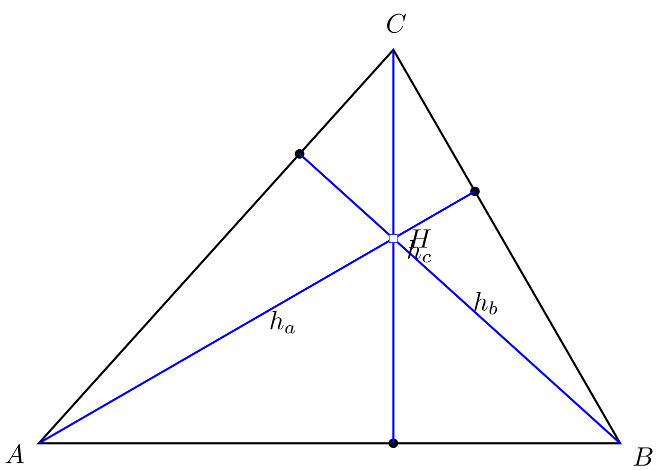
Seitenhalbierende-Schwerpunkt



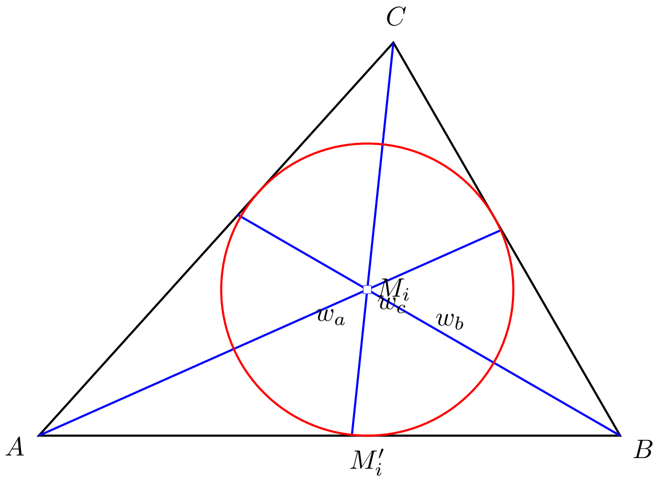
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

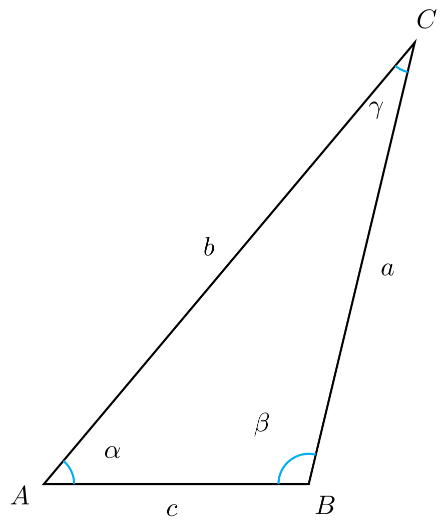

C
M_i
w_a
w_c
w_b
A
M'_i
B

Aufgabe (39)

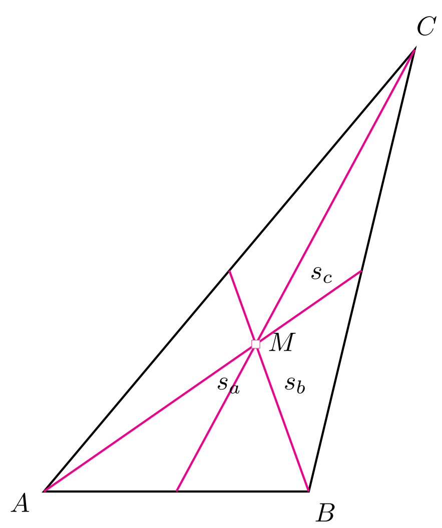
$Seite - Seite - Winkel$
$a = 6 \quad c = 3\frac{1}{2} \quad \alpha = 50°$

Sinus-Satz: $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$
$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \quad / \cdot \sin\gamma \quad / \cdot \sin\alpha$
$a \cdot \sin\gamma = c \cdot \sin\alpha \quad / : a$
$\sin\gamma = \frac{c \cdot \sin\alpha}{a}$
$\sin\gamma = \frac{3\frac{1}{2} \cdot \sin 50°}{6}$
$\sin\gamma = 0,447$
$\gamma = \arcsin(0,447)$
$\gamma = 26,542°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \quad / - \alpha \quad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 50° - 26,542°$
$\beta = 103,458°$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{6^2 + 3\frac{1}{2}^2 - 2 \cdot 6 \cdot 3\frac{1}{2} \cdot \cos 103,458°}$
$b = 7,617$
Umfang: $U = a + b + c$
$U = 6 + 7,617 + 3\frac{1}{2}$
$U = 17,117$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3\frac{1}{2} \cdot \sin 103,458°$
$h_a = 3,404$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 3,404$
$A = 10,212$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 26,542°$
$h_b = 2,681$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,617 \cdot \sin 50°$
$h_c = 5,835$
Winkelhalbierende α

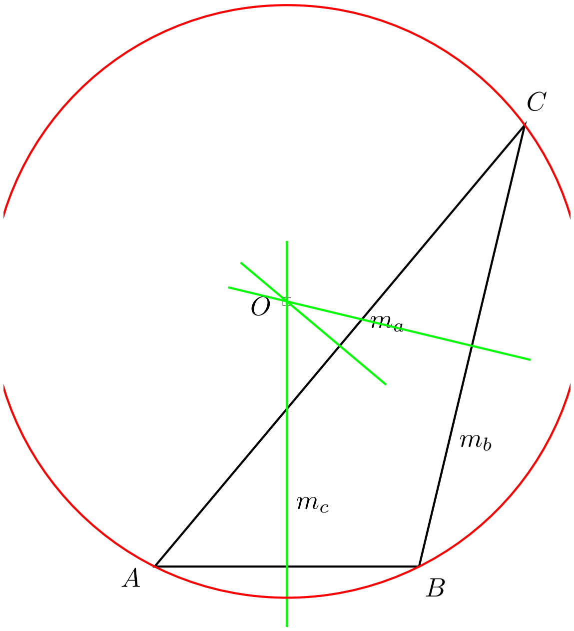
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{3\frac{1}{2} \cdot \sin 103,458}{\sin 51,542}$
$wha = 4,347$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 26,542}{\sin 101,729}$
$whb = 2,738$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,617 \cdot \sin 50}{\sin 51,542}$
$whc = 5,87$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,617^2 + 3\frac{1}{2}^2) - 6^2}$
$s_a = 5,112$
Seitenhalbierende:$s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 3\frac{1}{2}^2) - 7,617^2}$
$s_b = 3,101$
Seitenhalbierende:$s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 7,617^2) - 3\frac{1}{2}^2}$
$s_c = 5,701$
Umkreisradius:$2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 50^\circ}$
$r_u = 3,916$
Innkreisradius:$r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 10,212}{17,117}$
$r_i = 1,193$



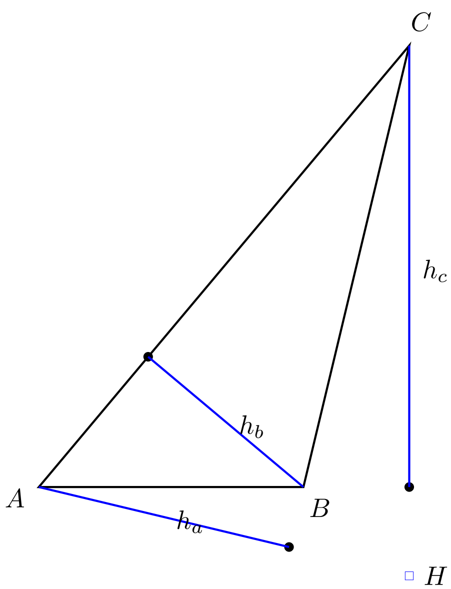
Seitenhalbierende-Schwerpunkt



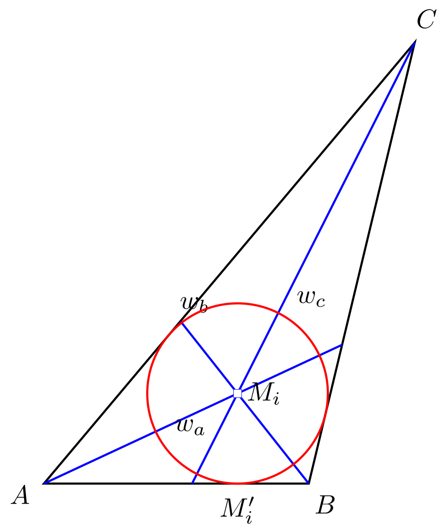
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

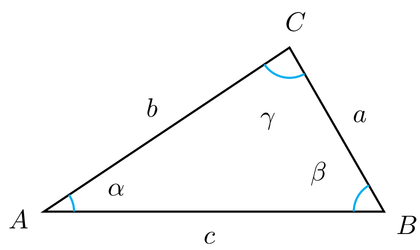

C
w_b
w_c
M_i
w_a
A
M_i'
B

Aufgabe (40)

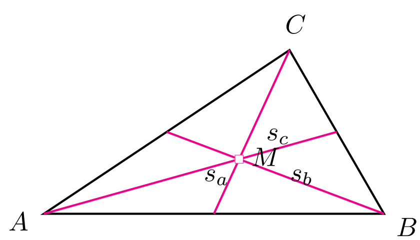
$Seite - Winkel - Seite$
$a = 2\frac{1}{2} \quad c = 4\frac{1}{2} \quad \beta = 60°$

Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{2\frac{1}{2}^2 + 4\frac{1}{2}^2 - 2 \cdot 2\frac{1}{2} \cdot 4\frac{1}{2} \cdot \cos 60°}$
$b = 3,905$
Umfang:$U = a + b + c$
$U = 2\frac{1}{2} + 3,905 + 4\frac{1}{2}$
$U = 10,905$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha \qquad / - a^2 \qquad / + 2 \cdot b \cdot c \cdot \cos\alpha$
$2 \cdot b \cdot c \cdot \cos\alpha = b^2 + c^2 - a^2 \qquad / : (2 \cdot b \cdot c)$
$\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$
$\cos\alpha = \dfrac{3,905^2 + 4\frac{1}{2}^2 - 2\frac{1}{2}^2}{2 \cdot 3,905 \cdot 4\frac{1}{2}}$
$\cos\alpha = 0,832$
$\alpha = \arccos(0,832)$
$\alpha = 33,67°$
Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 33,67° - 60°$
$\gamma = 86,33°$
Höhe h_a
$\sin\beta = \dfrac{h_a}{c}$
$\sin\beta = \dfrac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4\dfrac{1}{2} \cdot \sin 60°$
$h_a = 3,897$
Flaeche: $A = \dfrac{1}{2} \cdot a \cdot h_a$
$A = \dfrac{1}{2} \cdot 2\dfrac{1}{2} \cdot 3,897$
$A = 4,871$
Höhe h_b
$\sin\gamma = \dfrac{h_b}{a}$
$\sin\gamma = \dfrac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 2\dfrac{1}{2} \cdot \sin 86,33°$
$h_b = 2,495$
Höhe h_c
$\sin\alpha = \dfrac{h_c}{b}$
$\sin\alpha = \dfrac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3,905 \cdot \sin 33,67°$
$h_c = 2,165$
Winkelhalbierende α
$\delta = 180 - \beta - \dfrac{\alpha}{2}$

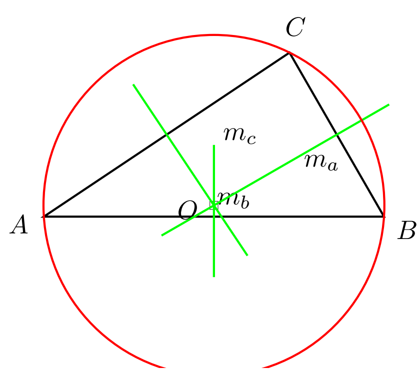
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4\frac{1}{2} \cdot \sin 60}{\sin 103,165}$
$wha = 4,002$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{2\frac{1}{2} \cdot \sin 86,33}{\sin 63,67}$
$whb = 2,784$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3,905 \cdot \sin 33,67}{\sin 103,165}$
$whc = 1,423$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3,905^2 + 4\frac{1}{2}^2) - 2\frac{1}{2}^2}$
$s_a = 4,023$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(2\frac{1}{2}^2 + 4\frac{1}{2}^2) - 3,905^2}$
$s_b = 3,072$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(2\frac{1}{2}^2 + 3,905^2) - 4\frac{1}{2}^2}$
$s_c = 2,634$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{2\frac{1}{2}}{2 \cdot \sin 33,67^\circ}$
$r_u = 2,255$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 4,871}{10,905}$
$r_i = 0,893$



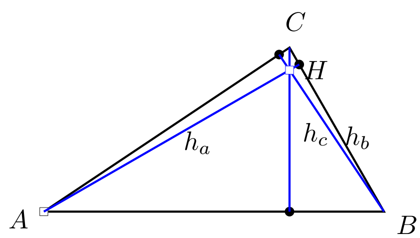
Seitenhalbierende-Schwerpunkt



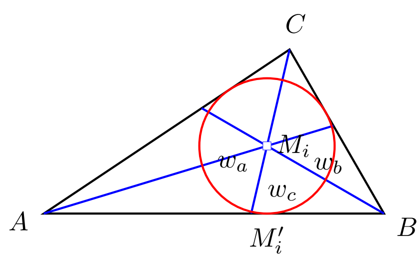
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

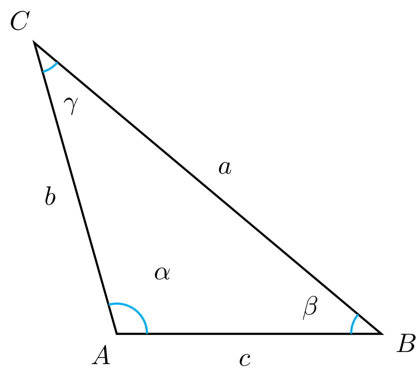


Aufgabe (41)

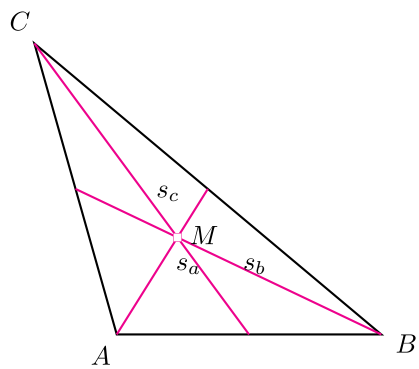
$Seite - Seite - Winkel$
$b = 4 \quad c = 3\frac{1}{2} \quad \beta = 40^\circ$

Sinus-Satz: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\beta \qquad / \cdot \sin\gamma$
$b \cdot \sin\gamma = c \cdot \sin\beta \qquad / : b$
$\sin\gamma = \frac{c \cdot \sin\beta}{b}$
$\sin\gamma = \frac{3\frac{1}{2} \cdot \sin 40^\circ}{4}$
$\sin\gamma = 0,562$
$\gamma = \arcsin(0,562)$
$\gamma = 34,225^\circ$
Winkelsumme: $\alpha + \beta + \gamma = 180^\circ$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180^\circ - \beta - \gamma$
$\alpha = 180^\circ - 40^\circ - 34,225^\circ$
$\alpha = 105,775^\circ$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{4^2 + 3\frac{1}{2}^2 - 2 \cdot 4 \cdot 3\frac{1}{2} \cdot \cos 105,775^\circ}$
$a = 5,989$
Umfang: $U = a + b + c$
$U = 5,989 + 4 + 3\frac{1}{2}$
$U = 13,489$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \qquad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 3\frac{1}{2} \cdot \sin 40^\circ$
$h_a = 2,25$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 5,989 \cdot 2,25$
$A = 6,736$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \qquad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 5,989 \cdot \sin 34,225^\circ$
$h_b = 3,368$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \qquad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 4 \cdot \sin 105,775^\circ$
$h_c = 3,849$

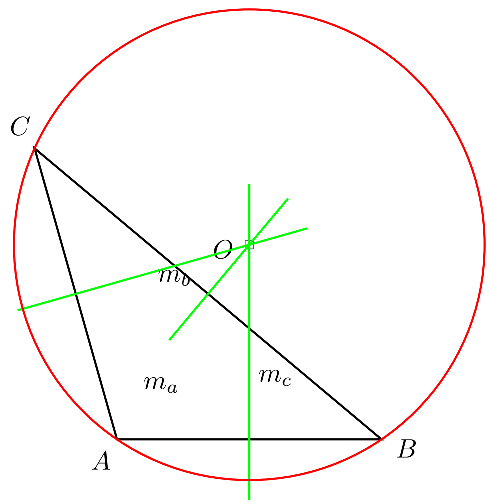
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{3\frac{1}{2} \cdot \sin 40}{\sin 87,112}$
$wha = 2,253$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{5,989 \cdot \sin 34,225}{\sin 125,775}$
$whb = 4,151$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{4 \cdot \sin 105,775}{\sin 87,112}$
$whc = 5,77$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(4^2 + 3\frac{1}{2}^2) - 5,989^2}$
$s_a = 2,271$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(5,989^2 + 3\frac{1}{2}^2) - 4^2}$
$s_b = 4,478$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(5,989^2 + 4^2) - 3\frac{1}{2}^2}$
$s_c = 4,683$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{5,989}{2 \cdot \sin 105,775°}$
$r_u = 3,111$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 6,736}{13,489}$
$r_i = 0,999$



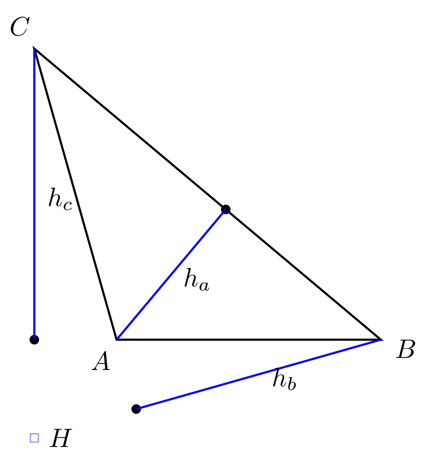
Seitenhalbierende-Schwerpunkt



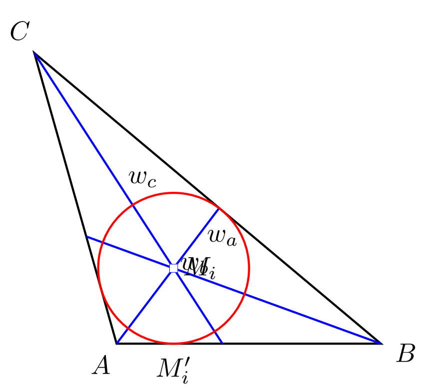
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

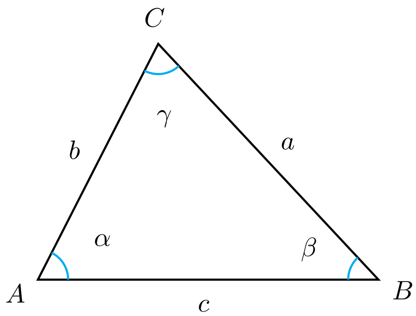


Aufgabe (42)

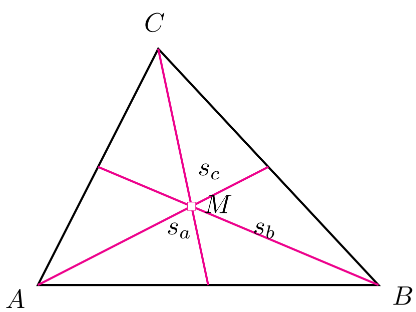
$Seite - Seite - Winkel$
$b = 3\frac{1}{2} \quad c = 4\frac{1}{2} \quad \gamma = 70°$

Sinus-Satz: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\beta \qquad / \cdot \sin\gamma$
$b \cdot \sin\gamma = c \cdot \sin\beta \qquad / : c$
$\sin\beta = \frac{b \cdot \sin\gamma}{c}$
$\sin\beta = \frac{3\frac{1}{2} \cdot \sin}{4\frac{1}{2}}$
$\sin\beta = 0,731$
$\beta = \arcsin(0,731)$
$\beta = 46,96°$
Winkelsumme: $\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 46,96° - 70°$
$\alpha = 63,04°$
Kosinus-Satz: $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$
$a = \sqrt{b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha}$
$a = \sqrt{3\frac{1}{2}^2 + 4\frac{1}{2}^2 - 2 \cdot 3\frac{1}{2} \cdot 4\frac{1}{2} \cdot \cos 63,04°}$
$a = 4,268$
Umfang: $U = a + b + c$
$U = 4,268 + 3\frac{1}{2} + 4\frac{1}{2}$
$U = 12,268$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 4\frac{1}{2} \cdot \sin 46,96°$
$h_a = 3,289$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,268 \cdot 3,289$
$A = 7,019$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,268 \cdot \sin 70°$
$h_b = 4,011$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 3\frac{1}{2} \cdot \sin 63,04°$
$h_c = 3,12$

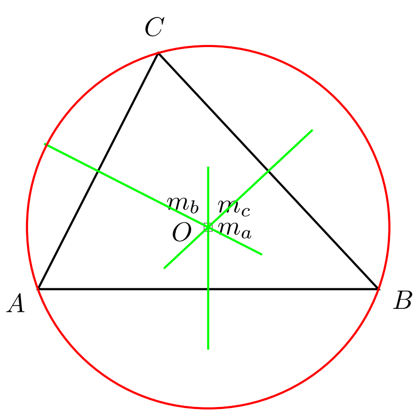
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz: $\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{4\frac{1}{2} \cdot \sin 46,96}{\sin 101,52}$
$wha = 3,357$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{4,268 \cdot \sin 70}{\sin 86,52}$
$whb = 4,018$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{3\frac{1}{2} \cdot \sin 63,04}{\sin 101,52}$
$whc = 3,883$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(3\frac{1}{2}^2 + 4\frac{1}{2}^2) - 4,268^2}$
$s_a = 3,42$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(4,268^2 + 4\frac{1}{2}^2) - 3\frac{1}{2}^2}$
$s_b = 4,021$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(4,268^2 + 3\frac{1}{2}^2) - 4\frac{1}{2}^2}$
$s_c = 3,489$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{4,268}{2 \cdot \sin 63,04^\circ}$
$r_u = 2,394$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 7,019}{12,268}$
$r_i = 1,144$



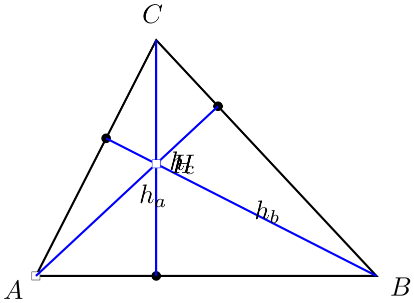
Seitenhalbierende-Schwerpunkt



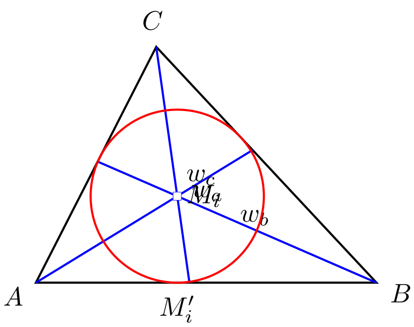
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis



Aufgabe (43)

$Winkel - Winkel - Seite$
$a = 6 \quad \alpha = 30° \quad \beta = 50°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 50°$
$\gamma = 100°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta$
$b = \frac{a \cdot \sin\beta}{\sin\alpha}$
$b = \frac{6 \cdot \sin 50}{\sin 30}$
$b = 9,193$
Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 9,193^2 - 2 \cdot 6 \cdot 9,193 \cdot \cos 100°}$
$c = 11,818$
Umfang:$U = a + b + c$
$U = 6 + 9,193 + 11,818$
$U = 27,01$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 11,818 \cdot \sin 50°$
$h_a = 9,053$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 9,053$
$A = 27,159$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 100°$
$h_b = 5,909$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 9,193 \cdot \sin 30°$
$h_c = 4,596$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{11,818 \cdot \sin 50}{\sin 115}$

$wha = 9,989$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{6 \cdot \sin 100}{\sin 55}$

$whb = 7,213$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{9,193 \cdot \sin 30}{\sin 115}$

$whc = 3,31$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(9,193^2 + 11,818^2) - 6^2}$

$s_a = 10,153$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(6^2 + 11,818^2) - 9,193^2}$

$s_b = 8,167$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(6^2 + 9,193^2) - 11,818^2}$

$s_c = 6,255$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

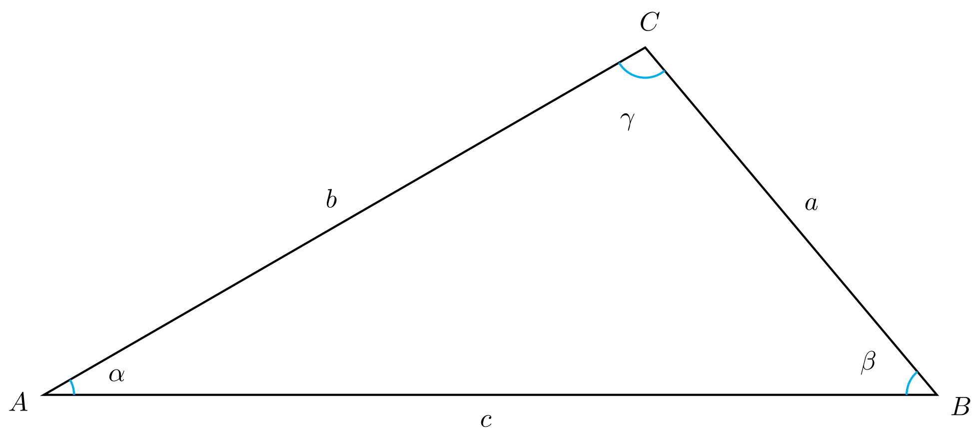
$r_u = \frac{6}{2 \cdot \sin 30^\circ}$

$r_u = 6$

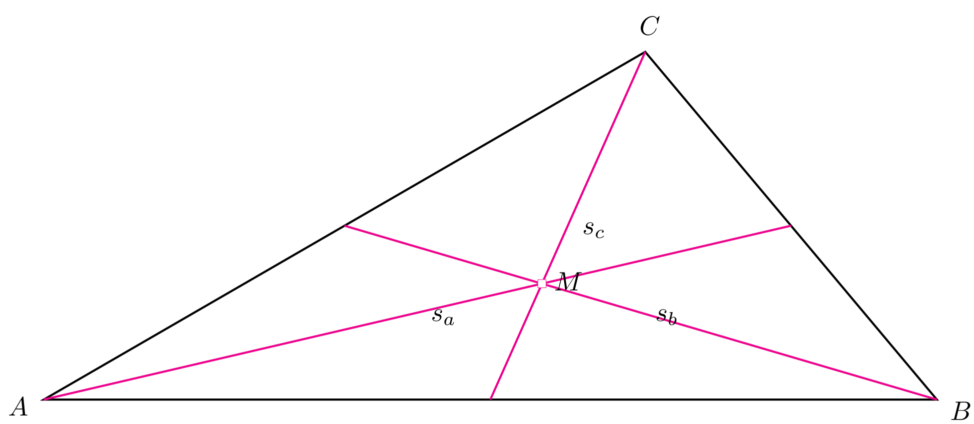
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 27,159}{27,01}$

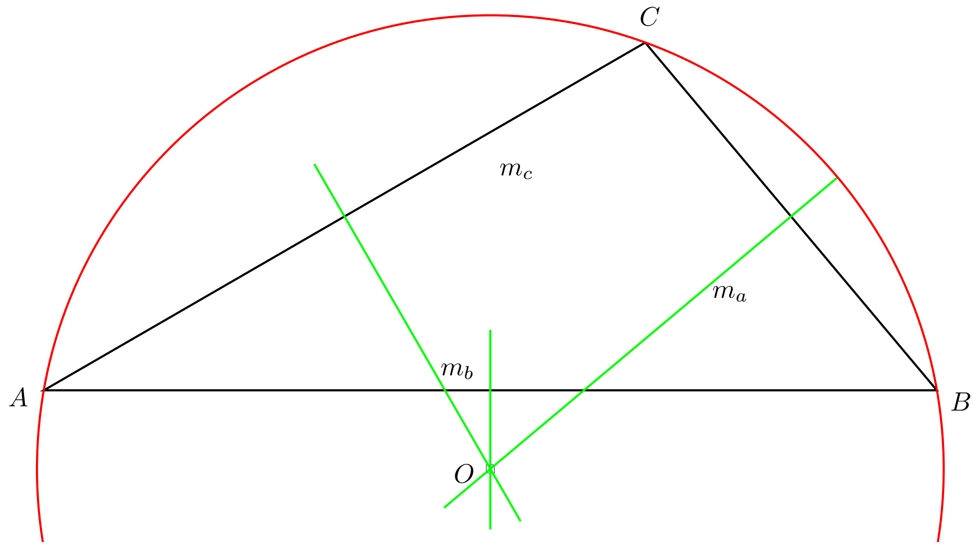
$r_i = 2\frac{1}{91}$



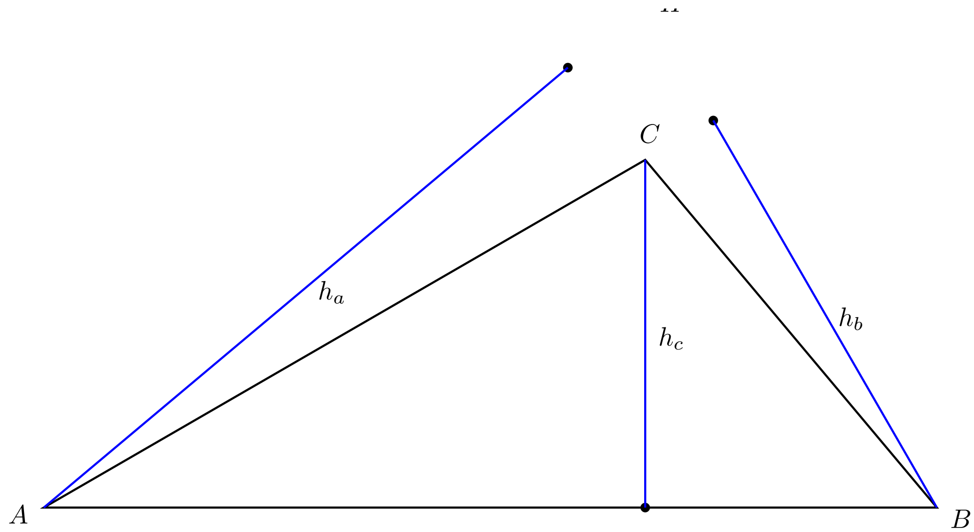
Seitenhalbierende-Schwerpunkt



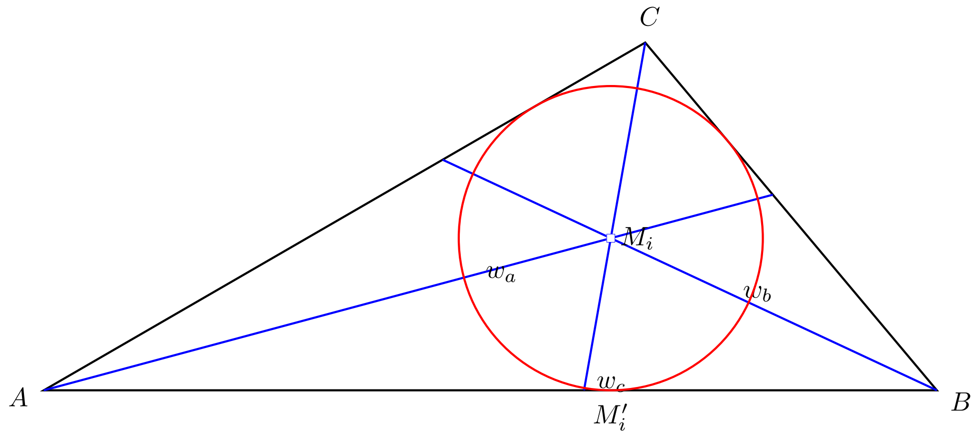
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

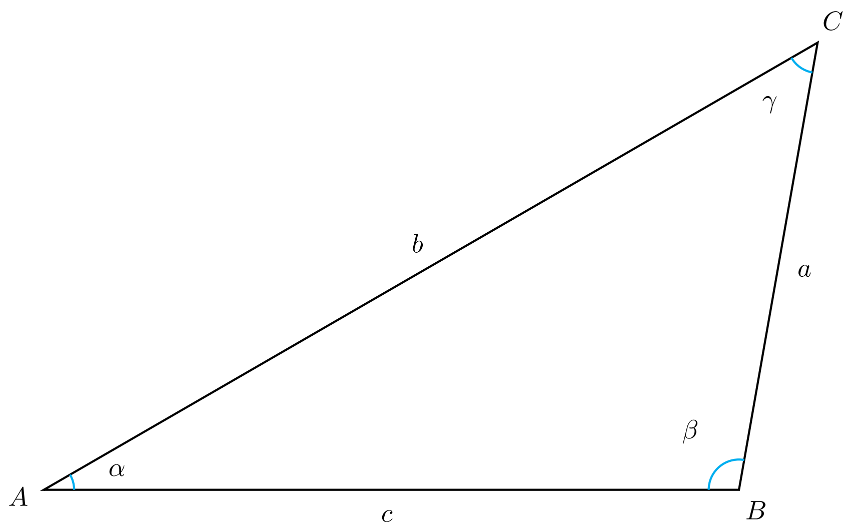

C
M_i
w_a
w_b
w_c
A
B
M_i'

Aufgabe (44)

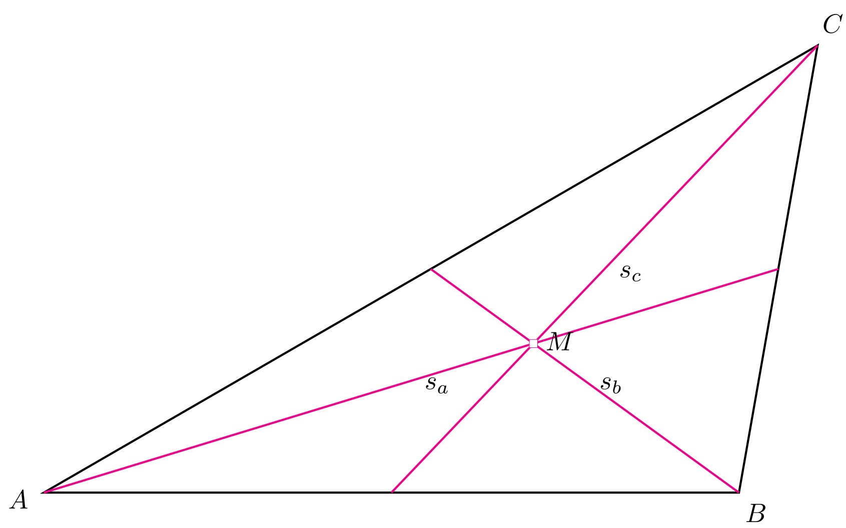
$Winkel - Winkel - Seite$
$a = 6 \quad \alpha = 30° \quad \gamma = 50°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 30° - 50°$
$\beta = 100°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\beta$
$b = \frac{a \cdot \sin\beta}{\sin\alpha}$
$b = \frac{6 \cdot \sin 100}{\sin 30}$
$b = 11,818$
Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{6^2 + 11,818^2 - 2 \cdot 6 \cdot 11,818 \cdot \cos 50°}$
$c = 9,193$
Umfang:$U = a + b + c$
$U = 6 + 11,818 + 9,193$
$U = 27,01$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 9,193 \cdot \sin 100°$
$h_a = 9,053$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 6 \cdot 9,053$
$A = 27,159$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 6 \cdot \sin 50°$
$h_b = 4,596$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 11,818 \cdot \sin 30°$
$h_c = 5,909$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

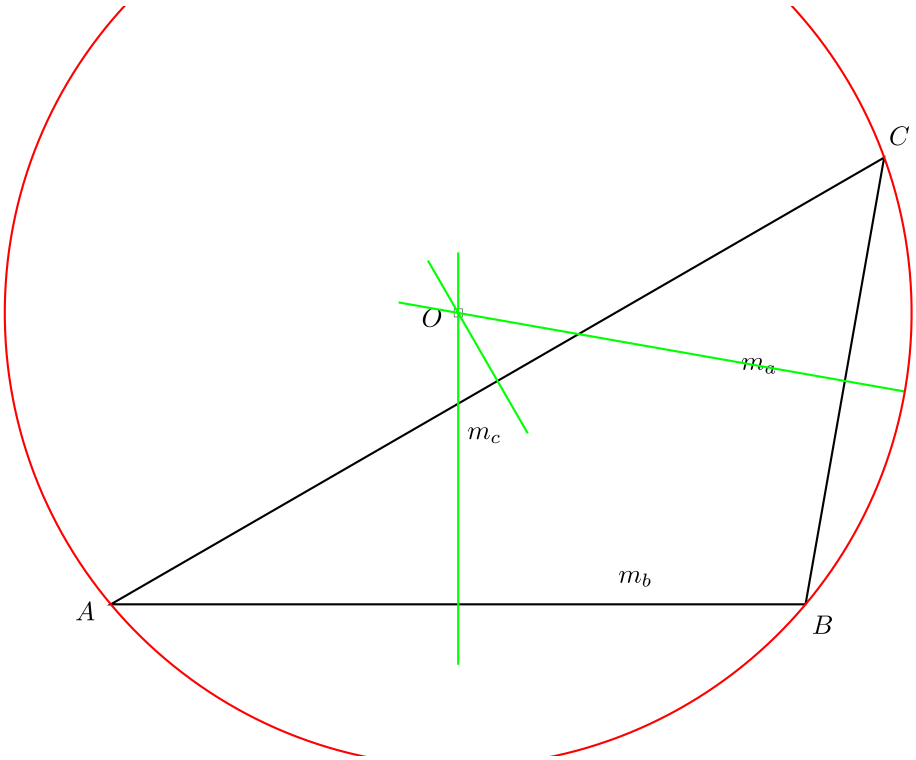
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{9,193 \cdot \sin 100}{\sin 65}$
$wha = 9,989$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz:$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{6 \cdot \sin 50}{\sin 80}$
$whb = 4,667$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz:$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{11,818 \cdot \sin 30}{\sin 65}$
$whc = 3,31$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(11,818^2 + 9,193^2) - 6^2}$
$s_a = 10,153$
Seitenhalbierende:$s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(6^2 + 9,193^2) - 11,818^2}$
$s_b = 5,034$
Seitenhalbierende:$s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(6^2 + 11,818^2) - 9,193^2}$
$s_c = 7,274$
Umkreisradius:$2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{6}{2 \cdot \sin 30^\circ}$
$r_u = 6$
Innkreisradius:$r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 27,159}{27,01}$
$r_i = 2\frac{1}{91}$



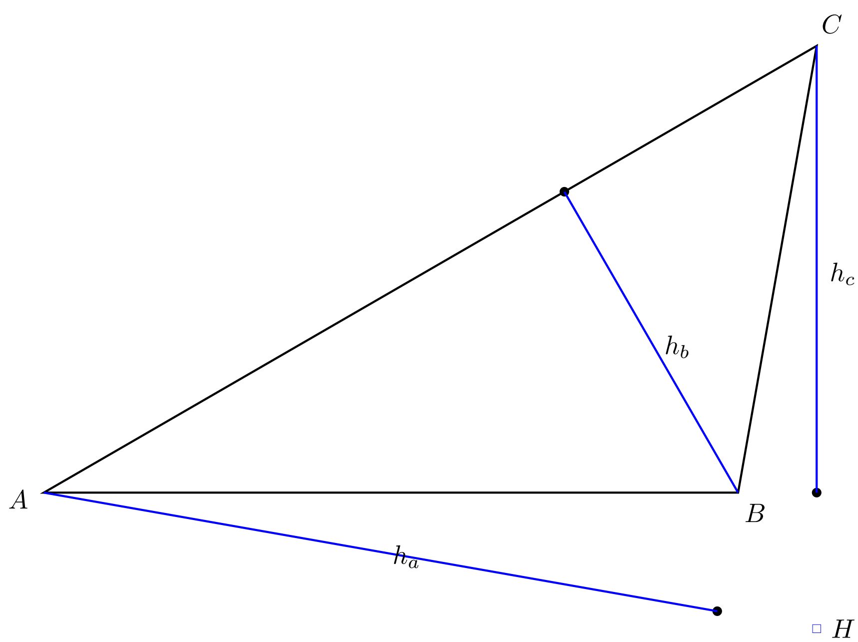
Seitenhalbierende-Schwerpunkt



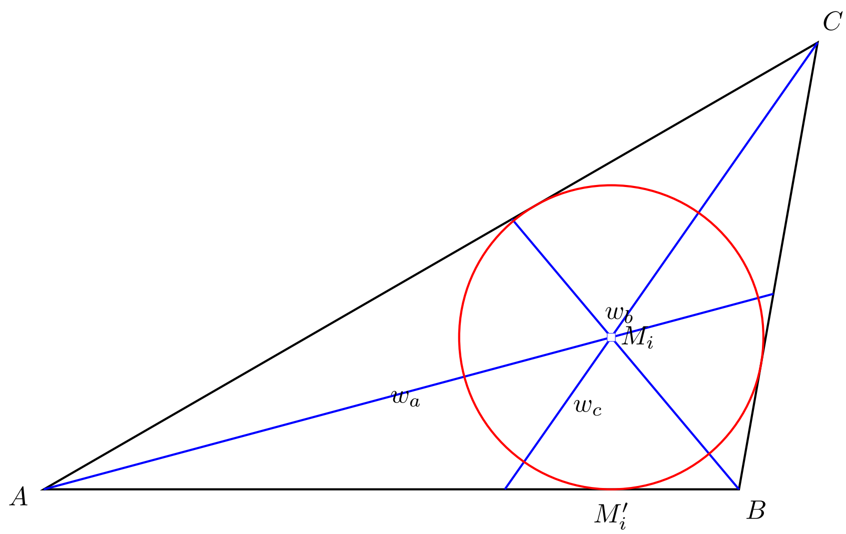
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_b
M_i
w_a
w_c
A
M_i'
B

Aufgabe (45)

$Winkel - Winkel - Seite$
$b = 7 \quad \alpha = 30° \quad \beta = 50°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \beta$
$\gamma = 180° - \alpha - \beta$
$\gamma = 180° - 30° - 50°$
$\gamma = 100°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{7 \cdot \sin 30}{\sin 50}$
$a = 4,569$
Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{4,569^2 + 7^2 - 2 \cdot 4,569 \cdot 7 \cdot \cos 100°}$
$c = 8,999$
Umfang:$U = a + b + c$
$U = 4,569 + 7 + 8,999$
$U = 20,568$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,999 \cdot \sin 50°$
$h_a = 6,894$
Flaeche: $\quad A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 4,569 \cdot 6,894$
$A = 15,748$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 4,569 \cdot \sin 100°$
$h_b = 4,5$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 30°$
$h_c = 3\frac{1}{2}$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$

$wha = \frac{c \cdot \sin\beta}{\sin\delta}$

$wha = \frac{8,999 \cdot \sin 50}{\sin 115}$

$wha = 7,606$

Winkelhalbierende β

$\delta = 180 - \frac{\beta}{2} - \gamma$

Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$

$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$

$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$

$whb = \frac{4,569 \cdot \sin 100}{\sin 55}$

$whb = 5,493$

Winkelhalbierende γ

$\delta = 180 - \alpha - \frac{\gamma}{2}$

Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$

$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$

$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$

$whc = \frac{7 \cdot \sin 30}{\sin 115}$

$whc = 2,521$

Seitenhalbierende:

$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

$s_a = \frac{1}{2}\sqrt{2(7^2 + 8,999^2) - 4,569^2}$

$s_a = 7,731$

Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$

$s_b = \frac{1}{2}\sqrt{2(4,569^2 + 8,999^2) - 7^2}$

$s_b = 6,219$

Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$

$s_c = \frac{1}{2}\sqrt{2(4,569^2 + 7^2) - 8,999^2}$

$s_c = 4,763$

Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$

$r_u = \frac{a}{2 \cdot \sin\alpha}$

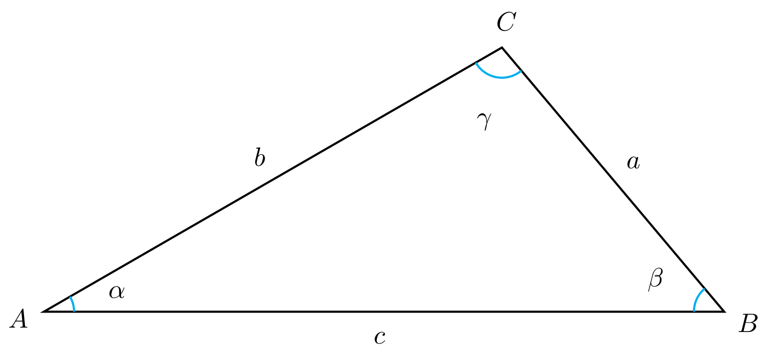
$r_u = \frac{4,569}{2 \cdot \sin 30^\circ}$

$r_u = 4,569$

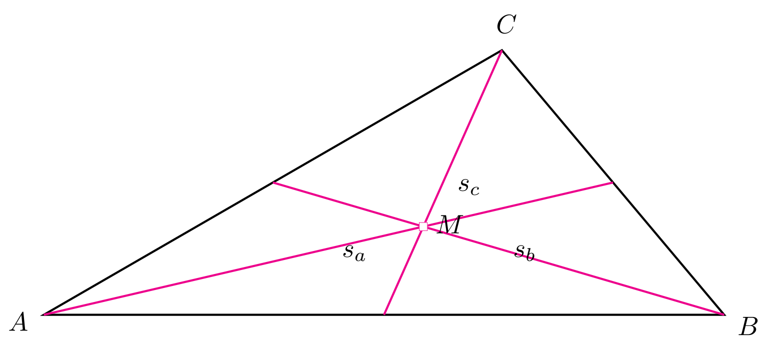
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$

$r_i = \frac{2 \cdot 15,748}{20,568}$

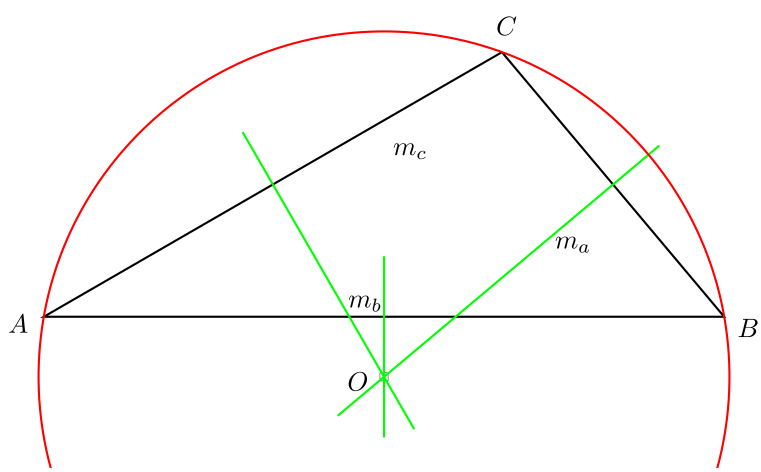
$r_i = 1,531$



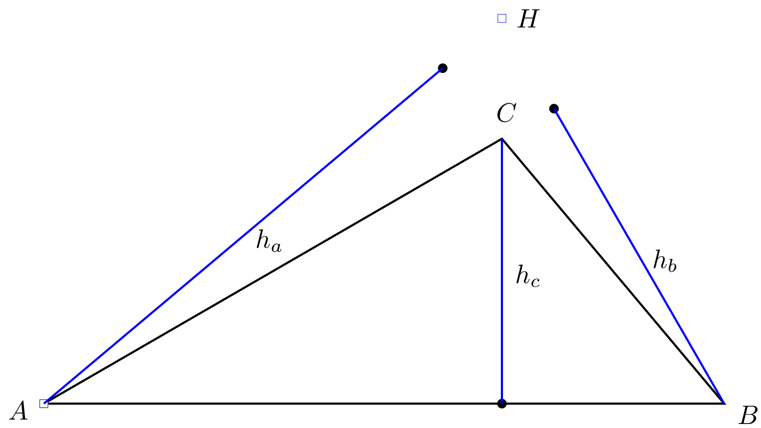
Seitenhalbierende-Schwerpunkt



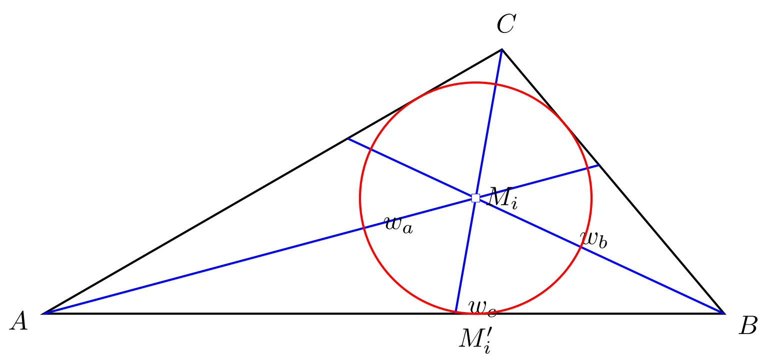
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

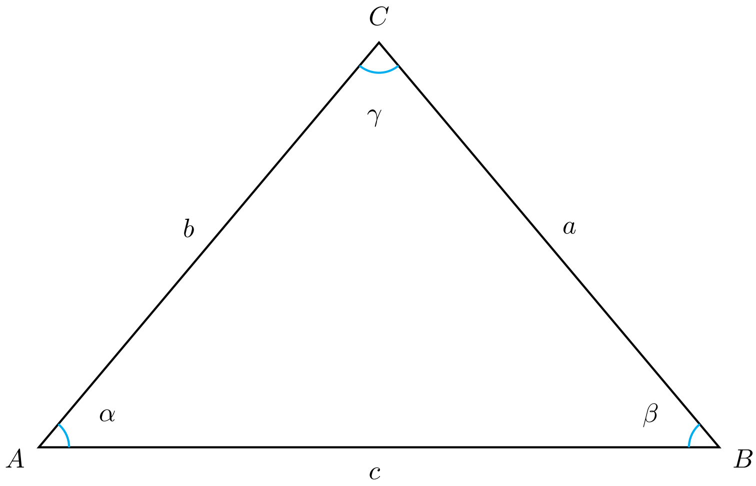


Aufgabe (46)

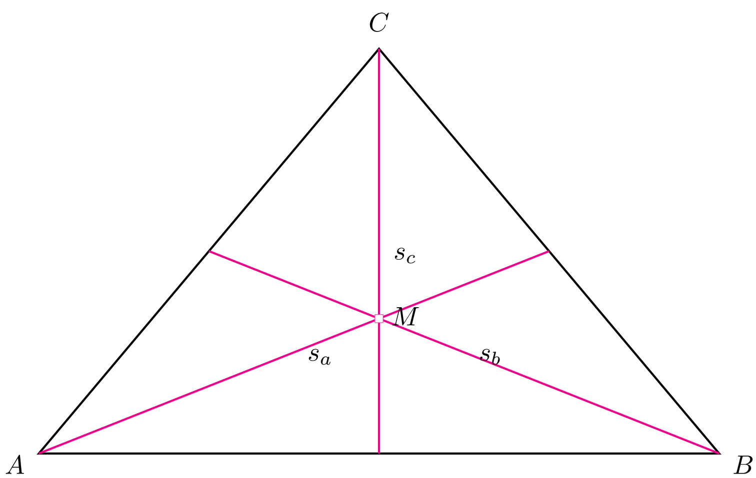
$Winkel - Winkel - Seite$
$b = 7 \quad \gamma = 80° \quad \beta = 50°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 50° - 80°$
$\alpha = 50°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \qquad / \cdot \sin\alpha$
$a = \frac{b \cdot \sin\alpha}{\sin\beta}$
$a = \frac{7 \cdot \sin 50}{\sin 50}$
$a = 7$
Kosinus-Satz:$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma$
$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos\gamma}$
$c = \sqrt{7^2 + 7^2 - 2 \cdot 7 \cdot 7 \cdot \cos 80°}$
$c = 8,999$
Umfang:$U = a + b + c$
$U = 7 + 7 + 8,999$
$U = 22,999$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 8,999 \cdot \sin 50°$
$h_a = 6,894$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 7 \cdot 6,894$
$A = 24,128$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 7 \cdot \sin 80°$
$h_b = 6,894$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7 \cdot \sin 50°$
$h_c = 5,362$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

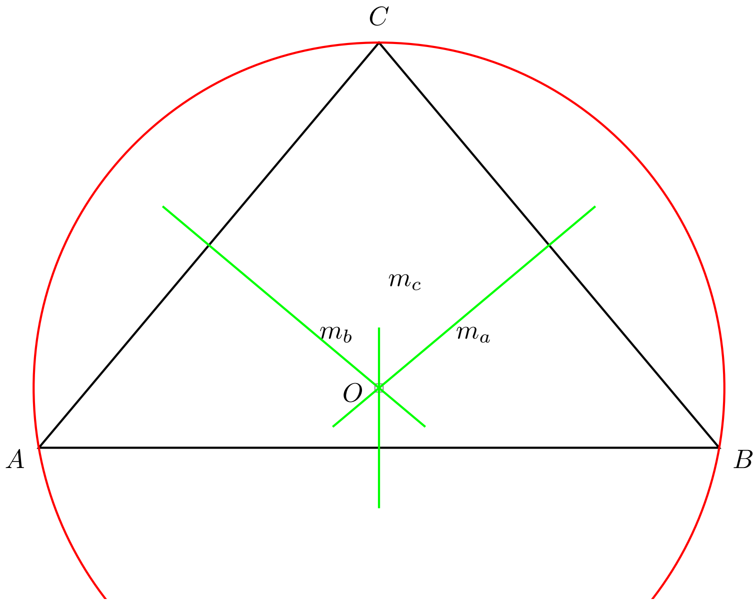
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{8,999 \cdot \sin 50}{\sin 105}$
$wha = 7,137$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{7 \cdot \sin 80}{\sin 75}$
$whb = 7,137$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7 \cdot \sin 50}{\sin 105}$
$whc = 5,551$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7^2 + 8,999^2) - 7^2}$
$s_a = 7,262$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(7^2 + 8,999^2) - 7^2}$
$s_b = 7,262$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(7^2 + 7^2) - 8,999^2}$
$s_c = 6,062$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{7}{2 \cdot \sin 50^\circ}$
$r_u = 4,569$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 24,128}{22,999}$
$r_i = 2,098$



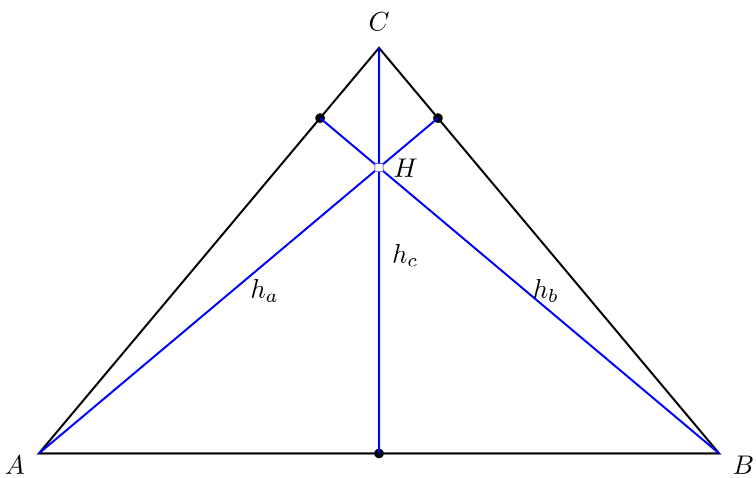
Seitenhalbierende-Schwerpunkt



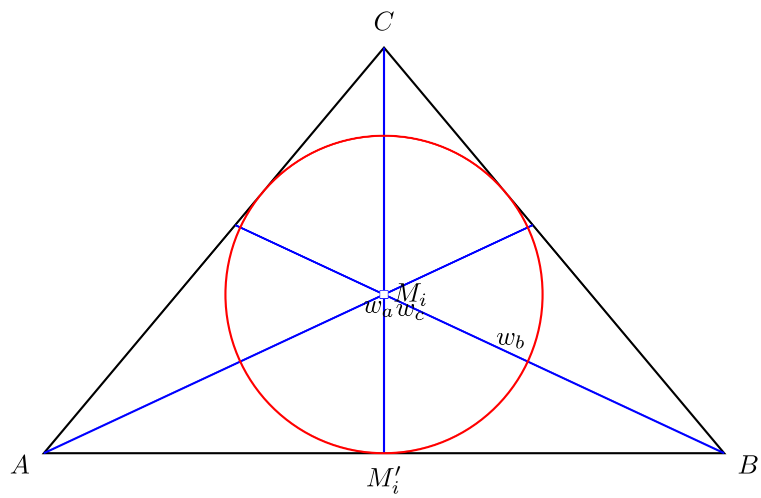
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

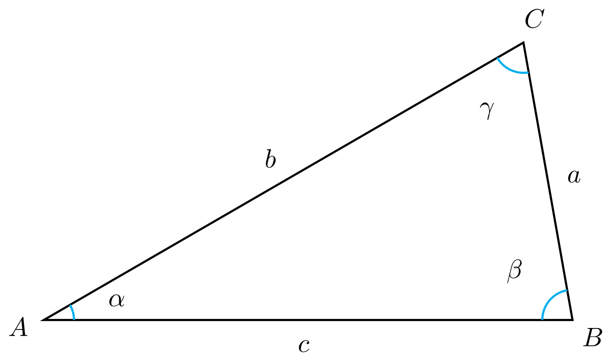

C
M_i
w_a
w_c
w_b
A
B
M_i'

Aufgabe (47)

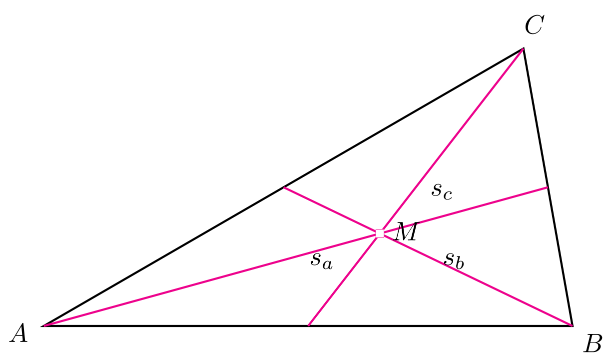
$Winkel - Winkel - Seite$
$c = 7 \quad \gamma = 70° \quad \alpha = 30°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\beta = 180° - \alpha - \gamma$
$\beta = 180° - 30° - 70°$
$\beta = 80°$
Sinus-Satz:$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$
$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad / \cdot \sin\alpha$
$a = \frac{c \cdot \sin\alpha}{\sin\gamma}$
$a = \frac{7 \cdot \sin 30}{\sin 70}$
$a = 3,725$
Kosinus-Satz:$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\beta$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$
$b = \sqrt{a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta}$
$b = \sqrt{3,725^2 + 7^2 - 2 \cdot 3,725 \cdot 7 \cdot \cos 80°}$
$b = 7,336$
Umfang:$U = a + b + c$
$U = 3,725 + 7,336 + 7$
$U = 18,061$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 7 \cdot \sin 80°$
$h_a = 6,894$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 3,725 \cdot 6,894$
$A = 12,838$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 3,725 \cdot \sin 70°$
$h_b = 3\frac{1}{2}$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,336 \cdot \sin 30°$
$h_c = 3,668$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

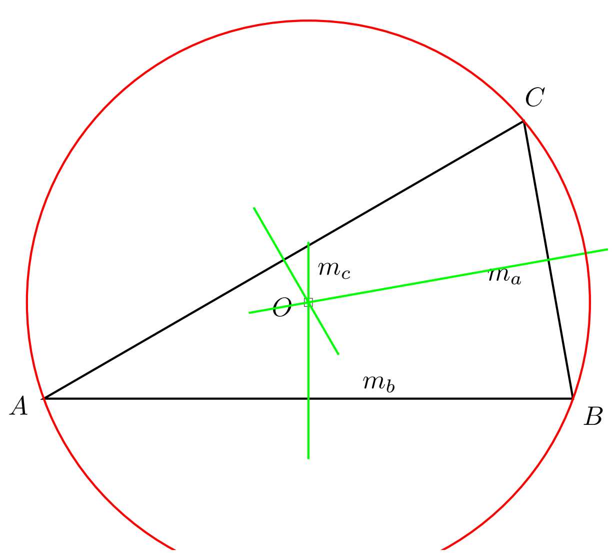
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{7 \cdot \sin 80}{\sin 85}$
$wha = 6,92$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{3,725 \cdot \sin 70}{\sin 70}$
$whb = 3,725$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,336 \cdot \sin 30}{\sin 85}$
$whc = 1,869$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,336^2 + 7^2) - 3,725^2}$
$s_a = 6,924$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(3,725^2 + 7^2) - 7,336^2}$
$s_b = 4,241$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(3,725^2 + 7,336^2) - 7^2}$
$s_c = 4,516$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{3,725}{2 \cdot \sin 30^\circ}$
$r_u = 3,725$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 12,838}{18,061}$
$r_i = 1,422$



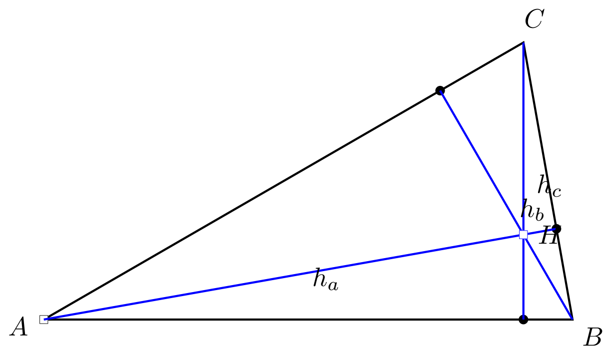
Seitenhalbierende-Schwerpunkt



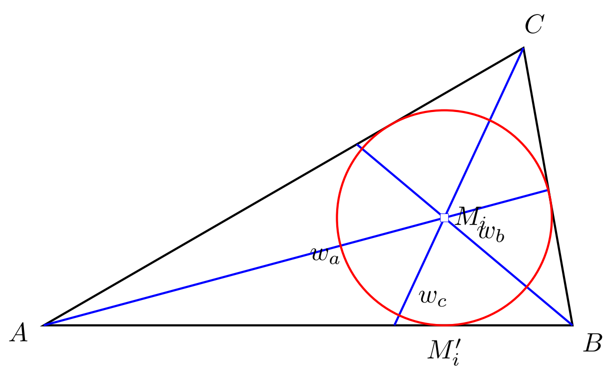
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis

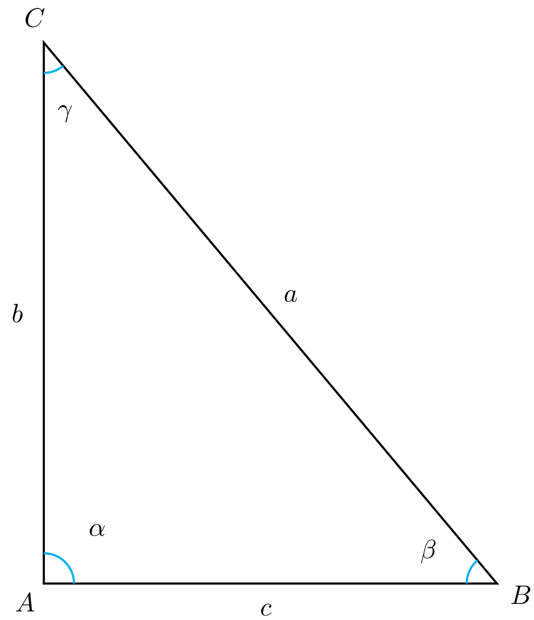


Aufgabe (48)

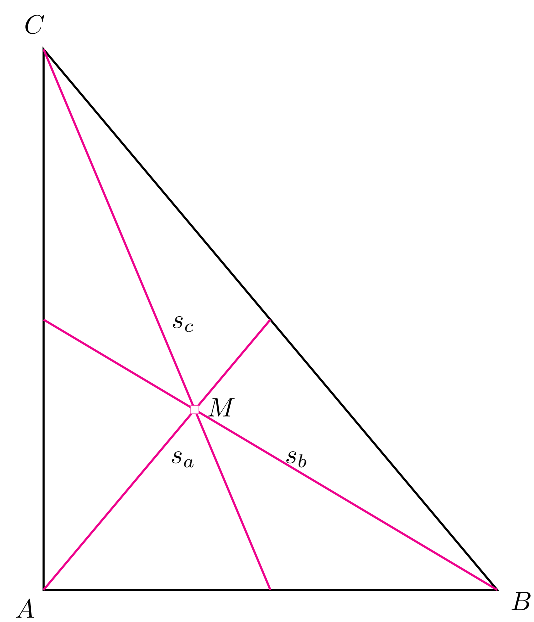
$Winkel - Winkel - Seite$
$c = 6 \quad \gamma = 40° \quad \beta = 50°$

Winkelsumme:$\alpha + \beta + \gamma = 180°$
$\alpha + \beta + \gamma = 180 \qquad / - \alpha \qquad / - \gamma$
$\alpha = 180° - \beta - \gamma$
$\alpha = 180° - 50° - 40°$
$\alpha = 90°$
Cosinus: $\cos\beta = \frac{c}{a}$
$\cos\beta = \frac{c}{a} \quad / \cdot a$
$a \cdot \cos\beta = c \quad / : cos\beta$
$a = \frac{c}{\cos\beta}$
$a = \frac{6}{\cos 50}$
$a = 9,334$
Pythagoras$a^2 = b^2 + c^2 \quad / - c^2$
$b^2 = a^2 - c^2$
$b = \sqrt{a^2 - c^2}$
$b = \sqrt{9,334^2 - 6^2}$
$b = 7,151$
Umfang:$U = a + b + c$
$U = 9,334 + 7,151 + 6$
$U = 22,485$
Höhe h_a
$\sin\beta = \frac{h_a}{c}$
$\sin\beta = \frac{h_a}{c} \quad / \cdot c$
$h_a = c \cdot \sin\beta$
$h_a = 6 \cdot \sin 50°$
$h_a = 4,596$
Flaeche: $A = \frac{1}{2} \cdot a \cdot h_a$
$A = \frac{1}{2} \cdot 9,334 \cdot 4,596$
$A = 21,452$
Höhe h_b
$\sin\gamma = \frac{h_b}{a}$
$\sin\gamma = \frac{h_b}{a} \quad / \cdot a$
$h_b = a \cdot \sin\gamma$
$h_b = 9,334 \cdot \sin 40°$
$h_b = 6$
Höhe h_c
$\sin\alpha = \frac{h_c}{b}$
$\sin\alpha = \frac{h_c}{b} \quad / \cdot b$
$h_c = b \cdot \sin\alpha$
$h_c = 7,151 \cdot \sin 90°$
$h_c = 7,151$
Winkelhalbierende α
$\delta = 180 - \beta - \frac{\alpha}{2}$
Sinus-Satz:$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta}$

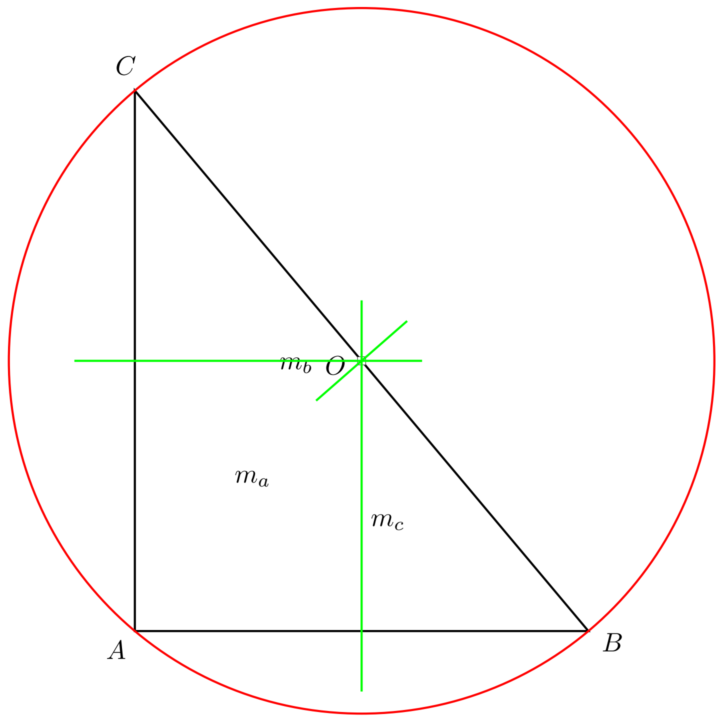
$\frac{wha}{\sin\beta} = \frac{c}{\sin\delta} \qquad / \cdot \sin\beta$
$wha = \frac{c \cdot \sin\beta}{\sin\delta}$
$wha = \frac{6 \cdot \sin 50}{\sin 85}$
$wha = 4,614$
Winkelhalbierende β
$\delta = 180 - \frac{\beta}{2} - \gamma$
Sinus-Satz: $\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta}$
$\frac{whb}{\sin\gamma} = \frac{a}{\sin\delta} \qquad / \cdot \sin\gamma$
$whb = \frac{a \cdot \sin\gamma}{\sin\delta}$
$whb = \frac{9,334 \cdot \sin 40}{\sin 115}$
$whb = 6,62$
Winkelhalbierende γ
$\delta = 180 - \alpha - \frac{\gamma}{2}$
Sinus-Satz: $\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta}$
$\frac{whc}{\sin\alpha} = \frac{b}{\sin\delta} \qquad / \cdot \sin\alpha$
$whc = \frac{b \cdot \sin\alpha}{\sin\delta}$
$whc = \frac{7,151 \cdot \sin 90}{\sin 85}$
$whc = 9,37$
Seitenhalbierende:
$s_a = \frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$
$s_a = \frac{1}{2}\sqrt{2(7,151^2 + 6^2) - 9,334^2}$
$s_a = 4,667$
Seitenhalbierende: $s_b = \frac{1}{2}\sqrt{2(a^2 + c^2) - b^2}$
$s_b = \frac{1}{2}\sqrt{2(9,334^2 + 6^2) - 7,151^2}$
$s_b = 6,984$
Seitenhalbierende: $s_c = \frac{1}{2}\sqrt{2(a^2 + b^2) - c^2}$
$s_c = \frac{1}{2}\sqrt{2(9,334^2 + 7,151^2) - 6^2}$
$s_c = 7,506$
Umkreisradius: $2 \cdot r_u = \frac{a}{\sin\alpha}$
$r_u = \frac{a}{2 \cdot \sin\alpha}$
$r_u = \frac{9,334}{2 \cdot \sin 90^\circ}$
$r_u = 4,667$
Innkreisradius: $r_i = \frac{2 \cdot A}{U}$
$r_i = \frac{2 \cdot 21,452}{22,485}$
$r_i = 1,908$



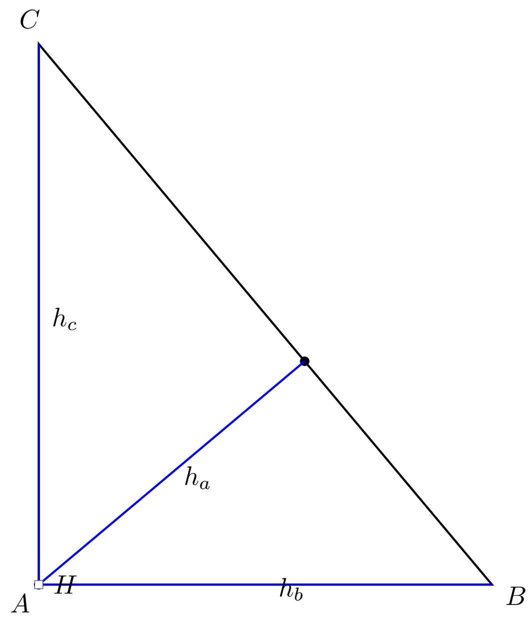
Seitenhalbierende-Schwerpunkt



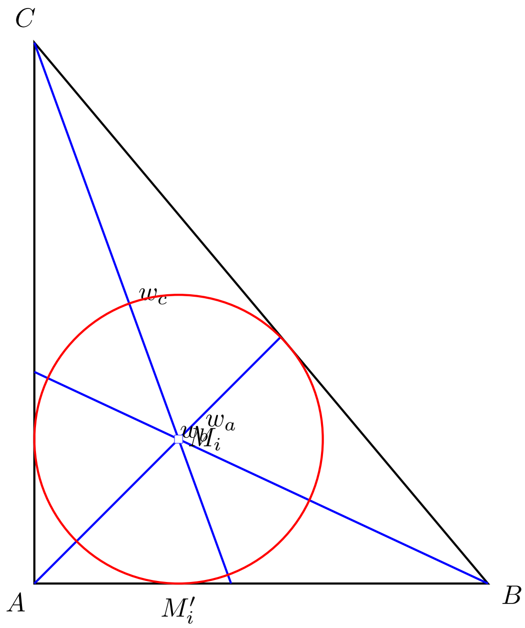
Mittelsenkrechte - Umkreis



Höhen



Winkelhalbierende-Innkreis


C
w_c
w_a
w_b
M_i
A
M_i'
B