

# Lineares Gleichungssystem

## Aufgaben und Lösungen

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### Inhaltsverzeichnis

<b>1</b>	<b>Einsetzverfahren (2)</b>	<b>2</b>
1.1	Aufgaben .....	2
1.2	Lösungen .....	3
<b>2</b>	<b>Gleichsetzungsverfahren (2)</b>	<b>14</b>
2.1	Aufgaben .....	14
2.2	Lösungen .....	15
<b>3</b>	<b>Additionssverfahren (2)</b>	<b>27</b>
3.1	Aufgaben .....	27
3.2	Lösungen .....	28
<b>4</b>	<b>Determinantenverfahren (2)</b>	<b>39</b>
4.1	Aufgaben .....	39
4.2	Lösungen .....	40
<b>5</b>	<b>Determinantenverfahren (3)</b>	<b>47</b>
5.1	Aufgaben .....	47
5.2	Lösungen .....	47
<b>6</b>	<b>Gaußsches Eliminationsverfahren</b>	<b>52</b>
6.1	Aufgaben .....	52
6.2	Lösungen .....	53

# 1 Einsetzverfahren (2)

## 1.1 Aufgaben

Gesucht: Die Lösung des linearen Gleichungssystems

$$a1 \cdot x + b1 \cdot y = c1$$

$$a2 \cdot x + b2 \cdot y = c2$$

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

$$(1) \\ 3x + 5y = 19 \\ 7x + 5y = 31$$

$$(10) \\ -x + 1y = 3 \\ \frac{1}{2}x - 4y = 5$$

$$(2) \\ 1x + 1y = 10 \\ 1x - 1y = 4$$

$$(11) \\ 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\ 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$$

$$(3) \\ 9x - 2y = 5 \\ 5x - 2y = 1$$

$$(12) \\ \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\ 1x + 1y = 10\frac{2}{3}$$

$$(4) \\ 9x - 2y = 1 \\ -3x - 3y = -7$$

$$(13) \\ 1\frac{1}{2}x - 2y = 9 \\ \frac{2}{5}x + \frac{1}{3}y = 5$$

$$(5) \\ 1x + 3y = 9 \\ 3x - 2y = -6$$

$$(14) \\ 2x + 3y = 4 \\ \frac{1}{3}x - \frac{1}{5}y = 12$$

$$(6) \\ 7x + 1\frac{1}{2}y = 1 \\ 2x - 5y = 22$$

$$(15) \\ 1x + 2y = -4 \\ -x + 2y = 5$$

$$(7) \\ 4x - 5y = 43 \\ -3x - 7y = 0$$

$$(16) \\ 2x + 4y = 4 \\ -\frac{1}{2}x + 3y = 3$$

$$(8) \\ 8x - 3y = 10 \\ 12x - 5y = 24$$

$$(17) \\ -\frac{1}{2}x + 4y = 6 \\ -2x - 8y = 2$$

$$(9) \\ -\frac{1}{2}x + 1y = 2 \\ \frac{1}{2}x - 3y = -3$$

$$(18) \\ 1x + 1y = 1 \\ 2x + 2y = 2$$

(19)

$$1x + 1y = 2$$

$$3x + 3y = 3$$

$$2x - 7y = -8$$

$$7x - 1y = -9$$

(20)

$$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$$

$$-\frac{2}{3}x + \frac{1}{9}y = 9$$

(22)

$$-7x + 9y = -3$$

$$5x - 6y = -4$$

(21)

## 1.2 Lösungen

Aufgabe (1)

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

I nach x auflösen

$$3x + 5y = 19$$

$$3x + 5y = 19 \quad / - 5y$$

$$3x = 19 - 5y \quad / : 3$$

$$x = 6\frac{1}{3} - 1\frac{2}{3}y$$

I in II

$$7(6\frac{1}{3} - 1\frac{2}{3}y) + 5y = 31$$

$$44\frac{1}{3} - 11\frac{2}{3}y + 5y = 31 \quad / - 44\frac{1}{3}$$

$$-11\frac{2}{3}y + 5y = 31 - 44\frac{1}{3}$$

$$-6\frac{2}{3}y = -13\frac{1}{3} \quad / : (-6\frac{2}{3})$$

$$y = \frac{-13\frac{1}{3}}{-6\frac{2}{3}}$$

$$y = 2$$

$$x = 6\frac{1}{3} - 1\frac{2}{3}y$$

$$x = 6\frac{1}{3} - 1\frac{2}{3} \cdot 2$$

$$x = 3$$

$$L = \{3/2\}$$

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

I nach y auflösen

$$3x + 5y = 19$$

$$3x + 5y = 19 \quad / - 3x$$

$$5y = 19 - 3x \quad / : 5$$

$$y = 3\frac{4}{5} - \frac{3}{5}x$$

I in II

$$7x + 5(3\frac{4}{5} - \frac{3}{5}x) = 31$$

$$19 - 3x + 5x = 31 \quad / - 19$$

$$-3x + 5x = 31 - 19$$

$$4x = 12 \quad / : 4$$

$$x = \frac{12}{4}$$

$$x = 3$$

$$y = 3\frac{4}{5} - \frac{3}{5}x$$

$$y = 3\frac{4}{5} - \frac{3}{5} \cdot 3$$

$$y = 2$$

$$L = \{3/2\}$$

Aufgabe (2)

$$I \quad 1x + 1y = 10$$

$$II \quad 1x - 1y = 4$$

I nach x auflösen

$$1x + 1y = 10$$

$$1x + 1y = 10 \quad / - 1y$$

$$1x = 10 - 1y \quad / : 1$$

$$x = 10 - 1y$$

I in II

$$1(10 - 1y) + -1y = 4$$

$$10 - 1y - 1y = 4 \quad / - 10$$

$$-1y - 1y = 4 - 10$$

$$-2y = -6 \quad / : (-2)$$

$$y = \frac{-6}{-2}$$

$$y = 3$$

$$x = 10 - 1y$$

$$x = 10 - 1 \cdot 3$$

$$x = 7$$

$$L = \{7/3\}$$

$$I \quad 1x + 1y = 10$$

$$II \quad 1x - 1y = 4$$

I nach y auflösen

$$1x + 1y = 10$$

$$1x + 1y = 10 \quad / - 1x$$

$$1y = 10 - x \quad / : 1$$

$$y = 10 - x$$

I in II

$$1x + -(10 - 1x) = 4$$

$$-10 + 1x - 1x = 4 \quad / - (-10)$$

$$+1x - 1x = 4 - (-10)$$

$$2x = 14 \quad / : 2$$

$$x = \frac{14}{2}$$

$$x = 7$$

$$y = 10 - 1x$$

$$y = 10 - 1 \cdot 7$$

$$y = 3$$

$$L = \{7/3\}$$

## Aufgabe (3)

$  \begin{aligned}  I \quad & 9x - 2y = 5 \\  II \quad & 5x - 2y = 1 \\  & \text{I nach x auflösen} \\  & 9x - 2y = 5 \\  & 9x - 2y = 5 \quad / + 2y \\  & 9x = 5 + 2y \quad / : 9 \\  & x = \frac{5}{9} + \frac{2}{9}y \\  & \text{I in II} \\  & 5\left(\frac{5}{9} + \frac{2}{9}y\right) - 2y = 1 \\  & 2\frac{7}{9} + 1\frac{1}{9}y - 2y = 1 \quad / - 2\frac{7}{9} \\  & + 1\frac{1}{9}y - 2y = 1 - 2\frac{7}{9} \\  & -\frac{8}{9}y = -1\frac{7}{9} \quad / : \left(-\frac{8}{9}\right) \\  & y = \frac{-1\frac{7}{9}}{-\frac{8}{9}} \\  & y = 2 \\  & x = \frac{5}{9} + \frac{2}{9}y \\  & x = \frac{5}{9} + \frac{2}{9} \cdot 2 \\  & x = 1 \\  & L = \{1/2\}  \end{aligned}  $	$  \begin{aligned}  I \quad & 9x - 2y = 5 \\  II \quad & 5x - 2y = 1 \\  & \text{I nach y auflösen} \\  & 9x - 2y = 5 \\  & 9x - 2y = 5 \quad / - 9x \\  & -2y = 5 - 9x \quad / : (-2) \\  & y = -2\frac{1}{2} + 4\frac{1}{2}x \\  & \text{I in II} \\  & 5x + -2\left(-2\frac{1}{2} + 4\frac{1}{2}x\right) = 1 \\  & 5 - 9x - 2x = 1 \quad / - 5 \\  & -9x - 2x = 1 - 5 \\  & -4x = -4 \quad / : (-4) \\  & x = \frac{-4}{-4} \\  & x = 1 \\  & y = -2\frac{1}{2} + 4\frac{1}{2}x \\  & y = -2\frac{1}{2} + 4\frac{1}{2} \cdot 1 \\  & y = 2 \\  & L = \{1/2\}  \end{aligned}  $
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## Aufgabe (4)

$  \begin{aligned}  I \quad & 9x - 2y = 1 \\  II \quad & -3x - 3y = -7 \\  & \text{I nach x auflösen} \\  & 9x - 2y = 1 \\  & 9x - 2y = 1 \quad / + 2y \\  & 9x = 1 + 2y \quad / : 9 \\  & x = \frac{1}{9} + \frac{2}{9}y \\  & \text{I in II} \\  & -3\left(\frac{1}{9} + \frac{2}{9}y\right) - 3y = -7 \\  & -\frac{1}{3} - \frac{2}{3}y - 3y = -7 \quad / - \left(-\frac{1}{3}\right) \\  & -\frac{2}{3}y - 3y = -7 - \left(-\frac{1}{3}\right) \\  & -3\frac{2}{3}y = -6\frac{2}{3} \quad / : \left(-3\frac{2}{3}\right) \\  & y = \frac{-6\frac{2}{3}}{-3\frac{2}{3}} \\  & y = 1\frac{9}{11} \\  & x = \frac{1}{9} + \frac{2}{9}y \\  & x = \frac{1}{9} + \frac{2}{9} \cdot 1\frac{9}{11} \\  & x = \frac{17}{33} \\  & L = \left\{\frac{17}{33}/1\frac{9}{11}\right\}  \end{aligned}  $	$  \begin{aligned}  I \quad & 9x - 2y = 1 \\  II \quad & -3x - 3y = -7 \\  & \text{I nach y auflösen} \\  & 9x - 2y = 1 \\  & 9x - 2y = 1 \quad / - 9x \\  & -2y = 1 - 9x \quad / : (-2) \\  & y = -\frac{1}{2} + 4\frac{1}{2}x \\  & \text{I in II} \\  & -3x + -3\left(-\frac{1}{2} + 4\frac{1}{2}x\right) = -7 \\  & 1\frac{1}{2} - 13\frac{1}{2}x - 3x = -7 \quad / - 1\frac{1}{2} \\  & -13\frac{1}{2}x - 3x = -7 - 1\frac{1}{2} \\  & -16\frac{1}{2}x = -8\frac{1}{2} \quad / : \left(-16\frac{1}{2}\right) \\  & x = \frac{-8\frac{1}{2}}{-16\frac{1}{2}} \\  & x = \frac{17}{33} \\  & y = -\frac{1}{2} + 4\frac{1}{2}x \\  & y = -\frac{1}{2} + 4\frac{1}{2} \cdot \frac{17}{33} \\  & y = 1\frac{9}{11} \\  & L = \left\{\frac{17}{33}/1\frac{9}{11}\right\}  \end{aligned}  $
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## Aufgabe (5)

$$\begin{array}{l}
 I \quad 1x + 3y = 9 \\
 II \quad 3x - 2y = -6 \\
 \text{I nach x auflösen} \\
 1x + 3y = 9 \\
 1x + 3y = 9 \quad / - 3y \\
 1x = 9 - 3y \quad / : 1 \\
 x = 9 - 3y \\
 \text{I in II} \\
 3(9 - 3y) + -2y = -6 \\
 27 - 9y - 2y = -6 \quad / - 27 \\
 -9y - 2y = -6 - 27 \\
 -11y = -33 \quad / : (-11) \\
 y = \frac{-33}{-11} \\
 y = 3 \\
 x = 9 - 3y \\
 x = 9 - 3 \cdot 3 \\
 x = 0 \\
 L = \{0/3\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 3y = 9 \\
 II \quad 3x - 2y = -6 \\
 \text{I nach y auflösen} \\
 1x + 3y = 9 \\
 1x + 3y = 9 \quad / - 1x \\
 3y = 9 - x \quad / : 3 \\
 y = 3 - \frac{1}{3}x \\
 \text{I in II} \\
 3x + -2(3 - \frac{1}{3}x) = -6 \\
 -6 + \frac{2}{3}x - 2x = -6 \quad / - (-6) \\
 +\frac{2}{3}x - 2x = -6 - (-6) \\
 3\frac{2}{3}x = 0 \quad / : 3\frac{2}{3} \\
 x = \frac{0}{3\frac{2}{3}} \\
 x = 0 \\
 y = 3 - \frac{1}{3}x \\
 y = 3 - \frac{1}{3} \cdot 0 \\
 y = 3 \\
 L = \{0/3\}
 \end{array}$$

Aufgabe (6)

$$\begin{array}{l}
 I \quad 7x + 1\frac{1}{2}y = 1 \\
 II \quad 2x - 5y = 22 \\
 \text{I nach x auflösen} \\
 7x + 1\frac{1}{2}y = 1 \\
 7x + 1\frac{1}{2}y = 1 \quad / - 1\frac{1}{2}y \\
 7x = 1 - 1\frac{1}{2}y \quad / : 7 \\
 x = \frac{1}{7} - \frac{3}{14}y \\
 \text{I in II} \\
 2(\frac{1}{7} - \frac{3}{14}y) + -5y = 22 \\
 \frac{2}{7} - \frac{3}{7}y - 5y = 22 \quad / - \frac{2}{7} \\
 -\frac{3}{7}y - 5y = 22 - \frac{2}{7} \\
 -5\frac{3}{7}y = 21\frac{5}{7} \quad / : (-5\frac{3}{7}) \\
 y = \frac{21\frac{5}{7}}{-5\frac{3}{7}} \\
 y = -4 \\
 x = \frac{1}{7} - \frac{3}{14}y \\
 x = \frac{1}{7} - \frac{3}{14} \cdot (-4) \\
 x = 1 \\
 L = \{1/-4\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 7x + 1\frac{1}{2}y = 1 \\
 II \quad 2x - 5y = 22 \\
 \text{I nach y auflösen} \\
 7x + 1\frac{1}{2}y = 1 \\
 7x + 1\frac{1}{2}y = 1 \quad / - 7x \\
 1\frac{1}{2}y = 1 - 7x \quad / : 1\frac{1}{2} \\
 y = \frac{2}{3} - 4\frac{2}{3}x \\
 \text{I in II} \\
 2x + -5(\frac{2}{3} - 4\frac{2}{3}x) = 22 \\
 -3\frac{1}{3} + 23\frac{1}{3}x - 5x = 22 \quad / - (-3\frac{1}{3}) \\
 +23\frac{1}{3}x - 5x = 22 - (-3\frac{1}{3}) \\
 25\frac{1}{3}x = 25\frac{1}{3} \quad / : 25\frac{1}{3} \\
 x = \frac{25\frac{1}{3}}{25\frac{1}{3}} \\
 x = 1 \\
 y = \frac{2}{3} - 4\frac{2}{3}x \\
 y = \frac{2}{3} - 4\frac{2}{3} \cdot 1 \\
 y = -4 \\
 L = \{1/-4\}
 \end{array}$$

Aufgabe (7)

$I \quad 4x - 5y = 43$ $II \quad -3x - 7y = 0$ <p>I nach x auflösen</p> $4x - 5y = 43$ $4x - 5y = 43 \quad / + 5y$ $4x = 43 + 5y \quad / : 4$ $x = 10\frac{3}{4} + 1\frac{1}{4}y$ <p>I in II</p> $-3(10\frac{3}{4} + 1\frac{1}{4}y) - 7y = 0$ $-32\frac{1}{4} - 3\frac{3}{4}y - 7y = 0 \quad / - (-32\frac{1}{4})$ $-3\frac{3}{4}y - 7y = 0 - (-32\frac{1}{4})$ $-10\frac{3}{4}y = 32\frac{1}{4} \quad / : (-10\frac{3}{4})$ $y = \frac{32\frac{1}{4}}{-10\frac{3}{4}}$ $y = -3$ $x = 10\frac{3}{4} + 1\frac{1}{4}y$ $x = 10\frac{3}{4} + 1\frac{1}{4} \cdot (-3)$ $x = 7$ $L = \{7 / -3\}$	$I \quad 4x - 5y = 43$ $II \quad -3x - 7y = 0$ <p>I nach y auflösen</p> $4x - 5y = 43$ $4x - 5y = 43 \quad / - 4x$ $-5y = 43 - 4x \quad / : (-5)$ $y = -8\frac{3}{5} + \frac{4}{5}x$ <p>I in II</p> $-3x + -7(-8\frac{3}{5} + \frac{4}{5}x) = 0$ $60\frac{1}{5} - 5\frac{3}{5}x - 7x = 0 \quad / - 60\frac{1}{5}$ $-5\frac{3}{5}x - 7x = 0 - 60\frac{1}{5}$ $-8\frac{3}{5}x = -60\frac{1}{5} \quad / : (-8\frac{3}{5})$ $x = \frac{-60\frac{1}{5}}{-8\frac{3}{5}}$ $x = 7$ $y = -8\frac{3}{5} + \frac{4}{5}x$ $y = -8\frac{3}{5} + \frac{4}{5} \cdot 7$ $y = -3$ $L = \{7 / -3\}$
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Aufgabe (8)

$I \quad 8x - 3y = 10$ $II \quad 12x - 5y = 24$ <p>I nach x auflösen</p> $8x - 3y = 10$ $8x - 3y = 10 \quad / + 3y$ $8x = 10 + 3y \quad / : 8$ $x = 1\frac{1}{4} + \frac{3}{8}y$ <p>I in II</p> $12(1\frac{1}{4} + \frac{3}{8}y) - 5y = 24$ $15 + 4\frac{1}{2}y - 5y = 24 \quad / - 15$ $+4\frac{1}{2}y - 5y = 24 - 15$ $-\frac{1}{2}y = 9 \quad / : (-\frac{1}{2})$ $y = \frac{9}{-\frac{1}{2}}$ $y = -18$ $x = 1\frac{1}{4} + \frac{3}{8}y$ $x = 1\frac{1}{4} + \frac{3}{8} \cdot (-18)$ $x = -5\frac{1}{2}$ $L = \{-5\frac{1}{2} / -18\}$	$I \quad 8x - 3y = 10$ $II \quad 12x - 5y = 24$ <p>I nach y auflösen</p> $8x - 3y = 10$ $8x - 3y = 10 \quad / - 8x$ $-3y = 10 - 8x \quad / : (-3)$ $y = -3\frac{1}{3} + 2\frac{2}{3}x$ <p>I in II</p> $12x + -5(-3\frac{1}{3} + 2\frac{2}{3}x) = 24$ $16\frac{2}{3} - 13\frac{1}{3}x - 5x = 24 \quad / - 16\frac{2}{3}$ $-13\frac{1}{3}x - 5x = 24 - 16\frac{2}{3}$ $-1\frac{1}{3}x = 7\frac{1}{3} \quad / : (-1\frac{1}{3})$ $x = \frac{7\frac{1}{3}}{-1\frac{1}{3}}$ $x = -5\frac{1}{2}$ $y = -3\frac{1}{3} + 2\frac{2}{3}x$ $y = -3\frac{1}{3} + 2\frac{2}{3} \cdot (-5\frac{1}{2})$ $y = -18$ $L = \{-5\frac{1}{2} / -18\}$
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Aufgabe (9)

$$I \quad -\frac{1}{2}x + 1y = 2$$

$$II \quad \frac{1}{2}x - 3y = -3$$

I nach x auflösen

$$-\frac{1}{2}x + 1y = 2$$

$$-\frac{1}{2}x + 1y = 2 \quad / -1y$$

$$-\frac{1}{2}x = 2 - 1y \quad / : \left(-\frac{1}{2}\right)$$

$$x = -4 + 2y$$

I in II

$$\frac{1}{2}(-4 + 2y) + -3y = -3$$

$$-2 + 1y - 3y = -3 \quad / -(-2)$$

$$+1y - 3y = -3 - (-2)$$

$$-2y = -1 \quad / : (-2)$$

$$y = \frac{-1}{-2}$$

$$y = \frac{1}{2}$$

$$x = -4 + 2y$$

$$x = -4 + 2 \cdot \frac{1}{2}$$

$$x = -3$$

$$L = \left\{-3/\frac{1}{2}\right\}$$

$$I \quad -\frac{1}{2}x + 1y = 2$$

$$II \quad \frac{1}{2}x - 3y = -3$$

I nach y auflösen

$$-\frac{1}{2}x + 1y = 2$$

$$-\frac{1}{2}x + 1y = 2 \quad / + \frac{1}{2}x$$

$$1y = 2 + \frac{1}{2}x \quad / : 1$$

$$y = 2 + \frac{1}{2}x$$

I in II

$$\frac{1}{2}x + -3\left(2 + \frac{1}{2}x\right) = -3$$

$$-6 - 1\frac{1}{2}x - 3x = -3 \quad / -(-6)$$

$$-1\frac{1}{2}x - 3x = -3 - (-6)$$

$$-1x = 3 \quad / : (-1)$$

$$x = \frac{3}{-1}$$

$$x = -3$$

$$y = 2 + \frac{1}{2}x$$

$$y = 2 + \frac{1}{2} \cdot (-3)$$

$$y = \frac{1}{2}$$

$$L = \left\{-3/\frac{1}{2}\right\}$$

Aufgabe (10)

$$I \quad -1x + 1y = 3$$

$$II \quad \frac{1}{2}x - 4y = 5$$

I nach x auflösen

$$-1x + 1y = 3$$

$$-1x + 1y = 3 \quad / -1y$$

$$-1x = 3 - 1y \quad / : (-1)$$

$$x = -3 + 1y$$

I in II

$$\frac{1}{2}(-3 + 1y) + -4y = 5$$

$$-1\frac{1}{2} + \frac{1}{2}y - 4y = 5 \quad / -(-1\frac{1}{2})$$

$$+\frac{1}{2}y - 4y = 5 - (-1\frac{1}{2})$$

$$-3\frac{1}{2}y = 6\frac{1}{2} \quad / : (-3\frac{1}{2})$$

$$y = \frac{6\frac{1}{2}}{-3\frac{1}{2}}$$

$$y = -1\frac{6}{7}$$

$$x = -3 + 1y$$

$$x = -3 + 1 \cdot \left(-1\frac{6}{7}\right)$$

$$x = -4\frac{6}{7}$$

$$L = \left\{-4\frac{6}{7}/-1\frac{6}{7}\right\}$$

$$I \quad -1x + 1y = 3$$

$$II \quad \frac{1}{2}x - 4y = 5$$

I nach y auflösen

$$-1x + 1y = 3$$

$$-1x + 1y = 3 \quad / + 1x$$

$$1y = 3 + x \quad / : 1$$

$$y = 3 + x$$

I in II

$$\frac{1}{2}x + -4(3 + 1x) = 5$$

$$-12 - 4x - 4x = 5 \quad / -(-12)$$

$$-4x - 4x = 5 - (-12)$$

$$-3\frac{1}{2}x = 17 \quad / : (-3\frac{1}{2})$$

$$x = \frac{17}{-3\frac{1}{2}}$$

$$x = -4\frac{6}{7}$$

$$y = 3 + 1x$$

$$y = 3 + 1 \cdot \left(-4\frac{6}{7}\right)$$

$$y = -1\frac{6}{7}$$

$$L = \left\{-4\frac{6}{7}/-1\frac{6}{7}\right\}$$

Aufgabe (11)

$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$ $II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$ <p>I nach x auflösen</p> $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$ $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / + 1\frac{1}{3}y$ $1\frac{1}{5}x = 5\frac{1}{3} + 1\frac{1}{3}y \quad / : 1\frac{1}{5}$ $x = 4\frac{4}{9} + 1\frac{1}{9}y$ <p>I in II</p> $2\frac{1}{2}(4\frac{4}{9} + 1\frac{1}{9}y) - \frac{1}{4}y = 12\frac{3}{8}$ $11\frac{1}{9} + 2\frac{7}{9}y - \frac{1}{4}y = 12\frac{3}{8} \quad / - 11\frac{1}{9}$ $+ 2\frac{7}{9}y - \frac{1}{4}y = 12\frac{3}{8} - 11\frac{1}{9}$ $2\frac{19}{36}y = 1\frac{19}{72} \quad / : 2\frac{19}{36}$ $y = \frac{1\frac{19}{72}}{2\frac{19}{36}}$ $y = \frac{1}{2}$ $x = 4\frac{4}{9} + 1\frac{1}{9}y$ $x = 4\frac{4}{9} + 1\frac{1}{9} \cdot \frac{1}{2}$ $x = 5$ $L = \{5/\frac{1}{2}\}$	$I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$ $II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$ <p>I nach y auflösen</p> $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$ $1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / - 1\frac{1}{5}x$ $-1\frac{1}{3}y = 5\frac{1}{3} - 1\frac{1}{5}x \quad / : (-1\frac{1}{3})$ $y = -4 + \frac{9}{10}x$ <p>I in II</p> $2\frac{1}{2}x + \frac{1}{4}(-4 + \frac{9}{10}x) = 12\frac{3}{8}$ $1 - \frac{9}{40}x - \frac{1}{4}x = 12\frac{3}{8} \quad / - 1$ $-\frac{9}{40}x - \frac{1}{4}x = 12\frac{3}{8} - 1$ $2\frac{11}{40}x = 11\frac{3}{8} \quad / : 2\frac{11}{40}$ $x = \frac{11\frac{3}{8}}{2\frac{11}{40}}$ $x = 5$ $y = -4 + \frac{9}{10}x$ $y = -4 + \frac{9}{10} \cdot 5$ $y = \frac{1}{2}$ $L = \{5/\frac{1}{2}\}$
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Aufgabe (12)

$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$ $II \quad 1x + 1y = 10\frac{2}{3}$ <p>I nach x auflösen</p> $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$ $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / + \frac{5}{7}y$ $\frac{2}{3}x = \frac{2}{3} + \frac{5}{7}y \quad / : \frac{2}{3}$ $x = 1 + 1\frac{5}{14}y$ <p>I in II</p> $1(1 + 1\frac{5}{14}y) + 1y = 10\frac{2}{3}$ $1 + 1\frac{5}{14}y + 1y = 10\frac{2}{3} \quad / - 1$ $+ 1\frac{5}{14}y + 1y = 10\frac{2}{3} - 1$ $2\frac{1}{14}y = 9\frac{2}{3} \quad / : 2\frac{1}{14}$ $y = \frac{9\frac{2}{3}}{2\frac{1}{14}}$ $y = 4\frac{2}{3}$ $x = 1 + 1\frac{5}{14}y$ $x = 1 + 1\frac{5}{14} \cdot 4\frac{2}{3}$ $x = 6$ $L = \{6/4\frac{2}{3}\}$	$I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$ $II \quad 1x + 1y = 10\frac{2}{3}$ <p>I nach y auflösen</p> $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$ $\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / - \frac{2}{3}x$ $-\frac{5}{7}y = \frac{2}{3} - \frac{2}{3}x \quad / : (-\frac{5}{7})$ $y = -\frac{14}{15} + \frac{14}{15}x$ <p>I in II</p> $1x + 1(-\frac{14}{15} + \frac{14}{15}x) = 10\frac{2}{3}$ $-\frac{14}{15} + \frac{14}{15}x + 1x = 10\frac{2}{3} \quad / - (-\frac{14}{15})$ $+\frac{14}{15}x + 1x = 10\frac{2}{3} - (-\frac{14}{15})$ $1\frac{14}{15}x = 11\frac{3}{5} \quad / : 1\frac{14}{15}$ $x = \frac{11\frac{3}{5}}{1\frac{14}{15}}$ $x = 6$ $y = -\frac{14}{15} + \frac{14}{15}x$ $y = -\frac{14}{15} + \frac{14}{15} \cdot 6$ $y = 4\frac{2}{3}$ $L = \{6/4\frac{2}{3}\}$
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Aufgabe (13)

$$\begin{array}{l}
 I \quad 1\frac{1}{2}x - 2y = 9 \\
 II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \text{I nach x auflösen} \\
 1\frac{1}{2}x - 2y = 9 \\
 1\frac{1}{2}x - 2y = 9 \quad / + 2y \\
 1\frac{1}{2}x = 9 + 2y \quad / : 1\frac{1}{2} \\
 x = 6 + 1\frac{1}{3}y \\
 \text{I in II} \\
 \frac{2}{5}(6 + 1\frac{1}{3}y) + \frac{1}{3}y = 5 \\
 2\frac{2}{5} + \frac{8}{15}y + \frac{1}{3}y = 5 \quad / - 2\frac{2}{5} \\
 + \frac{8}{15}y + \frac{1}{3}y = 5 - 2\frac{2}{5} \\
 \frac{13}{15}y = 2\frac{3}{5} \quad / : \frac{13}{15} \\
 y = \frac{2\frac{3}{5}}{\frac{13}{15}} \\
 y = 3 \\
 x = 6 + 1\frac{1}{3}y \\
 x = 6 + 1\frac{1}{3} \cdot 3 \\
 x = 10 \\
 L = \{10/3\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1\frac{1}{2}x - 2y = 9 \\
 II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \text{I nach y auflösen} \\
 1\frac{1}{2}x - 2y = 9 \\
 1\frac{1}{2}x - 2y = 9 \quad / - 1\frac{1}{2}x \\
 -2y = 9 - 1\frac{1}{2}x \quad / : (-2) \\
 y = -4\frac{1}{2} + \frac{3}{4}x \\
 \text{I in II} \\
 \frac{2}{5}x + \frac{1}{3}(-4\frac{1}{2} + \frac{3}{4}x) = 5 \\
 -1\frac{1}{2} + \frac{1}{4}x + \frac{1}{3}x = 5 \quad / - (-1\frac{1}{2}) \\
 + \frac{1}{4}x + \frac{1}{3}x = 5 - (-1\frac{1}{2}) \\
 \frac{13}{20}x = 6\frac{1}{2} \quad / : \frac{13}{20} \\
 x = \frac{6\frac{1}{2}}{\frac{13}{20}} \\
 x = 10 \\
 y = -4\frac{1}{2} + \frac{3}{4}x \\
 y = -4\frac{1}{2} + \frac{3}{4} \cdot 10 \\
 y = 3 \\
 L = \{10/3\}
 \end{array}$$

Aufgabe (14)

$$\begin{array}{l}
 I \quad 2x + 3y = 4 \\
 II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \text{I nach x auflösen} \\
 2x + 3y = 4 \\
 2x + 3y = 4 \quad / - 3y \\
 2x = 4 - 3y \quad / : 2 \\
 x = 2 - 1\frac{1}{2}y \\
 \text{I in II} \\
 \frac{1}{3}(2 - 1\frac{1}{2}y) - \frac{1}{5}y = 12 \\
 \frac{2}{3} - \frac{1}{2}y - \frac{1}{5}y = 12 \quad / - \frac{2}{3} \\
 -\frac{1}{2}y - \frac{1}{5}y = 12 - \frac{2}{3} \\
 -\frac{7}{10}y = 11\frac{1}{3} \quad / : (-\frac{7}{10}) \\
 y = \frac{11\frac{1}{3}}{-\frac{7}{10}} \\
 y = -16\frac{4}{21} \\
 x = 2 - 1\frac{1}{2}y \\
 x = 2 - 1\frac{1}{2} \cdot (-16\frac{4}{21}) \\
 x = 26\frac{2}{7} \\
 L = \{26\frac{2}{7} / -16\frac{4}{21}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 2x + 3y = 4 \\
 II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \text{I nach y auflösen} \\
 2x + 3y = 4 \\
 2x + 3y = 4 \quad / - 2x \\
 3y = 4 - 2x \quad / : 3 \\
 y = 1\frac{1}{3} - \frac{2}{3}x \\
 \text{I in II} \\
 \frac{1}{3}x + -\frac{1}{5}(1\frac{1}{3} - \frac{2}{3}x) = 12 \\
 -\frac{4}{15} + \frac{2}{15}x - \frac{1}{5}x = 12 \quad / - (-\frac{4}{15}) \\
 + \frac{2}{15}x - \frac{1}{5}x = 12 - (-\frac{4}{15}) \\
 \frac{7}{15}x = 12\frac{4}{15} \quad / : \frac{7}{15} \\
 x = \frac{12\frac{4}{15}}{\frac{7}{15}} \\
 x = 26\frac{2}{7} \\
 y = 1\frac{1}{3} - \frac{2}{3}x \\
 y = 1\frac{1}{3} - \frac{2}{3} \cdot 26\frac{2}{7} \\
 y = -16\frac{4}{21} \\
 L = \{26\frac{2}{7} / -16\frac{4}{21}\}
 \end{array}$$

Aufgabe (15)

$I \quad 1x + 2y = -4$ $II \quad -1x + 2y = 5$ <p>I nach x auflösen</p> $1x + 2y = -4$ $1x + 2y = -4 \quad / - 2y$ $1x = -4 - 2y \quad / : 1$ $x = -4 - 2y$ <p>I in II</p> $-1(-4 - 2y) + 2y = 5$ $4 + 2y + 2y = 5 \quad / - 4$ $+2y + 2y = 5 - 4$ $4y = 1 \quad / : 4$ $y = \frac{1}{4}$ $y = \frac{1}{4}$ $x = -4 - 2y$ $x = -4 - 2 \cdot \frac{1}{4}$ $x = -4 \frac{1}{2}$ $L = \{-4 \frac{1}{2} / \frac{1}{4}\}$	$I \quad 1x + 2y = -4$ $II \quad -1x + 2y = 5$ <p>I nach y auflösen</p> $1x + 2y = -4$ $1x + 2y = -4 \quad / - 1x$ $2y = -4 - x \quad / : 2$ $y = -2 - \frac{1}{2}x$ <p>I in II</p> $-x + 2(-2 - \frac{1}{2}x) = 5$ $-4 - 1x + 2x = 5 \quad / - (-4)$ $-1x + 2x = 5 - (-4)$ $-2x = 9 \quad / : (-2)$ $x = \frac{9}{-2}$ $x = -4 \frac{1}{2}$ $y = -2 - \frac{1}{2}x$ $y = -2 - \frac{1}{2} \cdot (-4 \frac{1}{2})$ $y = \frac{1}{4}$ $L = \{-4 \frac{1}{2} / \frac{1}{4}\}$
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Aufgabe (16)

$I \quad 2x + 4y = 4$ $II \quad -\frac{1}{2}x + 3y = 3$ <p>I nach x auflösen</p> $2x + 4y = 4$ $2x + 4y = 4 \quad / - 4y$ $2x = 4 - 4y \quad / : 2$ $x = 2 - 2y$ <p>I in II</p> $-\frac{1}{2}(2 - 2y) + 3y = 3$ $-1 + 1y + 3y = 3 \quad / - (-1)$ $+1y + 3y = 3 - (-1)$ $4y = 4 \quad / : 4$ $y = \frac{4}{4}$ $y = 1$ $x = 2 - 2y$ $x = 2 - 2 \cdot 1$ $x = 0$ $L = \{0/1\}$	$I \quad 2x + 4y = 4$ $II \quad -\frac{1}{2}x + 3y = 3$ <p>I nach y auflösen</p> $2x + 4y = 4$ $2x + 4y = 4 \quad / - 2x$ $4y = 4 - 2x \quad / : 4$ $y = 1 - \frac{1}{2}x$ <p>I in II</p> $-\frac{1}{2}x + 3(1 - \frac{1}{2}x) = 3$ $3 - 1\frac{1}{2}x + 3x = 3 \quad / - 3$ $-1\frac{1}{2}x + 3x = 3 - 3$ $-2x = 0 \quad / : (-2)$ $x = \frac{0}{-2}$ $x = 0$ $y = 1 - \frac{1}{2}x$ $y = 1 - \frac{1}{2} \cdot 0$ $y = 1$ $L = \{0/1\}$
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Aufgabe (17)

$I \quad -\frac{1}{2}x + 4y = 6$ $II \quad -2x - 8y = 2$ <p>I nach x auflösen</p> $-\frac{1}{2}x + 4y = 6$ $-\frac{1}{2}x + 4y = 6 \quad / -4y$ $-\frac{1}{2}x = 6 - 4y \quad / : (-\frac{1}{2})$ $x = -12 + 8y$ <p>I in II</p> $-2(-12 + 8y) + -8y = 2$ $24 - 16y - 8y = 2 \quad / -24$ $-16y - 8y = 2 - 24$ $-24y = -22 \quad / : (-24)$ $y = \frac{-22}{-24}$ $y = \frac{11}{12}$ $x = -12 + 8y$ $x = -12 + 8 \cdot \frac{11}{12}$ $x = -4\frac{2}{3}$ $L = \{-4\frac{2}{3} / \frac{11}{12}\}$	$I \quad -\frac{1}{2}x + 4y = 6$ $II \quad -2x - 8y = 2$ <p>I nach y auflösen</p> $-\frac{1}{2}x + 4y = 6$ $-\frac{1}{2}x + 4y = 6 \quad / + \frac{1}{2}x$ $4y = 6 + \frac{1}{2}x \quad / : 4$ $y = 1\frac{1}{2} + \frac{1}{8}x$ <p>I in II</p> $-2x + -8(1\frac{1}{2} + \frac{1}{8}x) = 2$ $-2x - 1x - 8x = 2 \quad / -(-12)$ $-1x - 8x = 2 - (-12)$ $-3x = 14 \quad / : (-3)$ $x = \frac{14}{-3}$ $x = -4\frac{2}{3}$ $y = 1\frac{1}{2} + \frac{1}{8}x$ $y = 1\frac{1}{2} + \frac{1}{8} \cdot (-4\frac{2}{3})$ $y = \frac{11}{12}$ $L = \{-4\frac{2}{3} / \frac{11}{12}\}$
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Aufgabe (18)

$I \quad 1x + 1y = 1$ $II \quad 2x + 2y = 2$ <p>I nach x auflösen</p> $1x + 1y = 1$ $1x + 1y = 1 \quad / -1y$ $1x = 1 - 1y \quad / : 1$ $x = 1 - 1y$ <p>I in II</p> $2(1 - 1y) + 2y = 2$ $2 - 2y + 2y = 2 \quad / -2$ $-2y + 2y = 2 - 2$ <p><math>L = \text{unendlich}</math></p>	$I \quad 1x + 1y = 1$ $II \quad 2x + 2y = 2$ <p>I nach y auflösen</p> $1x + 1y = 1$ $1x + 1y = 1 \quad / -1x$ $1y = 1 - x \quad / : 1$ $y = 1 - x$ <p>I in II</p> $2x + 2(1 - 1x) = 2$ $2 - 2x + 2x = 2 \quad / -2$ $-2x + 2x = 2 - 2$ <p><math>L = \text{unendlich}</math></p>
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Aufgabe (19)

$I \quad 1x + 1y = 2$ $II \quad 3x + 3y = 3$ <p>I nach x auflösen</p> $1x + 1y = 2$ $1x + 1y = 2 \quad / -1y$ $1x = 2 - 1y \quad / : 1$ $x = 2 - 1y$ <p>I in II</p> $3(2 - 1y) + 3y = 3$ $6 - 3y + 3y = 3 \quad / -6$ $-3y + 3y = 3 - 6$ <p><math>L = \{\}</math></p>	$I \quad 1x + 1y = 2$ $II \quad 3x + 3y = 3$ <p>I nach y auflösen</p> $1x + 1y = 2$ $1x + 1y = 2 \quad / -1x$ $1y = 2 - x \quad / : 1$ $y = 2 - x$ <p>I in II</p> $3x + 3(2 - 1x) = 3$ $6 - 3x + 3x = 3 \quad / -6$ $-3x + 3x = 3 - 6$ <p><math>L = \{\}</math></p>
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Aufgabe (20)

$$\begin{array}{l}
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 II \quad -\frac{3}{3}x + \frac{1}{9}y = 9 \\
 \text{I nach x auflösen} \\
 -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \quad / -1\frac{1}{3}y \\
 -1\frac{4}{3}x = -1 - 1\frac{1}{3}y \quad / : (-1\frac{4}{3}) \\
 x = \frac{5}{9} + \frac{20}{27}y \\
 \text{I in II} \\
 -\frac{2}{3}(\frac{5}{9} + \frac{20}{27}y) + \frac{1}{9}y = 9 \\
 -\frac{10}{27} - \frac{40}{81}y + \frac{1}{9}y = 9 \quad / -(-\frac{10}{27}) \\
 -\frac{40}{81}y + \frac{1}{9}y = 9 - (-\frac{10}{27}) \\
 -\frac{31}{81}y = 9\frac{10}{27} \quad / : (-\frac{31}{81}) \\
 y = \frac{9\frac{10}{27}}{-\frac{31}{81}} \\
 y = -24\frac{15}{31} \\
 x = \frac{5}{9} + \frac{20}{27}y \\
 x = \frac{5}{9} + \frac{20}{27} \cdot (-24\frac{15}{31}) \\
 x = -17\frac{18}{31} \\
 L = \{-17\frac{18}{31} / -24\frac{15}{31}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 II \quad -\frac{3}{3}x + \frac{1}{9}y = 9 \\
 \text{I nach y auflösen} \\
 -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \quad / +1\frac{4}{3}x \\
 1\frac{1}{3}y = -1 + 1\frac{4}{3}x \quad / : 1\frac{1}{3} \\
 y = -\frac{3}{4} + 1\frac{7}{20}x \\
 \text{I in II} \\
 -\frac{2}{3}x + \frac{1}{9}(-\frac{3}{4} + 1\frac{7}{20}x) = 9 \\
 -\frac{1}{12} + \frac{3}{20}x + \frac{1}{9}x = 9 \quad / -(-\frac{1}{12}) \\
 +\frac{3}{20}x + \frac{1}{9}x = 9 - (-\frac{1}{12}) \\
 -\frac{31}{60}x = 9\frac{1}{12} \quad / : (-\frac{31}{60}) \\
 x = \frac{9\frac{1}{12}}{-\frac{31}{60}} \\
 x = -17\frac{18}{31} \\
 y = -\frac{3}{4} + 1\frac{7}{20}x \\
 y = -\frac{3}{4} + 1\frac{7}{20} \cdot (-17\frac{18}{31}) \\
 y = -24\frac{15}{31} \\
 L = \{-17\frac{18}{31} / -24\frac{15}{31}\}
 \end{array}$$

Aufgabe (21)

$$\begin{array}{l}
 I \quad 2x - 7y = -8 \\
 II \quad 7x - 1y = -9 \\
 \text{I nach x auflösen} \\
 2x - 7y = -8 \\
 2x - 7y = -8 \quad / +7y \\
 2x = -8 + 7y \quad / : 2 \\
 x = -4 + 3\frac{1}{2}y \\
 \text{I in II} \\
 7(-4 + 3\frac{1}{2}y) - 1y = -9 \\
 -28 + 24\frac{1}{2}y - 1y = -9 \quad / -(-28) \\
 +24\frac{1}{2}y - 1y = -9 - (-28) \\
 23\frac{1}{2}y = 19 \quad / : 23\frac{1}{2} \\
 y = \frac{19}{23\frac{1}{2}} \\
 y = \frac{38}{47} \\
 x = -4 + 3\frac{1}{2}y \\
 x = -4 + 3\frac{1}{2} \cdot \frac{38}{47} \\
 x = -1\frac{8}{47} \\
 L = \{-1\frac{8}{47} / \frac{38}{47}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 2x - 7y = -8 \\
 II \quad 7x - 1y = -9 \\
 \text{I nach y auflösen} \\
 2x - 7y = -8 \\
 2x - 7y = -8 \quad / -2x \\
 -7y = -8 - 2x \quad / : (-7) \\
 y = 1\frac{1}{7} + \frac{2}{7}x \\
 \text{I in II} \\
 7x + -(1\frac{1}{7} + \frac{2}{7}x) = -9 \\
 -1\frac{1}{7} - \frac{2}{7}x - 1x = -9 \quad / -(-1\frac{1}{7}) \\
 -\frac{2}{7}x - 1x = -9 - (-1\frac{1}{7}) \\
 6\frac{5}{7}x = -7\frac{6}{7} \quad / : 6\frac{5}{7} \\
 x = \frac{-7\frac{6}{7}}{6\frac{5}{7}} \\
 x = -1\frac{8}{47} \\
 y = 1\frac{1}{7} + \frac{2}{7}x \\
 y = 1\frac{1}{7} + \frac{2}{7} \cdot (-1\frac{8}{47}) \\
 y = \frac{38}{47} \\
 L = \{-1\frac{8}{47} / \frac{38}{47}\}
 \end{array}$$

Aufgabe (22)

$$I \quad -7x + 9y = -3$$

$$II \quad 5x - 6y = -4$$

I nach x auflösen

$$-7x + 9y = -3$$

$$-7x + 9y = -3 \quad / -9y$$

$$-7x = -3 - 9y \quad / : (-7)$$

$$x = \frac{3}{7} + 1\frac{2}{7}y$$

I in II

$$5\left(\frac{3}{7} + 1\frac{2}{7}y\right) - 6y = -4$$

$$2\frac{1}{7} + 6\frac{3}{7}y - 6y = -4 \quad / -2\frac{1}{7}$$

$$+6\frac{3}{7}y - 6y = -4 - 2\frac{1}{7}$$

$$\frac{3}{7}y = -6\frac{1}{7} \quad / : \frac{3}{7}$$

$$y = -6\frac{1}{3}$$

$$y = -14\frac{1}{3}$$

$$x = \frac{3}{7} + 1\frac{2}{7}y$$

$$x = \frac{3}{7} + 1\frac{2}{7} \cdot \left(-14\frac{1}{3}\right)$$

$$x = -18$$

$$L = \{-18 / -14\frac{1}{3}\}$$

$$I \quad -7x + 9y = -3$$

$$II \quad 5x - 6y = -4$$

I nach y auflösen

$$-7x + 9y = -3$$

$$-7x + 9y = -3 \quad / +7x$$

$$9y = -3 + 7x \quad / : 9$$

$$y = -\frac{1}{3} + \frac{7}{9}x$$

I in II

$$5x + -6\left(-\frac{1}{3} + \frac{7}{9}x\right) = -4$$

$$2 - 4\frac{2}{3}x - 6x = -4 \quad / -2$$

$$-4\frac{2}{3}x - 6x = -4 - 2$$

$$\frac{1}{3}x = -6 \quad / : \frac{1}{3}$$

$$x = -18$$

$$x = -18$$

$$y = -\frac{1}{3} + \frac{7}{9}x$$

$$y = -\frac{1}{3} + \frac{7}{9} \cdot (-18)$$

$$y = -14\frac{1}{3}$$

$$L = \{-18 / -14\frac{1}{3}\}$$

## 2 Gleichsetzungsverfahren (2)

### 2.1 Aufgaben

Gesucht: Die Lösung des linearen Gleichungssystems

$$a1 \cdot x + b1 \cdot y = c1$$

$$a2 \cdot x + b2 \cdot y = c2$$

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

(1)

$$3x + 5y = 19$$

$$7x + 5y = 31$$

(10)

$$-1x + 1y = 3$$

$$\frac{1}{2}x - 4y = 5$$

(2)

$$1x + 1y = 10$$

$$1x - 1y = 4$$

(11)

$$1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3}$$

$$2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$$

(3)

$$9x - 2y = 5$$

$$5x - 2y = 1$$

(12)

$$\frac{2}{3}x - \frac{5}{7}y = \frac{2}{3}$$

$$1x + 1y = 10\frac{2}{3}$$

(4)

$$9x - 2y = 1$$

$$-3x - 3y = -7$$

(13)

$$1\frac{1}{2}x - 2y = 9$$

$$\frac{2}{5}x + \frac{1}{3}y = 5$$

(5)

$$1x + 3y = 9$$

$$3x - 2y = -6$$

(14)

$$2x + 3y = 4$$

$$\frac{1}{3}x - \frac{1}{5}y = 12$$

(6)

$$7x + 1\frac{1}{2}y = 1$$

$$2x - 5y = 22$$

(15)

$$1x + 2y = -4$$

$$-1x + 2y = 5$$

(7)

$$4x - 5y = 43$$

$$-3x - 7y = 0$$

(16)

$$2x + 4y = 4$$

$$-\frac{1}{2}x + 3y = 3$$

(8)

$$8x - 3y = 10$$

$$12x - 5y = 24$$

(17)

$$-\frac{1}{2}x + 4y = 6$$

$$-2x - 8y = 2$$

(9)

$$-\frac{1}{2}x + 1y = 2$$

$$\frac{1}{2}x - 3y = -3$$

(18)

$$1x + 1y = 1$$

$$2x + 2y = 2$$

(19)

$$1x + 1y = 2$$

$$3x + 3y = 3$$

$$2x - 7y = -8$$

$$7x - 1y = -9$$

(20)

$$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$$

$$-\frac{2}{3}x + \frac{1}{9}y = 9$$

(22)

$$-7x + 9y = -3$$

$$5x - 6y = -4$$

(21)

## 2.2 Lösungen

Aufgabe (1)

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

I nach y auflösen

$$3x + 5y = 19$$

$$3x + 5y = 19 \quad / - 5y$$

$$5y = 19 - 3x \quad / : 5$$

$$y = 3\frac{4}{5} - \frac{3}{5}x$$

II nach y auflösen

$$7x + 5y = 31$$

$$7x + 5y = 31 \quad / - 5y$$

$$5y = 31 - 7x \quad / : 5$$

$$y = 6\frac{1}{5} - 1\frac{2}{5}x$$

I = II

$$3\frac{4}{5} - \frac{3}{5}x = 6\frac{1}{5} - 1\frac{2}{5}x \quad / + \frac{3}{5}x / - 6\frac{1}{5}$$

$$3\frac{4}{5} - 6\frac{1}{5} = -1\frac{2}{5}x + \frac{3}{5}x$$

$$-2\frac{2}{5} = -\frac{4}{5}x \quad / : (-\frac{4}{5})$$

$$x = 3$$

x in I

$$y = 3\frac{4}{5} - \frac{3}{5} \cdot 3$$

$$y = 2$$

$$L = \{3/2\}$$

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

I nach x auflösen

$$3x + 5y = 19$$

$$3x + 5y = 19 \quad / - 5y$$

$$3x = 19 - 5y \quad / : 3$$

$$x = 6\frac{1}{3} - 1\frac{2}{3}y$$

II nach x auflösen

$$7x + 5y = 31$$

$$7x + 5y = 31 \quad / - 5y$$

$$7x = 31 - 5y \quad / : 7$$

$$x = 4\frac{3}{7} - \frac{5}{7}y$$

I = II

$$6\frac{1}{3} - 1\frac{2}{3}y = 4\frac{3}{7} - \frac{5}{7}y \quad / + 1\frac{2}{3}x / - 4\frac{3}{7}$$

$$6\frac{1}{3} - 4\frac{3}{7} = -\frac{5}{7}y + 1\frac{2}{3}y$$

$$1\frac{19}{21} = \frac{20}{21}y \quad / : \frac{20}{21}$$

$$y = 2$$

y in I

$$x = 6\frac{1}{3} - 1\frac{2}{3} \cdot 2$$

$$x = 3$$

$$L = \{3/2\}$$

Aufgabe (2)

$I \quad 1x + 1y = 10$ $II \quad 1x - 1y = 4$ <p>I nach y auflösen</p> $1x + 1y = 10$ $1x + 1y = 10 \quad / - 1y$ $1y = 10 - 1x \quad / : 1$ $y = 10 - 1x$ <p>II nach y auflösen</p> $1x - 1y = 4$ $1x - 1y = 4 \quad / + 1y$ $-1y = 4 - 1x \quad / : (-1)$ $y = -4 + 1x$ <p>I = II</p> $10 - 1x = -4 + 1x \quad / + 1x / + 4$ $10 + 4 = 1x + 1x$ $14 = 2x \quad / : 2$ $x = 7$ <p>x in I</p> $y = 10 - 17$ $y = 3$ $L = \{7/3\}$	$I \quad 1x + 1y = 10$ $II \quad 1x - 1y = 4$ <p>I nach x auflösen</p> $1x + 1y = 10$ $1x + 1y = 10 \quad / - 1y$ $1x = 10 - 1y \quad / : 1$ $x = 10 - 1y$ <p>II nach x auflösen</p> $1x - 1y = 4$ $1x - 1y = 4 \quad / + 1y$ $1x = 4 + 1y \quad / : 1$ $x = 4 + 1y$ <p>I = II</p> $10 - 1y = 4 + 1y \quad / + 1x / - 4$ $10 - 4 = 1y + 1y$ $6 = 2y \quad / : 2$ $y = 3$ <p>y in I</p> $x = 10 - 13$ $x = 7$ $L = \{7/3\}$
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Aufgabe (3)

$I \quad 9x - 2y = 5$ $II \quad 5x - 2y = 1$ <p>I nach y auflösen</p> $9x - 2y = 5$ $9x - 2y = 5 \quad / + 2y$ $-2y = 5 - 9x \quad / : (-2)$ $y = -2\frac{1}{2} + 4\frac{1}{2}x$ <p>II nach y auflösen</p> $5x - 2y = 1$ $5x - 2y = 1 \quad / + 2y$ $-2y = 1 - 5x \quad / : (-2)$ $y = -\frac{1}{2} + 2\frac{1}{2}x$ <p>I = II</p> $-2\frac{1}{2} + 4\frac{1}{2}x = -\frac{1}{2} + 2\frac{1}{2}x \quad / - 4\frac{1}{2}x / + \frac{1}{2}$ $-2\frac{1}{2} + \frac{1}{2} = 2\frac{1}{2}x - 4\frac{1}{2}x$ $-2 = -2x \quad / : (-2)$ $x = 1$ <p>x in I</p> $y = -2\frac{1}{2} + 4\frac{1}{2} \cdot 1$ $y = 2$ $L = \{1/2\}$	$I \quad 9x - 2y = 5$ $II \quad 5x - 2y = 1$ <p>I nach x auflösen</p> $9x - 2y = 5$ $9x - 2y = 5 \quad / + 2y$ $9x = 5 + 2y \quad / : 9$ $x = \frac{5}{9} + \frac{2}{9}y$ <p>II nach x auflösen</p> $5x - 2y = 1$ $5x - 2y = 1 \quad / + 2y$ $5x = 1 + 2y \quad / : 5$ $x = \frac{1}{5} + \frac{2}{5}y$ <p>I = II</p> $\frac{5}{9} + \frac{2}{9}y = \frac{1}{5} + \frac{2}{5}y \quad / - \frac{2}{9}x / - \frac{1}{5}$ $\frac{5}{9} - \frac{1}{5} = \frac{2}{5}y - \frac{2}{9}y$ $\frac{16}{45} = \frac{8}{45}y \quad / : \frac{8}{45}$ $y = 2$ <p>y in I</p> $x = \frac{5}{9} + \frac{2}{9} \cdot 2$ $x = 1$ $L = \{1/2\}$
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Aufgabe (4)

$$\begin{array}{l}
 I \quad 9x - 2y = 1 \\
 II \quad -3x - 3y = -7 \\
 \text{I nach y auflösen} \\
 9x - 2y = 1 \\
 9x - 2y = 1 \quad / + 2y \\
 -2y = 1 - 9x \quad / : (-2) \\
 y = -\frac{1}{2} + 4\frac{1}{2}x \\
 \text{II nach y auflösen} \\
 -3x - 3y = -7 \\
 -3x - 3y = -7 \quad / + 3y \\
 -3y = -7 + 3x \quad / : (-3) \\
 y = 2\frac{1}{3} - 1x \\
 \text{I} = \text{II} \\
 -\frac{1}{2} + 4\frac{1}{2}x = 2\frac{1}{3} - 1x \quad / - 4\frac{1}{2}x / - 2\frac{1}{3} \\
 -\frac{1}{2} - 2\frac{1}{3} = -1x - 4\frac{1}{2}x \\
 -2\frac{5}{6} = -5\frac{1}{2}x \quad / : (-5\frac{1}{2}) \\
 x = \frac{17}{33} \\
 \text{x in I} \\
 y = -\frac{1}{2} + 4\frac{1}{2} \frac{17}{33} \\
 y = 1\frac{9}{11} \\
 L = \left\{ \frac{17}{33} / 1\frac{9}{11} \right\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 9x - 2y = 1 \\
 II \quad -3x - 3y = -7 \\
 \text{I nach x auflösen} \\
 9x - 2y = 1 \\
 9x - 2y = 1 \quad / + 2y \\
 9x = 1 + 2y \quad / : 9 \\
 x = \frac{1}{9} + \frac{2}{9}y \\
 \text{II nach x auflösen} \\
 -3x - 3y = -7 \\
 -3x - 3y = -7 \quad / + 3y \\
 -3x = -7 + 3y \quad / : (-3) \\
 x = 2\frac{1}{3} - 1y \\
 \text{I} = \text{II} \\
 \frac{1}{9} + \frac{2}{9}y = 2\frac{1}{3} - 1y \quad / - \frac{2}{9}y / - 2\frac{1}{3} \\
 \frac{1}{9} - 2\frac{1}{3} = -1y - \frac{2}{9}y \\
 -2\frac{2}{9} = -1\frac{2}{9}y \quad / : (-1\frac{2}{9}) \\
 y = 1\frac{9}{11} \\
 \text{y in I} \\
 x = \frac{1}{9} + \frac{2}{9} 1\frac{9}{11} \\
 x = \frac{17}{33} \\
 L = \left\{ \frac{17}{33} / 1\frac{9}{11} \right\}
 \end{array}$$

Aufgabe (5)

$$\begin{array}{l}
 I \quad 1x + 3y = 9 \\
 II \quad 3x - 2y = -6 \\
 \text{I nach y auflösen} \\
 1x + 3y = 9 \\
 1x + 3y = 9 \quad / - 3y \\
 3y = 9 - 1x \quad / : 3 \\
 y = 3 - \frac{1}{3}x \\
 \text{II nach y auflösen} \\
 3x - 2y = -6 \\
 3x - 2y = -6 \quad / + 2y \\
 -2y = -6 - 3x \quad / : (-2) \\
 y = 3 + 1\frac{1}{2}x \\
 \text{I} = \text{II} \\
 3 - \frac{1}{3}x = 3 + 1\frac{1}{2}x \quad / + \frac{1}{3}x / - 3 \\
 3 - 3 = 1\frac{1}{2}x + \frac{1}{3}x \\
 0 = 1\frac{5}{6}x \quad / : 1\frac{5}{6} \\
 x = 0 \\
 \text{x in I} \\
 y = 3 - \frac{1}{3} \cdot 0 \\
 y = 3 \\
 L = \{0/3\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 3y = 9 \\
 II \quad 3x - 2y = -6 \\
 \text{I nach x auflösen} \\
 1x + 3y = 9 \\
 1x + 3y = 9 \quad / - 3y \\
 1x = 9 - 3y \quad / : 1 \\
 x = 9 - 3y \\
 \text{II nach x auflösen} \\
 3x - 2y = -6 \\
 3x - 2y = -6 \quad / + 2y \\
 3x = -6 + 2y \quad / : 3 \\
 x = -2 + \frac{2}{3}y \\
 \text{I} = \text{II} \\
 9 - 3y = -2 + \frac{2}{3}y \quad / + 3x / + 2 \\
 9 + 2 = \frac{2}{3}y + 3y \\
 11 = 3\frac{2}{3}y \quad / : 3\frac{2}{3} \\
 y = 3 \\
 \text{y in I} \\
 x = 9 - 3 \cdot 3 \\
 x = 0 \\
 L = \{0/3\}
 \end{array}$$

Aufgabe (6)

$$I \quad 7x + 1\frac{1}{2}y = 1$$

$$II \quad 2x - 5y = 22$$

I nach y auflösen

$$7x + 1\frac{1}{2}y = 1$$

$$7x + 1\frac{1}{2}y = 1 \quad / - 1\frac{1}{2}y$$

$$1\frac{1}{2}y = 1 - 7x \quad / : 1\frac{1}{2}$$

$$y = \frac{2}{3} - 4\frac{2}{3}x$$

II nach y auflösen

$$2x - 5y = 22$$

$$2x - 5y = 22 \quad / + 5y$$

$$-5y = 22 - 2x \quad / : (-5)$$

$$y = -4\frac{2}{5} + \frac{2}{5}x$$

I = II

$$\frac{2}{3} - 4\frac{2}{3}x = -4\frac{2}{5} + \frac{2}{5}x \quad / + 4\frac{2}{3}x / + 4\frac{2}{5}$$

$$\frac{2}{3} + 4\frac{2}{5} = \frac{2}{5}x + 4\frac{2}{3}x$$

$$5\frac{1}{15} = 5\frac{1}{15}x \quad / : 5\frac{1}{15}$$

$$x = 1$$

x in I

$$y = \frac{2}{3} - 4\frac{2}{3} \cdot 1$$

$$y = -4$$

$$L = \{1 / -4\}$$

$$I \quad 7x + 1\frac{1}{2}y = 1$$

$$II \quad 2x - 5y = 22$$

I nach x auflösen

$$7x + 1\frac{1}{2}y = 1$$

$$7x + 1\frac{1}{2}y = 1 \quad / - 1\frac{1}{2}y$$

$$7x = 1 - 1\frac{1}{2}y \quad / : 7$$

$$x = \frac{1}{7} - \frac{3}{14}y$$

II nach x auflösen

$$2x - 5y = 22$$

$$2x - 5y = 22 \quad / + 5y$$

$$2x = 22 + 5y \quad / : 2$$

$$x = 11 + 2\frac{1}{2}y$$

I = II

$$\frac{1}{7} - \frac{3}{14}y = 11 + 2\frac{1}{2}y \quad / + \frac{3}{14}x / - 11$$

$$\frac{1}{7} - 11 = 2\frac{1}{2}y + \frac{3}{14}y$$

$$-10\frac{6}{7} = 2\frac{5}{7}y \quad / : 2\frac{5}{7}$$

$$y = -4$$

y in I

$$x = \frac{1}{7} - \frac{3}{14} \cdot -4$$

$$x = 1$$

$$L = \{1 / -4\}$$

Aufgabe (7)

$$I \quad 4x - 5y = 43$$

$$II \quad -3x - 7y = 0$$

I nach y auflösen

$$4x - 5y = 43$$

$$4x - 5y = 43 \quad / + 5y$$

$$-5y = 43 - 4x \quad / : (-5)$$

$$y = -8\frac{3}{5} + \frac{4}{5}x$$

II nach y auflösen

$$-3x - 7y = 0$$

$$-3x - 7y = 0 \quad / + 7y$$

$$-7y = 0 + 3x \quad / : (-7)$$

$$y = 0 - \frac{3}{7}x$$

I = II

$$-8\frac{3}{5} + \frac{4}{5}x = 0 - \frac{3}{7}x \quad / - \frac{4}{5}x / - 0$$

$$-8\frac{3}{5} - 0 = -\frac{3}{7}x - \frac{4}{5}x$$

$$-8\frac{3}{5} = -1\frac{8}{35}x \quad / : (-1\frac{8}{35})$$

$$x = 7$$

x in I

$$y = -8\frac{3}{5} + \frac{4}{5} \cdot 7$$

$$y = -3$$

$$L = \{7 / -3\}$$

$$I \quad 4x - 5y = 43$$

$$II \quad -3x - 7y = 0$$

I nach x auflösen

$$4x - 5y = 43$$

$$4x - 5y = 43 \quad / + 5y$$

$$4x = 43 + 5y \quad / : 4$$

$$x = 10\frac{3}{4} + 1\frac{1}{4}y$$

II nach x auflösen

$$-3x - 7y = 0$$

$$-3x - 7y = 0 \quad / + 7y$$

$$-3x = 0 + 7y \quad / : (-3)$$

$$x = 0 - 2\frac{1}{3}y$$

I = II

$$10\frac{3}{4} + 1\frac{1}{4}y = 0 - 2\frac{1}{3}y \quad / - 1\frac{1}{4}x / - 0$$

$$10\frac{3}{4} - 0 = -2\frac{1}{3}y - 1\frac{1}{4}y$$

$$10\frac{3}{4} = -3\frac{7}{12}y \quad / : (-3\frac{7}{12})$$

$$y = -3$$

y in I

$$x = 10\frac{3}{4} + 1\frac{1}{4} \cdot -3$$

$$x = 7$$

$$L = \{7 / -3\}$$

Aufgabe (8)

$I \quad 8x - 3y = 10$ $II \quad 12x - 5y = 24$ <p>I nach y auflösen</p> $8x - 3y = 10$ $8x - 3y = 10 \quad / + 3y$ $-3y = 10 - 8x \quad / : (-3)$ $y = -3\frac{1}{3} + 2\frac{2}{3}x$ <p>II nach y auflösen</p> $12x - 5y = 24$ $12x - 5y = 24 \quad / + 5y$ $-5y = 24 - 12x \quad / : (-5)$ $y = -4\frac{4}{5} + 2\frac{2}{5}x$ <p>I = II</p> $-3\frac{1}{3} + 2\frac{2}{3}x = -4\frac{4}{5} + 2\frac{2}{5}x \quad / - 2\frac{2}{3}x / + 4\frac{4}{5}$ $-3\frac{1}{3} + 4\frac{4}{5} = 2\frac{2}{5}x - 2\frac{2}{3}x$ $1\frac{7}{15} = -\frac{4}{15}x \quad / : (-\frac{4}{15})$ $x = -5\frac{1}{2}$ <p>x in I</p> $y = -3\frac{1}{3} + 2\frac{2}{3} \cdot -5\frac{1}{2}$ $y = -18$ $L = \{-5\frac{1}{2} / -18\}$	$I \quad 8x - 3y = 10$ $II \quad 12x - 5y = 24$ <p>I nach x auflösen</p> $8x - 3y = 10$ $8x - 3y = 10 \quad / + 3y$ $8x = 10 + 3y \quad / : 8$ $x = 1\frac{1}{4} + \frac{3}{8}y$ <p>II nach x auflösen</p> $12x - 5y = 24$ $12x - 5y = 24 \quad / + 5y$ $12x = 24 + 5y \quad / : 12$ $x = 2 + \frac{5}{12}y$ <p>I = II</p> $1\frac{1}{4} + \frac{3}{8}y = 2 + \frac{5}{12}y \quad / - \frac{3}{8}y / - 2$ $1\frac{1}{4} - 2 = \frac{5}{12}y - \frac{3}{8}y$ $-\frac{3}{4} = \frac{1}{24}y \quad / : \frac{1}{24}$ $y = -18$ <p>y in I</p> $x = 1\frac{1}{4} + \frac{3}{8} \cdot -18$ $x = -5\frac{1}{2}$ $L = \{-5\frac{1}{2} / -18\}$
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Aufgabe (9)

$I \quad -\frac{1}{2}x + 1y = 2$ $II \quad \frac{1}{2}x - 3y = -3$ <p>I nach y auflösen</p> $-\frac{1}{2}x + 1y = 2$ $-\frac{1}{2}x + 1y = 2 \quad / - 1y$ $1y = 2 + \frac{1}{2}x \quad / : 1$ $y = 2 + \frac{1}{2}x$ <p>II nach y auflösen</p> $\frac{1}{2}x - 3y = -3$ $\frac{1}{2}x - 3y = -3 \quad / + 3y$ $-3y = -3 - \frac{1}{2}x \quad / : (-3)$ $y = 1 + \frac{1}{6}x$ <p>I = II</p> $2 + \frac{1}{2}x = 1 + \frac{1}{6}x \quad / - \frac{1}{2}x / - 1$ $2 - 1 = \frac{1}{6}x - \frac{1}{2}x$ $1 = -\frac{1}{3}x \quad / : (-\frac{1}{3})$ $x = -3$ <p>x in I</p> $y = 2 + \frac{1}{2} \cdot -3$ $y = \frac{1}{2}$ $L = \{-3 / \frac{1}{2}\}$	$I \quad -\frac{1}{2}x + 1y = 2$ $II \quad \frac{1}{2}x - 3y = -3$ <p>I nach x auflösen</p> $-\frac{1}{2}x + 1y = 2$ $-\frac{1}{2}x + 1y = 2 \quad / - 1y$ $-\frac{1}{2}x = 2 - 1y \quad / : (-\frac{1}{2})$ $x = -4 + 2y$ <p>II nach x auflösen</p> $\frac{1}{2}x - 3y = -3$ $\frac{1}{2}x - 3y = -3 \quad / + 3y$ $\frac{1}{2}x = -3 + 3y \quad / : \frac{1}{2}$ $x = -6 + 6y$ <p>I = II</p> $-4 + 2y = -6 + 6y \quad / - 2x / + 6$ $-4 + 6 = 6y - 2y$ $2 = 4y \quad / : 4$ $y = \frac{1}{2}$ <p>y in I</p> $x = -4 + 2\frac{1}{2}$ $x = -3$ $L = \{-3 / \frac{1}{2}\}$
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Aufgabe (10)

$$\begin{array}{l}
 I \quad -1x + 1y = 3 \\
 II \quad \frac{1}{2}x - 4y = 5 \\
 \text{I nach y auflösen} \\
 -1x + 1y = 3 \\
 -1x + 1y = 3 \quad / -1y \\
 1y = 3 + 1x \quad / : 1 \\
 y = 3 + 1x \\
 \text{II nach y auflösen} \\
 \frac{1}{2}x - 4y = 5 \\
 \frac{1}{2}x - 4y = 5 \quad / + 4y \\
 -4y = 5 - \frac{1}{2}x \quad / : (-4) \\
 y = -1\frac{1}{4} + \frac{1}{8}x \\
 \text{I} = \text{II} \\
 3 + 1x = -1\frac{1}{4} + \frac{1}{8}x \quad / -1x / + 1\frac{1}{4} \\
 3 + 1\frac{1}{4} = \frac{1}{8}x - 1x \\
 4\frac{1}{4} = -\frac{7}{8}x \quad / : (-\frac{7}{8}) \\
 x = -4\frac{6}{7} \\
 \text{x in I} \\
 y = 3 + 1 - 4\frac{6}{7} \\
 y = -1\frac{6}{7} \\
 L = \{-4\frac{6}{7} / -1\frac{6}{7}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad -1x + 1y = 3 \\
 II \quad \frac{1}{2}x - 4y = 5 \\
 \text{I nach x auflösen} \\
 -1x + 1y = 3 \\
 -1x + 1y = 3 \quad / -1y \\
 -1x = 3 - 1y \quad / : (-1) \\
 x = -3 + 1y \\
 \text{II nach x auflösen} \\
 \frac{1}{2}x - 4y = 5 \\
 \frac{1}{2}x - 4y = 5 \quad / + 4y \\
 \frac{1}{2}x = 5 + 4y \quad / : \frac{1}{2} \\
 x = 10 + 8y \\
 \text{I} = \text{II} \\
 -3 + 1y = 10 + 8y \quad / -1x / -10 \\
 -3 - 10 = 8y - 1y \\
 -13 = 7y \quad / : 7 \\
 y = -1\frac{6}{7} \\
 \text{y in I} \\
 x = -3 + 1 - 1\frac{6}{7} \\
 x = -4\frac{6}{7} \\
 L = \{-4\frac{6}{7} / -1\frac{6}{7}\}
 \end{array}$$

Aufgabe (11)

$$\begin{array}{l}
 I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
 II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
 \text{I nach y auflösen} \\
 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / + 1\frac{1}{3}y \\
 -1\frac{1}{3}y = 5\frac{1}{3} - 1\frac{1}{5}x \quad / : (-1\frac{1}{3}) \\
 y = -4 + \frac{9}{10}x \\
 \text{II nach y auflösen} \\
 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / + \frac{1}{4}y \\
 -\frac{1}{4}y = 12\frac{3}{8} - 2\frac{1}{2}x \quad / : (-\frac{1}{4}) \\
 y = -49\frac{1}{2} + 10x \\
 \text{I} = \text{II} \\
 -4 + \frac{9}{10}x = -49\frac{1}{2} + 10x \quad / -\frac{9}{10}x / + 49\frac{1}{2} \\
 -4 + 49\frac{1}{2} = 10x - \frac{9}{10}x \\
 45\frac{1}{2} = 9\frac{1}{10}x \quad / : 9\frac{1}{10} \\
 x = 5 \\
 \text{x in I} \\
 y = -4 + \frac{9}{10} \cdot 5 \\
 y = \frac{1}{2} \\
 L = \{5 / \frac{1}{2}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
 II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
 \text{I nach x auflösen} \\
 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / + 1\frac{1}{3}y \\
 1\frac{1}{5}x = 5\frac{1}{3} + 1\frac{1}{3}y \quad / : 1\frac{1}{5} \\
 x = 4\frac{4}{9} + 1\frac{1}{9}y \\
 \text{II nach x auflösen} \\
 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / + \frac{1}{4}y \\
 2\frac{1}{2}x = 12\frac{3}{8} + \frac{1}{4}y \quad / : 2\frac{1}{2} \\
 x = 4\frac{19}{20} + \frac{1}{10}y \\
 \text{I} = \text{II} \\
 4\frac{4}{9} + 1\frac{1}{9}y = 4\frac{19}{20} + \frac{1}{10}y \quad / -1\frac{1}{9}x / -4\frac{19}{20} \\
 4\frac{4}{9} - 4\frac{19}{20} = \frac{1}{10}y - 1\frac{1}{9}y \\
 -0,506 = -1\frac{1}{90}y \quad / : (-1\frac{1}{90}) \\
 y = \frac{1}{2} \\
 \text{y in I} \\
 x = 4\frac{4}{9} + 1\frac{1}{9} \cdot \frac{1}{2} \\
 x = 5 \\
 L = \{5 / \frac{1}{2}\}
 \end{array}$$

Aufgabe (12)

$$\begin{array}{l}
 I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
 II \quad 1x + 1y = 10\frac{2}{3} \\
 \text{I nach y auflösen} \\
 \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
 \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / + \frac{5}{7}y \\
 -\frac{5}{7}y = \frac{2}{3} - \frac{2}{3}x \quad / : (-\frac{5}{7}) \\
 y = -\frac{14}{15} + \frac{14}{15}x \\
 \text{II nach y auflösen} \\
 1x + 1y = 10\frac{2}{3} \\
 1x + 1y = 10\frac{2}{3} \quad / - 1y \\
 1y = 10\frac{2}{3} - 1x \quad / : 1 \\
 y = 10\frac{2}{3} - 1x \\
 \text{I} = \text{II} \\
 -\frac{14}{15} + \frac{14}{15}x = 10\frac{2}{3} - 1x \quad / - \frac{14}{15}x / - 10\frac{2}{3} \\
 -\frac{14}{15} - 10\frac{2}{3} = -1x - \frac{14}{15}x \\
 -11\frac{3}{5} = -1\frac{14}{15}x \quad / : (-1\frac{14}{15}) \\
 x = 6 \\
 \text{x in I} \\
 y = -\frac{14}{15} + \frac{14}{15}6 \\
 y = 4\frac{2}{3} \\
 L = \{6/4\frac{2}{3}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
 II \quad 1x + 1y = 10\frac{2}{3} \\
 \text{I nach x auflösen} \\
 \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
 \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / + \frac{5}{7}y \\
 \frac{2}{3}x = \frac{2}{3} + \frac{5}{7}y \quad / : \frac{2}{3} \\
 x = 1 + 1\frac{1}{14}y \\
 \text{II nach x auflösen} \\
 1x + 1y = 10\frac{2}{3} \\
 1x + 1y = 10\frac{2}{3} \quad / - 1y \\
 1x = 10\frac{2}{3} - 1y \quad / : 1 \\
 x = 10\frac{2}{3} - 1y \\
 \text{I} = \text{II} \\
 1 + 1\frac{1}{14}y = 10\frac{2}{3} - 1y \quad / - 1\frac{1}{14}y / - 10\frac{2}{3} \\
 1 - 10\frac{2}{3} = -1y - 1\frac{1}{14}y \\
 -9\frac{2}{3} = -2\frac{1}{14}y \quad / : (-2\frac{1}{14}) \\
 y = 4\frac{2}{3} \\
 \text{y in I} \\
 x = 1 + 1\frac{1}{14}4\frac{2}{3} \\
 x = 6 \\
 L = \{6/4\frac{2}{3}\}
 \end{array}$$

Aufgabe (13)

$$\begin{array}{l}
 I \quad 1\frac{1}{2}x - 2y = 9 \\
 II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \text{I nach y auflösen} \\
 1\frac{1}{2}x - 2y = 9 \\
 1\frac{1}{2}x - 2y = 9 \quad / + 2y \\
 -2y = 9 - 1\frac{1}{2}x \quad / : (-2) \\
 y = -4\frac{1}{2} + \frac{3}{4}x \\
 \text{II nach y auflösen} \\
 \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \frac{2}{5}x + \frac{1}{3}y = 5 \quad / - \frac{1}{3}y \\
 \frac{1}{3}y = 5 - \frac{2}{5}x \quad / : \frac{1}{3} \\
 y = 15 - 1\frac{1}{5}x \\
 \text{I} = \text{II} \\
 -4\frac{1}{2} + \frac{3}{4}x = 15 - 1\frac{1}{5}x \quad / - \frac{3}{4}x / - 15 \\
 -4\frac{1}{2} - 15 = -1\frac{1}{5}x - \frac{3}{4}x \\
 -19\frac{1}{2} = -1\frac{19}{20}x \quad / : (-1\frac{19}{20}) \\
 x = 10 \\
 \text{x in I} \\
 y = -4\frac{1}{2} + \frac{3}{4}10 \\
 y = 3 \\
 L = \{10/3\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1\frac{1}{2}x - 2y = 9 \\
 II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \text{I nach x auflösen} \\
 1\frac{1}{2}x - 2y = 9 \\
 1\frac{1}{2}x - 2y = 9 \quad / + 2y \\
 1\frac{1}{2}x = 9 + 2y \quad / : 1\frac{1}{2} \\
 x = 6 + 1\frac{1}{3}y \\
 \text{II nach x auflösen} \\
 \frac{2}{5}x + \frac{1}{3}y = 5 \\
 \frac{2}{5}x + \frac{1}{3}y = 5 \quad / - \frac{1}{3}y \\
 \frac{2}{5}x = 5 - \frac{1}{3}y \quad / : \frac{2}{5} \\
 x = 12\frac{1}{2} - \frac{5}{6}y \\
 \text{I} = \text{II} \\
 6 + 1\frac{1}{3}y = 12\frac{1}{2} - \frac{5}{6}y \quad / - 1\frac{1}{3}y / - 12\frac{1}{2} \\
 6 - 12\frac{1}{2} = -\frac{5}{6}y - 1\frac{1}{3}y \\
 -6\frac{1}{2} = -2\frac{1}{6}y \quad / : (-2\frac{1}{6}) \\
 y = 3 \\
 \text{y in I} \\
 x = 6 + 1\frac{1}{3}3 \\
 x = 10 \\
 L = \{10/3\}
 \end{array}$$

Aufgabe (14)

$$\begin{array}{l}
 I \quad 2x + 3y = 4 \\
 II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \text{I nach y auflösen} \\
 2x + 3y = 4 \\
 2x + 3y = 4 \quad / - 3y \\
 3y = 4 - 2x \quad / : 3 \\
 y = 1\frac{1}{3} - \frac{2}{3}x \\
 \text{II nach y auflösen} \\
 \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \frac{1}{3}x - \frac{1}{5}y = 12 \quad / + \frac{1}{5}y \\
 -\frac{1}{5}y = 12 - \frac{1}{3}x \quad / : (-\frac{1}{5}) \\
 y = -60 + 1\frac{2}{3}x \\
 \text{I} = \text{II} \\
 1\frac{1}{3} - \frac{2}{3}x = -60 + 1\frac{2}{3}x \quad / + \frac{2}{3}x / + 60 \\
 1\frac{1}{3} + 60 = 1\frac{2}{3}x + \frac{2}{3}x \\
 61\frac{1}{3} = 2\frac{1}{3}x \quad / : 2\frac{1}{3} \\
 x = 26\frac{2}{7} \\
 \text{x in I} \\
 y = 1\frac{1}{3} - \frac{2}{3} \cdot 26\frac{2}{7} \\
 y = -16\frac{4}{21} \\
 L = \{26\frac{2}{7} / -16\frac{4}{21}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 2x + 3y = 4 \\
 II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \text{I nach x auflösen} \\
 2x + 3y = 4 \\
 2x + 3y = 4 \quad / - 3y \\
 2x = 4 - 3y \quad / : 2 \\
 x = 2 - 1\frac{1}{2}y \\
 \text{II nach x auflösen} \\
 \frac{1}{3}x - \frac{1}{5}y = 12 \\
 \frac{1}{3}x - \frac{1}{5}y = 12 \quad / + \frac{1}{5}y \\
 \frac{1}{3}x = 12 + \frac{1}{5}y \quad / : \frac{1}{3} \\
 x = 36 + \frac{3}{5}y \\
 \text{I} = \text{II} \\
 2 - 1\frac{1}{2}y = 36 + \frac{3}{5}y \quad / + 1\frac{1}{2}x / - 36 \\
 2 - 36 = \frac{3}{5}y + 1\frac{1}{2}y \\
 -34 = 2\frac{1}{10}y \quad / : 2\frac{1}{10} \\
 y = -16\frac{4}{21} \\
 \text{y in I} \\
 x = 2 - 1\frac{1}{2} - 16\frac{4}{21} \\
 x = 26\frac{2}{7} \\
 L = \{26\frac{2}{7} / -16\frac{4}{21}\}
 \end{array}$$

Aufgabe (15)

$$\begin{array}{l}
 I \quad 1x + 2y = -4 \\
 II \quad -1x + 2y = 5 \\
 \text{I nach y auflösen} \\
 1x + 2y = -4 \\
 1x + 2y = -4 \quad / - 2y \\
 2y = -4 - 1x \quad / : 2 \\
 y = -2 - \frac{1}{2}x \\
 \text{II nach y auflösen} \\
 -1x + 2y = 5 \\
 -1x + 2y = 5 \quad / - 2y \\
 2y = 5 + 1x \quad / : 2 \\
 y = 2\frac{1}{2} + \frac{1}{2}x \\
 \text{I} = \text{II} \\
 -2 - \frac{1}{2}x = 2\frac{1}{2} + \frac{1}{2}x \quad / + \frac{1}{2}x / - 2\frac{1}{2} \\
 -2 - 2\frac{1}{2} = \frac{1}{2}x + \frac{1}{2}x \\
 -4\frac{1}{2} = 1x \quad / : 1 \\
 x = -4\frac{1}{2} \\
 \text{x in I} \\
 y = -2 - \frac{1}{2} - 4\frac{1}{2} \\
 y = \frac{1}{4} \\
 L = \{-4\frac{1}{2} / \frac{1}{4}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 2y = -4 \\
 II \quad -1x + 2y = 5 \\
 \text{I nach x auflösen} \\
 1x + 2y = -4 \\
 1x + 2y = -4 \quad / - 2y \\
 1x = -4 - 2y \quad / : 1 \\
 x = -4 - 2y \\
 \text{II nach x auflösen} \\
 -1x + 2y = 5 \\
 -1x + 2y = 5 \quad / - 2y \\
 -1x = 5 - 2y \quad / : (-1) \\
 x = -5 + 2y \\
 \text{I} = \text{II} \\
 -4 - 2y = -5 + 2y \quad / + 2x / + 5 \\
 -4 + 5 = 2y + 2y \\
 1 = 4y \quad / : 4 \\
 y = \frac{1}{4} \\
 \text{y in I} \\
 x = -4 - 2\frac{1}{4} \\
 x = -4\frac{1}{2} \\
 L = \{-4\frac{1}{2} / \frac{1}{4}\}
 \end{array}$$

Aufgabe (16)

$I \quad 2x + 4y = 4$ $II \quad -\frac{1}{2}x + 3y = 3$ <p>I nach y auflösen</p> $2x + 4y = 4$ $2x + 4y = 4 \quad / - 4y$ $4y = 4 - 2x \quad / : 4$ $y = 1 - \frac{1}{2}x$ <p>II nach y auflösen</p> $-\frac{1}{2}x + 3y = 3$ $-\frac{1}{2}x + 3y = 3 \quad / - 3y$ $3y = 3 + \frac{1}{2}x \quad / : 3$ $y = 1 + \frac{1}{6}x$ <p>I = II</p> $1 - \frac{1}{2}x = 1 + \frac{1}{6}x \quad / + \frac{1}{2}x / - 1$ $1 - 1 = \frac{1}{6}x + \frac{1}{2}x$ $0 = \frac{2}{3}x \quad / : \frac{2}{3}$ $x = 0$ <p>x in I</p> $y = 1 - \frac{1}{2} \cdot 0$ $y = 1$ $L = \{0/1\}$	$I \quad 2x + 4y = 4$ $II \quad -\frac{1}{2}x + 3y = 3$ <p>I nach x auflösen</p> $2x + 4y = 4$ $2x + 4y = 4 \quad / - 4y$ $2x = 4 - 4y \quad / : 2$ $x = 2 - 2y$ <p>II nach x auflösen</p> $-\frac{1}{2}x + 3y = 3$ $-\frac{1}{2}x + 3y = 3 \quad / - 3y$ $-\frac{1}{2}x = 3 - 3y \quad / : (-\frac{1}{2})$ $x = -6 + 6y$ <p>I = II</p> $2 - 2y = -6 + 6y \quad / + 2x / + 6$ $2 + 6 = 6y + 2y$ $8 = 8y \quad / : 8$ $y = 1$ <p>y in I</p> $x = 2 - 2 \cdot 1$ $x = 0$ $L = \{0/1\}$
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Aufgabe (17)

$I \quad -\frac{1}{2}x + 4y = 6$ $II \quad -2x - 8y = 2$ <p>I nach y auflösen</p> $-\frac{1}{2}x + 4y = 6$ $-\frac{1}{2}x + 4y = 6 \quad / - 4y$ $4y = 6 + \frac{1}{2}x \quad / : 4$ $y = 1\frac{1}{2} + \frac{1}{8}x$ <p>II nach y auflösen</p> $-2x - 8y = 2$ $-2x - 8y = 2 \quad / + 8y$ $-8y = 2 + 2x \quad / : (-8)$ $y = -\frac{1}{4} - \frac{1}{4}x$ <p>I = II</p> $1\frac{1}{2} + \frac{1}{8}x = -\frac{1}{4} - \frac{1}{4}x \quad / - \frac{1}{8}x / + \frac{1}{4}$ $1\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}x - \frac{1}{8}x$ $1\frac{3}{4} = -\frac{3}{8}x \quad / : (-\frac{3}{8})$ $x = -4\frac{2}{3}$ <p>x in I</p> $y = 1\frac{1}{2} + \frac{1}{8} \cdot -4\frac{2}{3}$ $y = \frac{11}{12}$ $L = \{-4\frac{2}{3} / \frac{11}{12}\}$	$I \quad -\frac{1}{2}x + 4y = 6$ $II \quad -2x - 8y = 2$ <p>I nach x auflösen</p> $-\frac{1}{2}x + 4y = 6$ $-\frac{1}{2}x + 4y = 6 \quad / - 4y$ $-\frac{1}{2}x = 6 - 4y \quad / : (-\frac{1}{2})$ $x = -12 + 8y$ <p>II nach x auflösen</p> $-2x - 8y = 2$ $-2x - 8y = 2 \quad / + 8y$ $-2x = 2 + 8y \quad / : (-2)$ $x = -1 - 4y$ <p>I = II</p> $-12 + 8y = -1 - 4y \quad / - 8x / + 1$ $-12 + 1 = -4y - 8y$ $-11 = -12y \quad / : (-12)$ $y = \frac{11}{12}$ <p>y in I</p> $x = -12 + 8\frac{11}{12}$ $x = -4\frac{2}{3}$ $L = \{-4\frac{2}{3} / \frac{11}{12}\}$
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Aufgabe (18)

$$\begin{array}{l}
 I \quad 1x + 1y = 1 \\
 II \quad 2x + 2y = 2 \\
 \text{I nach y auflösen} \\
 1x + 1y = 1 \\
 1x + 1y = 1 \quad / - 1y \\
 1y = 1 - 1x \quad / : 1 \\
 y = 1 - 1x \\
 \text{II nach y auflösen} \\
 2x + 2y = 2 \\
 2x + 2y = 2 \quad / - 2y \\
 2y = 2 - 2x \quad / : 2 \\
 y = 1 - 1x \\
 \text{I} = \text{II} \\
 1 - 1x = 1 - 1x \quad / + 1x / - 1 \\
 1 - 1 = -1x + 1x \\
 \\
 L = \text{unendlich}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 1y = 1 \\
 II \quad 2x + 2y = 2 \\
 \text{I nach x auflösen} \\
 1x + 1y = 1 \\
 1x + 1y = 1 \quad / - 1y \\
 1x = 1 - 1y \quad / : 1 \\
 x = 1 - 1y \\
 \text{II nach x auflösen} \\
 2x + 2y = 2 \\
 2x + 2y = 2 \quad / - 2y \\
 2x = 2 - 2y \quad / : 2 \\
 x = 1 - 1y \\
 \text{I} = \text{II} \\
 1 - 1y = 1 - 1y \quad / + 1x / - 1 \\
 1 - 1 = -1y + 1y \\
 \\
 L = \text{unendlich}
 \end{array}$$

Aufgabe (19)

$$\begin{array}{l}
 I \quad 1x + 1y = 2 \\
 II \quad 3x + 3y = 3 \\
 \text{I nach y auflösen} \\
 1x + 1y = 2 \\
 1x + 1y = 2 \quad / - 1y \\
 1y = 2 - 1x \quad / : 1 \\
 y = 2 - 1x \\
 \text{II nach y auflösen} \\
 3x + 3y = 3 \\
 3x + 3y = 3 \quad / - 3y \\
 3y = 3 - 3x \quad / : 3 \\
 y = 1 - 1x \\
 \text{I} = \text{II} \\
 2 - 1x = 1 - 1x \quad / + 1x / - 1 \\
 2 - 1 = -1x + 1x \\
 \\
 L = \{\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 1y = 2 \\
 II \quad 3x + 3y = 3 \\
 \text{I nach x auflösen} \\
 1x + 1y = 2 \\
 1x + 1y = 2 \quad / - 1y \\
 1x = 2 - 1y \quad / : 1 \\
 x = 2 - 1y \\
 \text{II nach x auflösen} \\
 3x + 3y = 3 \\
 3x + 3y = 3 \quad / - 3y \\
 3x = 3 - 3y \quad / : 3 \\
 x = 1 - 1y \\
 \text{I} = \text{II} \\
 2 - 1y = 1 - 1y \quad / + 1x / - 1 \\
 2 - 1 = -1y + 1y \\
 \\
 L = \{\}
 \end{array}$$

Aufgabe (20)

$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$ $II \quad -\frac{3}{3}x + \frac{1}{9}y = 9$ <p>I nach y auflösen</p> $-1\frac{4}{5}x + 1\frac{1}{3}y = -1$ $-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / -1\frac{1}{3}y$ $1\frac{1}{3}y = -1 + 1\frac{4}{5}x \quad / : 1\frac{1}{3}$ $y = -\frac{3}{4} + 1\frac{7}{20}x$ <p>II nach y auflösen</p> $-\frac{2}{3}x + \frac{1}{9}y = 9$ $-\frac{2}{3}x + \frac{1}{9}y = 9 \quad / -\frac{1}{9}y$ $\frac{1}{9}y = 9 + \frac{2}{3}x \quad / : \frac{1}{9}$ $y = 81 + 6x$ <p>I = II</p> $-\frac{3}{4} + 1\frac{7}{20}x = 81 + 6x \quad / -1\frac{7}{20}x / -81$ $-\frac{3}{4} - 81 = 6x - 1\frac{7}{20}x$ $-81\frac{3}{4} = 4\frac{13}{20}x \quad / : 4\frac{13}{20}$ $x = -17\frac{18}{31}$ <p>x in I</p> $y = -\frac{3}{4} + 1\frac{7}{20} - 17\frac{18}{31}$ $y = -24\frac{15}{31}$ $L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$	$I \quad -1\frac{4}{5}x + 1\frac{1}{3}y = -1$ $II \quad -\frac{3}{3}x + \frac{1}{9}y = 9$ <p>I nach x auflösen</p> $-1\frac{4}{5}x + 1\frac{1}{3}y = -1$ $-1\frac{4}{5}x + 1\frac{1}{3}y = -1 \quad / -1\frac{1}{3}y$ $-1\frac{4}{5}x = -1 - 1\frac{1}{3}y \quad / : (-1\frac{4}{5})$ $x = \frac{5}{9} + \frac{20}{27}y$ <p>II nach x auflösen</p> $-\frac{2}{3}x + \frac{1}{9}y = 9$ $-\frac{2}{3}x + \frac{1}{9}y = 9 \quad / -\frac{1}{9}y$ $-\frac{2}{3}x = 9 - \frac{1}{9}y \quad / : (-\frac{2}{3})$ $x = -13\frac{1}{2} + \frac{1}{6}y$ <p>I = II</p> $\frac{5}{9} + \frac{20}{27}y = -13\frac{1}{2} + \frac{1}{6}y \quad / -\frac{20}{27}y / +13\frac{1}{2}$ $\frac{5}{9} + 13\frac{1}{2} = \frac{1}{6}y - \frac{20}{27}y$ $14\frac{1}{18} = -\frac{31}{54}y \quad / : (-\frac{31}{54})$ $y = -24\frac{15}{31}$ <p>y in I</p> $x = \frac{5}{9} + \frac{20}{27} - 24\frac{15}{31}$ $x = -17\frac{18}{31}$ $L = \{-17\frac{18}{31} / -24\frac{15}{31}\}$
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Aufgabe (21)

$I \quad 2x - 7y = -8$ $II \quad 7x - 1y = -9$ <p>I nach y auflösen</p> $2x - 7y = -8$ $2x - 7y = -8 \quad / +7y$ $-7y = -8 - 2x \quad / : (-7)$ $y = 1\frac{1}{7} + \frac{2}{7}x$ <p>II nach y auflösen</p> $7x - 1y = -9$ $7x - 1y = -9 \quad / +1y$ $-1y = -9 - 7x \quad / : (-1)$ $y = 9 + 7x$ <p>I = II</p> $1\frac{1}{7} + \frac{2}{7}x = 9 + 7x \quad / -\frac{2}{7}x / -9$ $1\frac{1}{7} - 9 = 7x - \frac{2}{7}x$ $-7\frac{6}{7} = 6\frac{5}{7}x \quad / : 6\frac{5}{7}$ $x = -1\frac{8}{47}$ <p>x in I</p> $y = 1\frac{1}{7} + \frac{2}{7} - 1\frac{8}{47}$ $y = \frac{38}{47}$ $L = \{-1\frac{8}{47} / \frac{38}{47}\}$	$I \quad 2x - 7y = -8$ $II \quad 7x - 1y = -9$ <p>I nach x auflösen</p> $2x - 7y = -8$ $2x - 7y = -8 \quad / +7y$ $2x = -8 + 7y \quad / : 2$ $x = -4 + 3\frac{1}{2}y$ <p>II nach x auflösen</p> $7x - 1y = -9$ $7x - 1y = -9 \quad / +1y$ $7x = -9 + 1y \quad / : 7$ $x = -1\frac{2}{7} + \frac{1}{7}y$ <p>I = II</p> $-4 + 3\frac{1}{2}y = -1\frac{2}{7} + \frac{1}{7}y \quad / -3\frac{1}{2}y / +1\frac{2}{7}$ $-4 + 1\frac{2}{7} = \frac{1}{7}y - 3\frac{1}{2}y$ $-2\frac{5}{7} = -3\frac{5}{14}y \quad / : (-3\frac{5}{14})$ $y = \frac{38}{47}$ <p>y in I</p> $x = -4 + 3\frac{1}{2}\frac{38}{47}$ $x = -1\frac{8}{47}$ $L = \{-1\frac{8}{47} / \frac{38}{47}\}$
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Aufgabe (22)

$$I \quad -7x + 9y = -3$$

$$II \quad 5x - 6y = -4$$

I nach y auflösen

$$-7x + 9y = -3$$

$$-7x + 9y = -3 \quad / -9y$$

$$9y = -3 + 7x \quad / : 9$$

$$y = -\frac{1}{3} + \frac{7}{9}x$$

II nach y auflösen

$$5x - 6y = -4$$

$$5x - 6y = -4 \quad / +6y$$

$$-6y = -4 - 5x \quad / : (-6)$$

$$y = \frac{2}{3} + \frac{5}{6}x$$

I = II

$$-\frac{1}{3} + \frac{7}{9}x = \frac{2}{3} + \frac{5}{6}x \quad / -\frac{7}{9}x / -\frac{2}{3}$$

$$-\frac{1}{3} - \frac{2}{3} = \frac{5}{6}x - \frac{7}{9}x$$

$$-1 = \frac{1}{18}x \quad / : \frac{1}{18}$$

$$x = -18$$

x in I

$$y = -\frac{1}{3} + \frac{7}{9} \cdot (-18)$$

$$y = -14\frac{1}{3}$$

$$L = \{-18 / -14\frac{1}{3}\}$$

$$I \quad -7x + 9y = -3$$

$$II \quad 5x - 6y = -4$$

I nach x auflösen

$$-7x + 9y = -3$$

$$-7x + 9y = -3 \quad / -9y$$

$$-7x = -3 - 9y \quad / : (-7)$$

$$x = \frac{3}{7} + 1\frac{2}{7}y$$

II nach x auflösen

$$5x - 6y = -4$$

$$5x - 6y = -4 \quad / +6y$$

$$5x = -4 + 6y \quad / : 5$$

$$x = -\frac{4}{5} + 1\frac{1}{5}y$$

I = II

$$\frac{3}{7} + 1\frac{2}{7}y = -\frac{4}{5} + 1\frac{1}{5}y \quad / -1\frac{2}{7}y / +\frac{4}{5}$$

$$\frac{3}{7} + \frac{4}{5} = 1\frac{1}{5}y - 1\frac{2}{7}y$$

$$1\frac{8}{35} = -\frac{3}{35}y \quad / : (-\frac{3}{35})$$

$$y = -14\frac{1}{3}$$

y in I

$$x = \frac{3}{7} + 1\frac{2}{7} \cdot (-14\frac{1}{3})$$

$$x = -18$$

$$L = \{-18 / -14\frac{1}{3}\}$$

### 3 Additionssverfahren (2)

#### 3.1 Aufgaben

Gesucht: Die Lösung des linearen Gleichungssystems

$$a1 \cdot x + b1 \cdot y = c1$$

$$a2 \cdot x + b2 \cdot y = c2$$

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

$$(1) \\ 3x + 5y = 19 \\ 7x + 5y = 31$$

$$(10) \\ -1x + 1y = 3 \\ \frac{1}{2}x - 4y = 5$$

$$(2) \\ 1x + 1y = 10 \\ 1x - 1y = 4$$

$$(11) \\ 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\ 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8}$$

$$(3) \\ 9x - 2y = 5 \\ 5x - 2y = 1$$

$$(12) \\ \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\ 1x + 1y = 10\frac{2}{3}$$

$$(4) \\ 9x - 2y = 1 \\ -3x - 3y = -7$$

$$(13) \\ 1\frac{1}{2}x - 2y = 9 \\ \frac{2}{5}x + \frac{1}{3}y = 5$$

$$(5) \\ 1x + 3y = 9 \\ 3x - 2y = -6$$

$$(14) \\ 2x + 3y = 4 \\ \frac{1}{3}x - \frac{1}{5}y = 12$$

$$(6) \\ 7x + 1\frac{1}{2}y = 1 \\ 2x - 5y = 22$$

$$(15) \\ 1x + 2y = -4 \\ -1x + 2y = 5$$

$$(7) \\ 4x - 5y = 43 \\ -3x - 7y = 0$$

$$(16) \\ 2x + 4y = 4 \\ -\frac{1}{2}x + 3y = 3$$

$$(8) \\ 8x - 3y = 10 \\ 12x - 5y = 24$$

$$(17) \\ -\frac{1}{2}x + 4y = 6 \\ -2x - 8y = 2$$

$$(9) \\ -\frac{1}{2}x + 1y = 2 \\ \frac{1}{2}x - 3y = -3$$

$$(18) \\ 1x + 1y = 1 \\ 2x + 2y = 2$$

$$(19)$$

$$1x + 1y = 2$$

$$3x + 3y = 3$$

$$2x - 7y = -8$$

$$7x - 1y = -9$$

$$(20)$$

$$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$$

$$-\frac{2}{3}x + \frac{1}{9}y = 9$$

$$(22)$$

$$-7x + 9y = -3$$

$$5x - 6y = -4$$

(21)

### 3.2 Lösungen

Aufgabe (1)

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

$$I \quad 3x + 5y = 19 \quad / \cdot 7$$

$$II \quad 7x + 5y = 31 \quad / \cdot (-3)$$

$$I \quad 21x + 35y = 133$$

$$II \quad -21x - 15y = -93$$

$$I + II$$

$$I \quad 21x - 21x + 35y - 15y = 133 - 93$$

$$20y = 40 \quad / : 20$$

$$y = \frac{40}{20}$$

$$y = 2$$

$$y \text{ in I}$$

$$I \quad 3x + 5 \cdot 2 = 19$$

$$3x + 10 = 19 \quad / - 10$$

$$3x = 19 - 10$$

$$3x = 9 \quad / : 3$$

$$x = \frac{9}{3}$$

$$x = 3$$

$$L = \{3/2\}$$

$$I \quad 3x + 5y = 19$$

$$II \quad 7x + 5y = 31$$

$$I \quad 3x + 5y = 19 \quad / \cdot 1$$

$$II \quad 7x + 5y = 31 \quad / \cdot (-1)$$

$$I \quad 3x + 5y = 19$$

$$II \quad -7x - 5y = -31$$

$$I + II$$

$$I \quad 3x - 7x + 5y - 5y = 19 - 31$$

$$-4x = -12 \quad / : (-4)$$

$$x = \frac{-12}{-4}$$

$$x = 3$$

$$x \text{ in I}$$

$$I \quad 3 \cdot 3 + 5y = 19$$

$$5y + 9 = 19 \quad / - 9$$

$$5y = 19 - 9$$

$$5y = 10 \quad / : 5$$

$$y = \frac{10}{5}$$

$$y = 2$$

$$L = \{3/2\}$$

Aufgabe (2)

$$\begin{array}{l}
I \quad 1x + 1y = 10 \\
II \quad 1x - 1y = 4 \\
I \quad 1x + 1y = 10 \quad / \cdot 1 \\
II \quad 1x - 1y = 4 \quad / \cdot (-1) \\
I \quad 1x + 1y = 10 \\
II \quad -1x + 1y = -4 \\
I + II \\
I \quad 1x - 1x + 1y + 1y = 10 - 4 \\
2y = 6 \quad / : 2 \\
y = \frac{6}{2} \\
y = 3 \\
y \text{ in I} \\
I \quad 1x + 1 \cdot 3 = 10 \\
1x + 3 = 10 \quad / - 3 \\
1x = 10 - 3 \\
1x = 7 \quad / : 1 \\
x = \frac{7}{1} \\
x = 7 \\
L = \{7/3\}
\end{array}$$

$$\begin{array}{l}
I \quad 1x + 1y = 10 \\
II \quad 1x - 1y = 4 \\
I \quad 1x + 1y = 10 \quad / \cdot (-1) \\
II \quad 1x - 1y = 4 \quad / \cdot (-1) \\
I \quad -1x - 1y = -10 \\
II \quad -1x + 1y = -4 \\
I + II \\
I \quad -1x - 1x - 1y + 1y = -10 - 4 \\
-2x = -14 \quad / : (-2) \\
x = \frac{-14}{-2} \\
x = 7 \\
x \text{ in I} \\
I \quad 1 \cdot 7 + 1y = 10 \\
1y + 7 = 10 \quad / - 7 \\
1y = 10 - 7 \\
1y = 3 \quad / : 1 \\
y = \frac{3}{1} \\
y = 3 \\
L = \{7/3\}
\end{array}$$

Aufgabe (3)

$$\begin{array}{l}
I \quad 9x - 2y = 5 \\
II \quad 5x - 2y = 1 \\
I \quad 9x - 2y = 5 \quad / \cdot 5 \\
II \quad 5x - 2y = 1 \quad / \cdot (-9) \\
I \quad 45x - 10y = 25 \\
II \quad -45x + 18y = -9 \\
I + II \\
I \quad 45x - 45x - 10y + 18y = 25 - 9 \\
8y = 16 \quad / : 8 \\
y = \frac{16}{8} \\
y = 2 \\
y \text{ in I} \\
I \quad 9x - 2 \cdot 2 = 5 \\
9x - 4 = 5 \quad / + 4 \\
9x = 5 + 4 \\
9x = 9 \quad / : 9 \\
x = \frac{9}{9} \\
x = 1 \\
L = \{1/2\}
\end{array}$$

$$\begin{array}{l}
I \quad 9x - 2y = 5 \\
II \quad 5x - 2y = 1 \\
I \quad 9x - 2y = 5 \quad / \cdot (-1) \\
II \quad 5x - 2y = 1 \quad / \cdot 1 \\
I \quad -9x + 2y = -5 \\
II \quad 5x - 2y = 1 \\
I + II \\
I \quad -9x + 5x + 2y - 2y = -5 + 1 \\
-4x = -4 \quad / : (-4) \\
x = \frac{-4}{-4} \\
x = 1 \\
x \text{ in I} \\
I \quad 9 \cdot 1 - 2y = 5 \\
-2y + 9 = 5 \quad / - 9 \\
-2y = 5 - 9 \\
-2y = -4 \quad / : (-2) \\
y = \frac{-4}{-2} \\
y = 2 \\
L = \{1/2\}
\end{array}$$

Aufgabe (4)

$$\begin{array}{l}
I \quad 9x - 2y = 1 \\
II \quad -3x - 3y = -7 \\
I \quad 9x - 2y = 1 \quad / \cdot (-1) \\
II \quad -3x - 3y = -7 \quad / \cdot (-3) \\
I \quad -9x + 2y = -1 \\
II \quad 9x + 9y = 21 \\
I + II \\
I \quad -9x + 9x + 2y + 9y = -1 + 21 \\
11y = 20 \quad / : 11 \\
y = \frac{20}{11} \\
y = 1 \frac{9}{11} \\
y \text{ in I} \\
I \quad 9x - 2 \cdot 1 \frac{9}{11} = 1 \\
9x - 3 \frac{7}{11} = 1 \quad / + 3 \frac{7}{11} \\
9x = 1 + 3 \frac{7}{11} \\
9x = 4 \frac{7}{11} \quad / : 9 \\
x = \frac{4 \frac{7}{11}}{9} \\
x = \frac{17}{33} \\
L = \left\{ \frac{17}{33} / 1 \frac{9}{11} \right\}
\end{array}$$

$$\begin{array}{l}
I \quad 9x - 2y = 1 \\
II \quad -3x - 3y = -7 \\
I \quad 9x - 2y = 1 \quad / \cdot (-3) \\
II \quad -3x - 3y = -7 \quad / \cdot 2 \\
I \quad -27x + 6y = -3 \\
II \quad -6x - 6y = -14 \\
I + II \\
I \quad -27x - 6x + 6y - 6y = -3 - 14 \\
-33x = -17 \quad / : (-33) \\
x = \frac{-17}{-33} \\
x = \frac{17}{33} \\
x \text{ in I} \\
I \quad 9 \cdot \frac{17}{33} - 2y = 1 \\
-2y + 4 \frac{7}{11} = 1 \quad / - 4 \frac{7}{11} \\
-2y = 1 - 4 \frac{7}{11} \\
-2y = -3 \frac{7}{11} \quad / : (-2) \\
y = \frac{-3 \frac{7}{11}}{-2} \\
y = 1 \frac{9}{11} \\
L = \left\{ \frac{17}{33} / 1 \frac{9}{11} \right\}
\end{array}$$

Aufgabe (5)

$$\begin{array}{l}
I \quad 1x + 3y = 9 \\
II \quad 3x - 2y = -6 \\
I \quad 1x + 3y = 9 \quad / \cdot 3 \\
II \quad 3x - 2y = -6 \quad / \cdot (-1) \\
I \quad 3x + 9y = 27 \\
II \quad -3x + 2y = 6 \\
I + II \\
I \quad 3x - 3x + 9y + 2y = 27 + 6 \\
11y = 33 \quad / : 11 \\
y = \frac{33}{11} \\
y = 3 \\
y \text{ in I} \\
I \quad 1x + 3 \cdot 3 = 9 \\
1x + 9 = 9 \quad / - 9 \\
1x = 9 - 9 \\
1x = 0 \quad / : 1 \\
x = \frac{0}{1} \\
x = 0 \\
L = \{0/3\}
\end{array}$$

$$\begin{array}{l}
I \quad 1x + 3y = 9 \\
II \quad 3x - 2y = -6 \\
I \quad 1x + 3y = 9 \quad / \cdot (-2) \\
II \quad 3x - 2y = -6 \quad / \cdot (-3) \\
I \quad -2x - 6y = -18 \\
II \quad -9x + 6y = 18 \\
I + II \\
I \quad -2x - 9x - 6y + 6y = -18 + 18 \\
-11x = 0 \quad / : (-11) \\
x = \frac{0}{-11} \\
x = 0 \\
x \text{ in I} \\
I \quad 1 \cdot 0 + 3y = 9 \\
3y + 0 = 9 \quad / - 0 \\
3y = 9 - 0 \\
3y = 9 \quad / : 3 \\
y = \frac{9}{3} \\
y = 3 \\
L = \{0/3\}
\end{array}$$

Aufgabe (6)

$$\begin{array}{l}
I \quad 7x + 1\frac{1}{2}y = 1 \\
II \quad 2x - 5y = 22 \\
I \quad 7x + 1\frac{1}{2}y = 1 \quad / \cdot 2 \\
II \quad 2x - 5y = 22 \quad / \cdot (-7) \\
I \quad 14x + 3y = 2 \\
II \quad -14x + 35y = -154 \\
I + II \\
I \quad 14x - 14x + 3y + 35y = 2 - 154 \\
38y = -152 \quad / : 38 \\
y = \frac{-152}{38} \\
y = -4 \\
y \text{ in I} \\
I \quad 7x + 1\frac{1}{2} \cdot (-4) = 1 \\
7x - 6 = 1 \quad / + 6 \\
7x = 1 + 6 \\
7x = 7 \quad / : 7 \\
x = \frac{7}{7} \\
x = 1 \\
L = \{1/ - 4\}
\end{array}$$

$$\begin{array}{l}
I \quad 7x + 1\frac{1}{2}y = 1 \\
II \quad 2x - 5y = 22 \\
I \quad 7x + 1\frac{1}{2}y = 1 \quad / \cdot 5 \\
II \quad 2x - 5y = 22 \quad / \cdot 1\frac{1}{2} \\
I \quad 35x + 7\frac{1}{2}y = 5 \\
II \quad 3x - 7\frac{1}{2}y = 33 \\
I + II \\
I \quad 35x + 3x + 7\frac{1}{2}y - 7\frac{1}{2}y = 5 + 33 \\
38x = 38 \quad / : 38 \\
x = \frac{38}{38} \\
x = 1 \\
x \text{ in I} \\
I \quad 7 \cdot 1 + 1\frac{1}{2}y = 1 \\
1\frac{1}{2}y + 7 = 1 \quad / - 7 \\
1\frac{1}{2}y = 1 - 7 \\
1\frac{1}{2}y = -6 \quad / : 1\frac{1}{2} \\
y = \frac{-6}{1\frac{1}{2}} \\
y = -4 \\
L = \{1/ - 4\}
\end{array}$$

Aufgabe (7)

$$\begin{array}{l}
I \quad 4x - 5y = 43 \\
II \quad -3x - 7y = 0 \\
I \quad 4x - 5y = 43 \quad / \cdot (-3) \\
II \quad -3x - 7y = 0 \quad / \cdot (-4) \\
I \quad -12x + 15y = -129 \\
II \quad 12x + 28y = 0 \\
I + II \\
I \quad -12x + 12x + 15y + 28y = -129 + 0 \\
43y = -129 \quad / : 43 \\
y = \frac{-129}{43} \\
y = -3 \\
y \text{ in I} \\
I \quad 4x - 5 \cdot (-3) = 43 \\
4x + 15 = 43 \quad / - 15 \\
4x = 43 - 15 \\
4x = 28 \quad / : 4 \\
x = \frac{28}{4} \\
x = 7 \\
L = \{7/ - 3\}
\end{array}$$

$$\begin{array}{l}
I \quad 4x - 5y = 43 \\
II \quad -3x - 7y = 0 \\
I \quad 4x - 5y = 43 \quad / \cdot (-7) \\
II \quad -3x - 7y = 0 \quad / \cdot 5 \\
I \quad -28x + 35y = -301 \\
II \quad -15x - 35y = 0 \\
I + II \\
I \quad -28x - 15x + 35y - 35y = -301 + 0 \\
-43x = -301 \quad / : (-43) \\
x = \frac{-301}{-43} \\
x = 7 \\
x \text{ in I} \\
I \quad 4 \cdot 7 - 5y = 43 \\
-5y + 28 = 43 \quad / - 28 \\
-5y = 43 - 28 \\
-5y = 15 \quad / : (-5) \\
y = \frac{15}{-5} \\
y = -3 \\
L = \{7/ - 3\}
\end{array}$$

Aufgabe (8)

$$\begin{array}{l}
I \quad 8x - 3y = 10 \\
II \quad 12x - 5y = 24 \\
I \quad 8x - 3y = 10 \quad / \cdot 3 \\
II \quad 12x - 5y = 24 \quad / \cdot (-2) \\
I \quad 24x - 9y = 30 \\
II \quad -24x + 10y = -48 \\
I + II \\
I \quad 24x - 24x - 9y + 10y = 30 - 48 \\
1y = -18 \quad / : 1 \\
y = \frac{-18}{1} \\
y = -18 \\
y \text{ in I} \\
I \quad 8x - 3 \cdot (-18) = 10 \\
8x + 54 = 10 \quad / - 54 \\
8x = 10 - 54 \\
8x = -44 \quad / : 8 \\
x = \frac{-44}{8} \\
x = -5\frac{1}{2} \\
L = \{-5\frac{1}{2} / -18\}
\end{array}$$

$$\begin{array}{l}
I \quad 8x - 3y = 10 \\
II \quad 12x - 5y = 24 \\
I \quad 8x - 3y = 10 \quad / \cdot (-5) \\
II \quad 12x - 5y = 24 \quad / \cdot 3 \\
I \quad -40x + 15y = -50 \\
II \quad 36x - 15y = 72 \\
I + II \\
I \quad -40x + 36x + 15y - 15y = -50 + 72 \\
-4x = 22 \quad / : (-4) \\
x = \frac{22}{-4} \\
x = -5\frac{1}{2} \\
x \text{ in I} \\
I \quad 8 \cdot (-5\frac{1}{2}) - 3y = 10 \\
-3y - 44 = 10 \quad / + 44 \\
-3y = 10 + 44 \\
-3y = 54 \quad / : (-3) \\
y = \frac{54}{-3} \\
y = -18 \\
L = \{-5\frac{1}{2} / -18\}
\end{array}$$

Aufgabe (9)

$$\begin{array}{l}
I \quad -\frac{1}{2}x + 1y = 2 \\
II \quad \frac{1}{2}x - 3y = -3 \\
I \quad -\frac{1}{2}x + 1y = 2 \quad / \cdot (-\frac{1}{2}) \\
II \quad \frac{1}{2}x - 3y = -3 \quad / \cdot (-\frac{1}{2}) \\
I \quad \frac{1}{4}x - \frac{1}{2}y = -1 \\
II \quad -\frac{1}{4}x + 1\frac{1}{2}y = 1\frac{1}{2} \\
I + II \\
I \quad \frac{1}{4}x - \frac{1}{4}x - \frac{1}{2}y + 1\frac{1}{2}y = -1 + 1\frac{1}{2} \\
1y = \frac{1}{2} \quad / : 1 \\
y = \frac{1}{2} \\
y = \frac{1}{2} \\
y \text{ in I} \\
I \quad -\frac{1}{2}x + 1 \cdot \frac{1}{2} = 2 \\
-\frac{1}{2}x + \frac{1}{2} = 2 \quad / - \frac{1}{2} \\
-\frac{1}{2}x = 2 - \frac{1}{2} \\
-\frac{1}{2}x = 1\frac{1}{2} \quad / : (-\frac{1}{2}) \\
x = \frac{1\frac{1}{2}}{-\frac{1}{2}} \\
x = -3 \\
L = \{-3 / \frac{1}{2}\}
\end{array}$$

$$\begin{array}{l}
I \quad -\frac{1}{2}x + 1y = 2 \\
II \quad \frac{1}{2}x - 3y = -3 \\
I \quad -\frac{1}{2}x + 1y = 2 \quad / \cdot (-3) \\
II \quad \frac{1}{2}x - 3y = -3 \quad / \cdot (-1) \\
I \quad 1\frac{1}{2}x - 3y = -6 \\
II \quad -\frac{1}{2}x + 3y = 3 \\
I + II \\
I \quad 1\frac{1}{2}x - \frac{1}{2}x - 3y + 3y = -6 + 3 \\
1x = -3 \quad / : 1 \\
x = \frac{-3}{1} \\
x = -3 \\
x \text{ in I} \\
I \quad -\frac{1}{2} \cdot (-3) + 1y = 2 \\
1y + 1\frac{1}{2} = 2 \quad / - 1\frac{1}{2} \\
1y = 2 - 1\frac{1}{2} \\
1y = \frac{1}{2} \quad / : 1 \\
y = \frac{1}{2} \\
y = \frac{1}{2} \\
L = \{-3 / \frac{1}{2}\}
\end{array}$$

Aufgabe (10)

$$\begin{array}{l}
I \quad -1x + 1y = 3 \\
II \quad \frac{1}{2}x - 4y = 5 \\
I \quad -1x + 1y = 3 \quad / \cdot (-\frac{1}{2}) \\
II \quad \frac{1}{2}x - 4y = 5 \quad / \cdot (-1) \\
I \quad \frac{1}{2}x - \frac{1}{2}y = -1\frac{1}{2} \\
II \quad -\frac{1}{2}x + 4y = -5 \\
I + II \\
I \quad \frac{1}{2}x - \frac{1}{2}x - \frac{1}{2}y + 4y = -1\frac{1}{2} - 5 \\
3\frac{1}{2}y = -6\frac{1}{2} \quad / : 3\frac{1}{2} \\
y = \frac{-6\frac{1}{2}}{3\frac{1}{2}} \\
y = -1\frac{6}{7} \\
y \text{ in I} \\
I \quad -1x + 1 \cdot (-1\frac{6}{7}) = 3 \\
-1x - 1\frac{6}{7} = 3 \quad / + 1\frac{6}{7} \\
-1x = 3 + 1\frac{6}{7} \\
-1x = 4\frac{6}{7} \quad / : (-1) \\
x = \frac{4\frac{6}{7}}{-1} \\
x = -4\frac{6}{7} \\
L = \{-4\frac{6}{7} / -1\frac{6}{7}\}
\end{array}$$

$$\begin{array}{l}
I \quad -1x + 1y = 3 \\
II \quad \frac{1}{2}x - 4y = 5 \\
I \quad -1x + 1y = 3 \quad / \cdot (-4) \\
II \quad \frac{1}{2}x - 4y = 5 \quad / \cdot (-1) \\
I \quad 4x - 4y = -12 \\
II \quad -\frac{1}{2}x + 4y = -5 \\
I + II \\
I \quad 4x - \frac{1}{2}x - 4y + 4y = -12 - 5 \\
3\frac{1}{2}x = -17 \quad / : 3\frac{1}{2} \\
x = \frac{-17}{3\frac{1}{2}} \\
x = -4\frac{6}{7} \\
x \text{ in I} \\
I \quad -1 \cdot (-4\frac{6}{7}) + 1y = 3 \\
1y + 4\frac{6}{7} = 3 \quad / - 4\frac{6}{7} \\
1y = 3 - 4\frac{6}{7} \\
1y = -1\frac{6}{7} \quad / : 1 \\
y = \frac{-1\frac{6}{7}}{1} \\
y = -1\frac{6}{7} \\
L = \{-4\frac{6}{7} / -1\frac{6}{7}\}
\end{array}$$

Aufgabe (11)

$$\begin{array}{l}
I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / \cdot 2\frac{1}{2} \\
II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / \cdot (-1\frac{1}{5}) \\
I \quad 3x - 3\frac{1}{3}y = 13\frac{1}{3} \\
II \quad -3x + \frac{3}{10}y = -14\frac{17}{20} \\
I + II \\
I \quad 3x - 3x - 3\frac{1}{3}y + \frac{3}{10}y = 13\frac{1}{3} - 14\frac{17}{20} \\
-3\frac{1}{30}y = -1\frac{31}{60} \quad / : (-3\frac{1}{30}) \\
y = \frac{-1\frac{31}{60}}{-3\frac{1}{30}} \\
y = \frac{1}{2} \\
y \text{ in I} \\
I \quad 1\frac{1}{5}x - 1\frac{1}{3} \cdot \frac{1}{2} = 5\frac{1}{3} \\
1\frac{1}{5}x - \frac{2}{3} = 5\frac{1}{3} \quad / + \frac{2}{3} \\
1\frac{1}{5}x = 5\frac{1}{3} + \frac{2}{3} \\
1\frac{1}{5}x = 6 \quad / : 1\frac{1}{5} \\
x = \frac{6}{1\frac{1}{5}} \\
x = 5 \\
L = \{5 / \frac{1}{2}\}
\end{array}$$

$$\begin{array}{l}
I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\
II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \\
I \quad 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \quad / \cdot (-\frac{1}{4}) \\
II \quad 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \quad / \cdot 1\frac{1}{3} \\
I \quad -\frac{3}{10}x + \frac{1}{3}y = -1\frac{1}{3} \\
II \quad 3\frac{1}{3}x - \frac{1}{3}y = 16\frac{1}{2} \\
I + II \\
I \quad -\frac{3}{10}x + 3\frac{1}{3}x + \frac{1}{3}y - \frac{1}{3}y = -1\frac{1}{3} + 16\frac{1}{2} \\
3\frac{1}{30}x = 15\frac{1}{6} \quad / : 3\frac{1}{30} \\
x = \frac{15\frac{1}{6}}{3\frac{1}{30}} \\
x = 5 \\
x \text{ in I} \\
I \quad 1\frac{1}{5} \cdot 5 - 1\frac{1}{3}y = 5\frac{1}{3} \\
-1\frac{1}{3}y + 6 = 5\frac{1}{3} \quad / - 6 \\
-1\frac{1}{3}y = 5\frac{1}{3} - 6 \\
-1\frac{1}{3}y = -\frac{2}{3} \quad / : (-1\frac{1}{3}) \\
y = \frac{-\frac{2}{3}}{-1\frac{1}{3}} \\
y = \frac{1}{2} \\
L = \{5 / \frac{1}{2}\}
\end{array}$$

Aufgabe (12)

$$\begin{array}{l}
I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
II \quad 1x + 1y = 10\frac{2}{3} \\
I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / \cdot 1 \\
II \quad 1x + 1y = 10\frac{2}{3} \quad / \cdot (-\frac{2}{3}) \\
I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
II \quad -\frac{2}{3}x - \frac{2}{3}y = -7\frac{1}{9} \\
I + II \\
I \quad \frac{2}{3}x - \frac{2}{3}x - \frac{5}{7}y - \frac{2}{3}y = \frac{2}{3} - 7\frac{1}{9} \\
-1\frac{8}{21}y = -6\frac{4}{9} \quad / : (-1\frac{8}{21}) \\
y = \frac{-6\frac{4}{9}}{-1\frac{8}{21}} \\
y = 4\frac{2}{3} \\
y \text{ in I} \\
I \quad \frac{2}{3}x - \frac{5}{7} \cdot 4\frac{2}{3} = \frac{2}{3} \\
\frac{2}{3}x - 3\frac{1}{3} = \frac{2}{3} \quad / + 3\frac{1}{3} \\
\frac{2}{3}x = \frac{2}{3} + 3\frac{1}{3} \\
\frac{2}{3}x = 4 \quad / : \frac{2}{3} \\
x = \frac{4}{\frac{2}{3}} \\
x = 6 \\
L = \{6/4\frac{2}{3}\}
\end{array}$$

$$\begin{array}{l}
I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\
II \quad 1x + 1y = 10\frac{2}{3} \\
I \quad \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \quad / \cdot (-1) \\
II \quad 1x + 1y = 10\frac{2}{3} \quad / \cdot (-\frac{5}{7}) \\
I \quad -\frac{2}{3}x + \frac{5}{7}y = -\frac{2}{3} \\
II \quad -\frac{5}{7}x - \frac{5}{7}y = -7\frac{13}{21} \\
I + II \\
I \quad -\frac{2}{3}x - \frac{5}{7}x + \frac{5}{7}y - \frac{5}{7}y = -\frac{2}{3} - 7\frac{13}{21} \\
-1\frac{8}{21}x = -8\frac{2}{7} \quad / : (-1\frac{8}{21}) \\
x = \frac{-8\frac{2}{7}}{-1\frac{8}{21}} \\
x = 6 \\
x \text{ in I} \\
I \quad \frac{2}{3} \cdot 6 - \frac{5}{7}y = \frac{2}{3} \\
-\frac{5}{7}y + 4 = \frac{2}{3} \quad / - 4 \\
-\frac{5}{7}y = \frac{2}{3} - 4 \\
-\frac{5}{7}y = -3\frac{1}{3} \quad / : (-\frac{5}{7}) \\
y = \frac{-3\frac{1}{3}}{-\frac{5}{7}} \\
y = 4\frac{2}{3} \\
L = \{6/4\frac{2}{3}\}
\end{array}$$

Aufgabe (13)

$$\begin{array}{l}
I \quad 1\frac{1}{2}x - 2y = 9 \\
II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
I \quad 1\frac{1}{2}x - 2y = 9 \quad / \cdot \frac{2}{5} \\
II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \quad / \cdot (-1\frac{1}{2}) \\
I \quad \frac{3}{5}x - \frac{4}{5}y = 3\frac{3}{5} \\
II \quad -\frac{3}{5}x - \frac{1}{2}y = -7\frac{1}{2} \\
I + II \\
I \quad \frac{3}{5}x - \frac{3}{5}x - \frac{4}{5}y - \frac{1}{2}y = 3\frac{3}{5} - 7\frac{1}{2} \\
-1\frac{3}{10}y = -3\frac{9}{10} \quad / : (-1\frac{3}{10}) \\
y = \frac{-3\frac{9}{10}}{-1\frac{3}{10}} \\
y = 3 \\
y \text{ in I} \\
I \quad 1\frac{1}{2}x - 2 \cdot 3 = 9 \\
1\frac{1}{2}x - 6 = 9 \quad / + 6 \\
1\frac{1}{2}x = 9 + 6 \\
1\frac{1}{2}x = 15 \quad / : 1\frac{1}{2} \\
x = \frac{15}{1\frac{1}{2}} \\
x = 10 \\
L = \{10/3\}
\end{array}$$

$$\begin{array}{l}
I \quad 1\frac{1}{2}x - 2y = 9 \\
II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \\
I \quad 1\frac{1}{2}x - 2y = 9 \quad / \cdot (-\frac{1}{3}) \\
II \quad \frac{2}{5}x + \frac{1}{3}y = 5 \quad / \cdot (-2) \\
I \quad -\frac{1}{2}x + \frac{2}{3}y = -3 \\
II \quad -\frac{4}{5}x - \frac{2}{3}y = -10 \\
I + II \\
I \quad -\frac{1}{2}x - \frac{4}{5}x + \frac{2}{3}y - \frac{2}{3}y = -3 - 10 \\
-1\frac{3}{10}x = -13 \quad / : (-1\frac{3}{10}) \\
x = \frac{-13}{-1\frac{3}{10}} \\
x = 10 \\
x \text{ in I} \\
I \quad 1\frac{1}{2} \cdot 10 - 2y = 9 \\
-2y + 15 = 9 \quad / - 15 \\
-2y = 9 - 15 \\
-2y = -6 \quad / : (-2) \\
y = \frac{-6}{-2} \\
y = 3 \\
L = \{10/3\}
\end{array}$$

Aufgabe (14)

$$\begin{array}{l}
I \quad 2x + 3y = 4 \\
II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
I \quad 2x + 3y = 4 \quad / \cdot \frac{1}{3} \\
II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \quad / \cdot (-2) \\
I \quad \frac{2}{3}x + 1y = 1\frac{1}{3} \\
II \quad -\frac{2}{3}x + \frac{2}{5}y = -24 \\
I + II \\
I \quad \frac{2}{3}x - \frac{2}{3}x + 1y + \frac{2}{5}y = 1\frac{1}{3} - 24 \\
1\frac{2}{5}y = -22\frac{2}{3} \quad / : 1\frac{2}{5} \\
y = \frac{-22\frac{2}{3}}{1\frac{2}{5}} \\
y = -16\frac{4}{21} \\
y \text{ in I} \\
I \quad 2x + 3 \cdot (-16\frac{4}{21}) = 4 \\
2x - 48\frac{4}{7} = 4 \quad / + 48\frac{4}{7} \\
2x = 4 + 48\frac{4}{7} \\
2x = 52\frac{4}{7} \quad / : 2 \\
x = \frac{52\frac{4}{7}}{2} \\
x = 26\frac{2}{7} \\
L = \{26\frac{2}{7} / -16\frac{4}{21}\}
\end{array}$$

$$\begin{array}{l}
I \quad 2x + 3y = 4 \\
II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \\
I \quad 2x + 3y = 4 \quad / \cdot \frac{1}{5} \\
II \quad \frac{1}{3}x - \frac{1}{5}y = 12 \quad / \cdot 3 \\
I \quad \frac{2}{5}x + \frac{3}{5}y = \frac{4}{5} \\
II \quad 1x - \frac{3}{5}y = 36 \\
I + II \\
I \quad \frac{2}{5}x + 1x + \frac{3}{5}y - \frac{3}{5}y = \frac{4}{5} + 36 \\
1\frac{2}{5}x = 36\frac{4}{5} \quad / : 1\frac{2}{5} \\
x = \frac{36\frac{4}{5}}{1\frac{2}{5}} \\
x = 26\frac{2}{7} \\
x \text{ in I} \\
I \quad 2 \cdot 26\frac{2}{7} + 3y = 4 \\
3y + 52\frac{4}{7} = 4 \quad / - 52\frac{4}{7} \\
3y = 4 - 52\frac{4}{7} \\
3y = -48\frac{4}{7} \quad / : 3 \\
y = \frac{-48\frac{4}{7}}{3} \\
y = -16\frac{4}{21} \\
L = \{26\frac{2}{7} / -16\frac{4}{21}\}
\end{array}$$

Aufgabe (15)

$$\begin{array}{l}
I \quad 1x + 2y = -4 \\
II \quad -1x + 2y = 5 \\
I \quad 1x + 2y = -4 \quad / \cdot (-1) \\
II \quad -1x + 2y = 5 \quad / \cdot (-1) \\
I \quad -1x - 2y = 4 \\
II \quad 1x - 2y = -5 \\
I + II \\
I \quad -1x + 1x - 2y - 2y = 4 - 5 \\
-4y = -1 \quad / : (-4) \\
y = \frac{-1}{-4} \\
y = \frac{1}{4} \\
y \text{ in I} \\
I \quad 1x + 2 \cdot \frac{1}{4} = -4 \\
1x + \frac{1}{2} = -4 \quad / - \frac{1}{2} \\
1x = -4 - \frac{1}{2} \\
1x = -4\frac{1}{2} \quad / : 1 \\
x = \frac{-4\frac{1}{2}}{1} \\
x = -4\frac{1}{2} \\
L = \{-4\frac{1}{2} / \frac{1}{4}\}
\end{array}$$

$$\begin{array}{l}
I \quad 1x + 2y = -4 \\
II \quad -1x + 2y = 5 \\
I \quad 1x + 2y = -4 \quad / \cdot 1 \\
II \quad -1x + 2y = 5 \quad / \cdot (-1) \\
I \quad 1x + 2y = -4 \\
II \quad 1x - 2y = -5 \\
I + II \\
I \quad 1x + 1x + 2y - 2y = -4 - 5 \\
2x = -9 \quad / : 2 \\
x = \frac{-9}{2} \\
x = -4\frac{1}{2} \\
x \text{ in I} \\
I \quad 1 \cdot (-4\frac{1}{2}) + 2y = -4 \\
2y - 4\frac{1}{2} = -4 \quad / + 4\frac{1}{2} \\
2y = -4 + 4\frac{1}{2} \\
2y = \frac{1}{2} \quad / : 2 \\
y = \frac{\frac{1}{2}}{2} \\
y = \frac{1}{4} \\
L = \{-4\frac{1}{2} / \frac{1}{4}\}
\end{array}$$

Aufgabe (16)

$$\begin{array}{l}
I \quad 2x + 4y = 4 \\
II \quad -\frac{1}{2}x + 3y = 3 \\
I \quad 2x + 4y = 4 \quad / \cdot \frac{1}{2} \\
II \quad -\frac{1}{2}x + 3y = 3 \quad / \cdot 2 \\
I \quad 1x + 2y = 2 \\
II \quad -1x + 6y = 6 \\
I + II \\
I \quad 1x - 1x + 2y + 6y = 2 + 6 \\
8y = 8 \quad / : 8 \\
y = \frac{8}{8} \\
y = 1 \\
y \text{ in I} \\
I \quad 2x + 4 \cdot 1 = 4 \\
2x + 4 = 4 \quad / - 4 \\
2x = 4 - 4 \\
2x = 0 \quad / : 2 \\
x = \frac{0}{2} \\
x = 0 \\
L = \{0/1\}
\end{array}$$

$$\begin{array}{l}
I \quad 2x + 4y = 4 \\
II \quad -\frac{1}{2}x + 3y = 3 \\
I \quad 2x + 4y = 4 \quad / \cdot 3 \\
II \quad -\frac{1}{2}x + 3y = 3 \quad / \cdot (-4) \\
I \quad 6x + 12y = 12 \\
II \quad 2x - 12y = -12 \\
I + II \\
I \quad 6x + 2x + 12y - 12y = 12 - 12 \\
8x = 0 \quad / : 8 \\
x = \frac{0}{8} \\
x = 0 \\
x \text{ in I} \\
I \quad 2 \cdot 0 + 4y = 4 \\
4y + 0 = 4 \quad / - 0 \\
4y = 4 - 0 \\
4y = 4 \quad / : 4 \\
y = \frac{4}{4} \\
y = 1 \\
L = \{0/1\}
\end{array}$$

Aufgabe (17)

$$\begin{array}{l}
I \quad -\frac{1}{2}x + 4y = 6 \\
II \quad -2x - 8y = 2 \\
I \quad -\frac{1}{2}x + 4y = 6 \quad / \cdot (-2) \\
II \quad -2x - 8y = 2 \quad / \cdot \frac{1}{2} \\
I \quad 1x - 8y = -12 \\
II \quad -1x - 4y = 1 \\
I + II \\
I \quad 1x - 1x - 8y - 4y = -12 + 1 \\
-12y = -11 \quad / : (-12) \\
y = \frac{-11}{-12} \\
y = \frac{11}{12} \\
y \text{ in I} \\
I \quad -\frac{1}{2}x + 4 \cdot \frac{11}{12} = 6 \\
-\frac{1}{2}x + 3\frac{2}{3} = 6 \quad / - 3\frac{2}{3} \\
-\frac{1}{2}x = 6 - 3\frac{2}{3} \\
-\frac{1}{2}x = 2\frac{1}{3} \quad / : (-\frac{1}{2}) \\
x = \frac{2\frac{1}{3}}{-\frac{1}{2}} \\
x = -4\frac{2}{3} \\
L = \{-4\frac{2}{3}/\frac{11}{12}\}
\end{array}$$

$$\begin{array}{l}
I \quad -\frac{1}{2}x + 4y = 6 \\
II \quad -2x - 8y = 2 \\
I \quad -\frac{1}{2}x + 4y = 6 \quad / \cdot (-2) \\
II \quad -2x - 8y = 2 \quad / \cdot (-1) \\
I \quad 1x - 8y = -12 \\
II \quad 2x + 8y = -2 \\
I + II \\
I \quad 1x + 2x - 8y + 8y = -12 - 2 \\
3x = -14 \quad / : 3 \\
x = \frac{-14}{3} \\
x = -4\frac{2}{3} \\
x \text{ in I} \\
I \quad -\frac{1}{2} \cdot (-4\frac{2}{3}) + 4y = 6 \\
4y + 2\frac{1}{3} = 6 \quad / - 2\frac{1}{3} \\
4y = 6 - 2\frac{1}{3} \\
4y = 3\frac{2}{3} \quad / : 4 \\
y = \frac{3\frac{2}{3}}{4} \\
y = \frac{11}{12} \\
L = \{-4\frac{2}{3}/\frac{11}{12}\}
\end{array}$$

Aufgabe (18)

$$\begin{array}{l}
I \quad 1x + 1y = 1 \\
II \quad 2x + 2y = 2 \\
I \quad 1x + 1y = 1 \quad / \cdot 2 \\
II \quad 2x + 2y = 2 \quad / \cdot (-1) \\
I \quad 2x + 2y = 2 \\
II \quad -2x - 2y = -2 \\
I + II \\
I \quad 2x - 2x + 2y - 2y = 2 - 2 \\
0 = 0 \\
L = \text{unendlich}
\end{array}$$

$$\begin{array}{l}
I \quad 1x + 1y = 1 \\
II \quad 2x + 2y = 2 \\
I \quad 1x + 1y = 1 \quad / \cdot 2 \\
II \quad 2x + 2y = 2 \quad / \cdot (-1) \\
I \quad 2x + 2y = 2 \\
II \quad -2x - 2y = -2 \\
I + II \\
I \quad 2x - 2x + 2y - 2y = 2 - 2 \\
0 = 0 \\
L = \text{unendlich}
\end{array}$$

## Aufgabe (19)

$$\begin{array}{l}
 I \quad 1x + 1y = 2 \\
 II \quad 3x + 3y = 3 \\
 I \quad 1x + 1y = 2 \quad / \cdot 3 \\
 II \quad 3x + 3y = 3 \quad / \cdot (-1) \\
 I \quad 3x + 3y = 6 \\
 II \quad -3x - 3y = -3 \\
 I + II \\
 I \quad 3x - 3x + 3y - 3y = 6 - 3 \\
 \\
 L = \{\}
 \end{array}$$

$$\begin{array}{l}
 I \quad 1x + 1y = 2 \\
 II \quad 3x + 3y = 3 \\
 I \quad 1x + 1y = 2 \quad / \cdot 3 \\
 II \quad 3x + 3y = 3 \quad / \cdot (-1) \\
 I \quad 3x + 3y = 6 \\
 II \quad -3x - 3y = -3 \\
 I + II \\
 I \quad 3x - 3x + 3y - 3y = 6 - 3 \\
 \\
 L = \{\}
 \end{array}$$

## Aufgabe (20)

$$\begin{array}{l}
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \\
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \quad / \cdot \left(-\frac{2}{3}\right) \\
 II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \quad / \cdot 1\frac{4}{5} \\
 I \quad 1\frac{1}{5}x - \frac{8}{9}y = \frac{2}{3} \\
 II \quad -1\frac{1}{5}x + \frac{1}{5}y = 16\frac{1}{5} \\
 I + II \\
 I \quad 1\frac{1}{5}x - 1\frac{1}{5}x - \frac{8}{9}y + \frac{1}{5}y = \frac{2}{3} + 16\frac{1}{5} \\
 -\frac{31}{45}y = 16\frac{13}{15} \quad / : \left(-\frac{31}{45}\right) \\
 y = \frac{16\frac{13}{15}}{-\frac{31}{45}} \\
 y = -24\frac{15}{31} \\
 y \text{ in I} \\
 I \quad -1\frac{4}{5}x + 1\frac{1}{3} \cdot \left(-24\frac{15}{31}\right) = -1 \\
 -1\frac{4}{5}x - 32\frac{20}{31} = -1 \quad / + 32\frac{20}{31} \\
 -1\frac{4}{5}x = -1 + 32\frac{20}{31} \\
 -1\frac{4}{5}x = 31\frac{20}{31} \quad / : \left(-1\frac{4}{5}\right) \\
 x = \frac{31\frac{20}{31}}{-1\frac{4}{5}} \\
 x = -17\frac{18}{31} \\
 L = \{-17\frac{18}{31} / -24\frac{15}{31}\}
 \end{array}$$

$$\begin{array}{l}
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \\
 II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \\
 I \quad -1\frac{4}{3}x + 1\frac{1}{3}y = -1 \quad / \cdot \frac{1}{9} \\
 II \quad -\frac{2}{3}x + \frac{1}{9}y = 9 \quad / \cdot \left(-1\frac{1}{3}\right) \\
 I \quad -\frac{1}{5}x + \frac{4}{27}y = -\frac{1}{9} \\
 II \quad \frac{8}{9}x - \frac{4}{27}y = -12 \\
 I + II \\
 I \quad -\frac{1}{5}x + \frac{8}{9}x + \frac{4}{27}y - \frac{4}{27}y = -\frac{1}{9} - 12 \\
 \frac{31}{45}x = -12\frac{1}{9} \quad / : \frac{31}{45} \\
 x = \frac{-12\frac{1}{9}}{\frac{31}{45}} \\
 x = -17\frac{18}{31} \\
 x \text{ in I} \\
 I \quad -1\frac{4}{5} \cdot \left(-17\frac{18}{31}\right) + 1\frac{1}{3}y = -1 \\
 1\frac{1}{3}y + 31\frac{20}{31} = -1 \quad / - 31\frac{20}{31} \\
 1\frac{1}{3}y = -1 - 31\frac{20}{31} \\
 1\frac{1}{3}y = -32\frac{20}{31} \quad / : 1\frac{1}{3} \\
 y = \frac{-32\frac{20}{31}}{1\frac{1}{3}} \\
 y = -24\frac{15}{31} \\
 L = \{-17\frac{18}{31} / -24\frac{15}{31}\}
 \end{array}$$

## Aufgabe (21)

$$\begin{array}{l}
I \quad 2x - 7y = -8 \\
II \quad 7x - 1y = -9 \\
I \quad 2x - 7y = -8 \quad / \cdot 7 \\
II \quad 7x - 1y = -9 \quad / \cdot (-2) \\
I \quad 14x - 49y = -56 \\
II \quad -14x + 2y = 18 \\
I + II \\
I \quad 14x - 14x - 49y + 2y = -56 + 18 \\
-47y = -38 \quad / : (-47) \\
y = \frac{-38}{-47} \\
y = \frac{38}{47} \\
y \text{ in I} \\
I \quad 2x - 7 \cdot \frac{38}{47} = -8 \\
2x - 5\frac{31}{47} = -8 \quad / + 5\frac{31}{47} \\
2x = -8 + 5\frac{31}{47} \\
2x = -2\frac{16}{47} \quad / : 2 \\
x = \frac{-2\frac{16}{47}}{2} \\
x = -1\frac{8}{47} \\
L = \left\{ -1\frac{8}{47} / \frac{38}{47} \right\}
\end{array}$$

$$\begin{array}{l}
I \quad 2x - 7y = -8 \\
II \quad 7x - 1y = -9 \\
I \quad 2x - 7y = -8 \quad / \cdot (-1) \\
II \quad 7x - 1y = -9 \quad / \cdot 7 \\
I \quad -2x + 7y = 8 \\
II \quad 49x - 7y = -63 \\
I + II \\
I \quad -2x + 49x + 7y - 7y = 8 - 63 \\
47x = -55 \quad / : 47 \\
x = \frac{-55}{47} \\
x = -1\frac{8}{47} \\
x \text{ in I} \\
I \quad 2 \cdot \left(-1\frac{8}{47}\right) - 7y = -8 \\
-7y - 2\frac{16}{47} = -8 \quad / + 2\frac{16}{47} \\
-7y = -8 + 2\frac{16}{47} \\
-7y = -5\frac{31}{47} \quad / : (-7) \\
y = \frac{-5\frac{31}{47}}{-7} \\
y = \frac{38}{47} \\
L = \left\{ -1\frac{8}{47} / \frac{38}{47} \right\}
\end{array}$$

Aufgabe (22)

$$\begin{array}{l}
I \quad -7x + 9y = -3 \\
II \quad 5x - 6y = -4 \\
I \quad -7x + 9y = -3 \quad / \cdot 5 \\
II \quad 5x - 6y = -4 \quad / \cdot 7 \\
I \quad -35x + 45y = -15 \\
II \quad 35x - 42y = -28 \\
I + II \\
I \quad -35x + 35x + 45y - 42y = -15 - 28 \\
3y = -43 \quad / : 3 \\
y = \frac{-43}{3} \\
y = -14\frac{1}{3} \\
y \text{ in I} \\
I \quad -7x + 9 \cdot \left(-14\frac{1}{3}\right) = -3 \\
-7x - 129 = -3 \quad / + 129 \\
-7x = -3 + 129 \\
-7x = 126 \quad / : (-7) \\
x = \frac{126}{-7} \\
x = -18 \\
L = \{-18 / -14\frac{1}{3}\}
\end{array}$$

$$\begin{array}{l}
I \quad -7x + 9y = -3 \\
II \quad 5x - 6y = -4 \\
I \quad -7x + 9y = -3 \quad / \cdot (-2) \\
II \quad 5x - 6y = -4 \quad / \cdot (-3) \\
I \quad 14x - 18y = 6 \\
II \quad -15x + 18y = 12 \\
I + II \\
I \quad 14x - 15x - 18y + 18y = 6 + 12 \\
-1x = 18 \quad / : (-1) \\
x = \frac{18}{-1} \\
x = -18 \\
x \text{ in I} \\
I \quad -7 \cdot (-18) + 9y = -3 \\
9y + 126 = -3 \quad / - 126 \\
9y = -3 - 126 \\
9y = -129 \quad / : 9 \\
y = \frac{-129}{9} \\
y = -14\frac{1}{3} \\
L = \{-18 / -14\frac{1}{3}\}
\end{array}$$

## 4 Determinantenverfahren (2)

### 4.1 Aufgaben

Gesucht: Die Lösung des linearen Gleichungssystems

$$D_h = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 \cdot b_2 - b_1 \cdot a_2$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1 \cdot b_2 - b_1 \cdot c_2$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1 \cdot c_2 - c_1 \cdot a_2$$

$$x = \frac{D_x}{D_h}$$

$$y = \frac{D_y}{D_h}$$

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

$$\begin{array}{l} (1) \\ 3x + 5y = 19 \\ 7x + 5y = 31 \end{array} \qquad \begin{array}{l} -\frac{1}{2}x + 1y = 2 \\ \frac{1}{2}x - 3y = -3 \end{array}$$

$$\begin{array}{l} (2) \\ 1x + 1y = 10 \\ 1x - 1y = 4 \end{array} \qquad \begin{array}{l} (10) \\ -1x + 1y = 3 \\ \frac{1}{2}x - 4y = 5 \end{array}$$

$$\begin{array}{l} (3) \\ 9x - 2y = 5 \\ 5x - 2y = 1 \end{array} \qquad \begin{array}{l} (11) \\ 1\frac{1}{5}x - 1\frac{1}{3}y = 5\frac{1}{3} \\ 2\frac{1}{2}x - \frac{1}{4}y = 12\frac{3}{8} \end{array}$$

$$\begin{array}{l} (4) \\ 9x - 2y = 1 \\ -3x - 3y = -7 \end{array} \qquad \begin{array}{l} (12) \\ \frac{2}{3}x - \frac{5}{7}y = \frac{2}{3} \\ 1x + 1y = 10\frac{2}{3} \end{array}$$

$$\begin{array}{l} (5) \\ 1x + 3y = 9 \\ 3x - 2y = -6 \end{array} \qquad \begin{array}{l} (13) \\ 1\frac{1}{2}x - 2y = 9 \\ \frac{2}{5}x + \frac{1}{3}y = 5 \end{array}$$

$$\begin{array}{l} (6) \\ 7x + 1\frac{1}{2}y = 1 \\ 2x - 5y = 22 \end{array} \qquad \begin{array}{l} (14) \\ 2x + 3y = 4 \\ \frac{1}{3}x - \frac{1}{5}y = 12 \end{array}$$

$$\begin{array}{l} (7) \\ 4x - 5y = 43 \\ -3x - 7y = 0 \end{array} \qquad \begin{array}{l} (15) \\ 1x + 2y = -4 \\ -1x + 2y = 5 \end{array}$$

$$\begin{array}{l} (8) \\ 8x - 3y = 10 \\ 12x - 5y = 24 \end{array} \qquad \begin{array}{l} (16) \\ 2x + 4y = 4 \\ -\frac{1}{2}x + 3y = 3 \end{array}$$

$$\begin{array}{l} (9) \end{array} \qquad \begin{array}{l} (17) \\ -\frac{1}{2}x + 4y = 6 \end{array}$$

$$-2x - 8y = 2$$

$$-\frac{2}{3}x + \frac{1}{9}y = 9$$

(18)

$$1x + 1y = 1$$

$$2x + 2y = 2$$

(21)

$$2x - 7y = -8$$

$$7x - 1y = -9$$

(19)

$$1x + 1y = 2$$

$$3x + 3y = 3$$

(22)

$$-7x + 9y = -3$$

$$5x - 6y = -4$$

(20)

$$-1\frac{4}{5}x + 1\frac{1}{3}y = -1$$

## 4.2 Lösungen

Aufgabe (1)

$$D_h = \begin{vmatrix} 3 & 5 \\ 7 & 5 \end{vmatrix} = 3 \cdot 5 - 5 \cdot 7 = -20$$

$$D_x = \begin{vmatrix} 19 & 5 \\ 31 & 5 \end{vmatrix} = 19 \cdot 5 - 5 \cdot 31 = -60$$

$$D_y = \begin{vmatrix} 3 & 19 \\ 7 & 31 \end{vmatrix} = 3 \cdot 31 - 19 \cdot 7 = -40$$

$$x = \frac{-60}{-20}$$

$$x = 3$$

$$y = \frac{-40}{-20}$$

$$y = 2$$

$$L = \{3/2\}$$

Aufgabe (2)

$$D_h = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1 \cdot (-1) - 1 \cdot 1 = -2$$

$$D_x = \begin{vmatrix} 10 & 1 \\ 4 & -1 \end{vmatrix} = 10 \cdot (-1) - 1 \cdot 4 = -14$$

$$D_y = \begin{vmatrix} 1 & 10 \\ 1 & 4 \end{vmatrix} = 1 \cdot 4 - 10 \cdot 1 = -6$$

$$x = \frac{-14}{-2}$$

$$x = 7$$

$$y = \frac{-6}{-2}$$

$$y = 3$$

$$L = \{7/3\}$$

Aufgabe (3)

$$D_h = \begin{vmatrix} 9 & -2 \\ 5 & -2 \end{vmatrix} = 9 \cdot (-2) - (-2) \cdot 5 = -8$$

$$D_x = \begin{vmatrix} 5 & -2 \\ 1 & -2 \end{vmatrix} = 5 \cdot (-2) - (-2) \cdot 1 = -8$$

$$D_y = \begin{vmatrix} 9 & 5 \\ 5 & 1 \end{vmatrix} = 9 \cdot 1 - 5 \cdot 5 = -16$$

$$x = \frac{-8}{-8}$$

$$x = 1$$

$$y = \frac{-16}{-8}$$

$$y = 2$$

$$L = \{1/2\}$$

Aufgabe (4)

$$D_h = \begin{vmatrix} 9 & -2 \\ -3 & -3 \end{vmatrix} = 9 \cdot (-3) - (-2) \cdot (-3) = -33$$

$$D_x = \begin{vmatrix} 1 & -2 \\ -7 & -3 \end{vmatrix} = 1 \cdot (-3) - (-2) \cdot (-7) = -17$$

$$D_y = \begin{vmatrix} 9 & 1 \\ -3 & -7 \end{vmatrix} = 9 \cdot (-7) - 1 \cdot (-3) = -60$$

$$x = \frac{-17}{-33}$$

$$x = \frac{17}{33}$$

$$y = \frac{-60}{-33}$$

$$y = 1\frac{9}{11}$$

$$L = \left\{ \frac{17}{33} / 1\frac{9}{11} \right\}$$

Aufgabe (5)

$$D_h = \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = 1 \cdot (-2) - 3 \cdot 3 = -11$$

$$D_x = \begin{vmatrix} 9 & 3 \\ -6 & -2 \end{vmatrix} = 9 \cdot (-2) - 3 \cdot (-6) = 0$$

$$D_y = \begin{vmatrix} 1 & 9 \\ 3 & -6 \end{vmatrix} = 1 \cdot (-6) - 9 \cdot 3 = -33$$

$$x = \frac{0}{-11}$$

$$x = 0$$

$$y = \frac{-33}{-11}$$

$$y = 3$$

$$L = \{0/3\}$$

Aufgabe (6)

$$D_h = \begin{vmatrix} 7 & 1\frac{1}{2} \\ 2 & -5 \end{vmatrix} = 7 \cdot (-5) - 1\frac{1}{2} \cdot 2 = -38$$

$$D_x = \begin{vmatrix} 1 & 1\frac{1}{2} \\ 22 & -5 \end{vmatrix} = 1 \cdot (-5) - 1\frac{1}{2} \cdot 22 = -38$$

$$D_y = \begin{vmatrix} 7 & 1 \\ 2 & 22 \end{vmatrix} = 7 \cdot 22 - 1 \cdot 2 = 152$$

$$x = \frac{-38}{-38}$$

$$x = 1$$

$$y = \frac{152}{-38}$$

$$y = -4$$

$$L = \{1 / -4\}$$

Aufgabe (7)

$$D_h = \begin{vmatrix} 4 & -5 \\ -3 & -7 \end{vmatrix} = 4 \cdot (-7) - (-5) \cdot (-3) = -43$$

$$D_x = \begin{vmatrix} 43 & -5 \\ 0 & -7 \end{vmatrix} = 43 \cdot (-7) - (-5) \cdot 0 = -301$$

$$D_y = \begin{vmatrix} 4 & 43 \\ -3 & 0 \end{vmatrix} = 4 \cdot 0 - 43 \cdot (-3) = 129$$

$$x = \frac{-301}{-43}$$

$$x = 7$$

$$y = \frac{129}{-43}$$

$$y = -3$$

$$L = \{7 / -3\}$$

Aufgabe (8)

$$D_h = \begin{vmatrix} 8 & -3 \\ 12 & -5 \end{vmatrix} = 8 \cdot (-5) - (-3) \cdot 12 = -4$$

$$D_x = \begin{vmatrix} 10 & -3 \\ 24 & -5 \end{vmatrix} = 10 \cdot (-5) - (-3) \cdot 24 = 22$$

$$D_y = \begin{vmatrix} 8 & 10 \\ 12 & 24 \end{vmatrix} = 8 \cdot 24 - 10 \cdot 12 = 72$$

$$x = \frac{22}{-4}$$

$$x = -5\frac{1}{2}$$

$$y = \frac{72}{-4}$$

$$y = -18$$

$$L = \{-5\frac{1}{2} / -18\}$$

Aufgabe (9)

$$D_h = \begin{vmatrix} -\frac{1}{2} & 1 \\ \frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2} \cdot (-3) - 1 \cdot \frac{1}{2} = 1$$

$$D_x = \begin{vmatrix} 2 & 1 \\ -3 & -3 \end{vmatrix} = 2 \cdot (-3) - 1 \cdot (-3) = -3$$

$$D_y = \begin{vmatrix} -\frac{1}{2} & 2 \\ \frac{1}{2} & -3 \end{vmatrix} = -\frac{1}{2} \cdot (-3) - 2 \cdot \frac{1}{2} = \frac{1}{2}$$

$$x = \frac{-3}{1}$$

$$x = -3$$

$$y = \frac{\frac{1}{2}}{1}$$

$$y = \frac{1}{2}$$

$$L = \{-3 / \frac{1}{2}\}$$

Aufgabe (10)

$$\begin{aligned}
 D_h &= \begin{vmatrix} -1 & 1 \\ \frac{1}{2} & -4 \end{vmatrix} = -1 \cdot (-4) - 1 \cdot \frac{1}{2} = 3\frac{1}{2} \\
 D_x &= \begin{vmatrix} 3 & 1 \\ 5 & -4 \end{vmatrix} = 3 \cdot (-4) - 1 \cdot 5 = -17 \\
 D_y &= \begin{vmatrix} -1 & 3 \\ \frac{1}{2} & 5 \end{vmatrix} = -1 \cdot 5 - 3 \cdot \frac{1}{2} = -6\frac{1}{2} \\
 x &= \frac{-17}{3\frac{1}{2}} \\
 x &= -4\frac{6}{7} \\
 y &= \frac{-6\frac{1}{2}}{3\frac{1}{2}} \\
 y &= -1\frac{6}{7} \\
 L &= \{-4\frac{6}{7} / -1\frac{6}{7}\}
 \end{aligned}$$

Aufgabe (11)

$$\begin{aligned}
 D_h &= \begin{vmatrix} 1\frac{1}{5} & -1\frac{1}{3} \\ 2\frac{1}{2} & -\frac{1}{4} \end{vmatrix} = 1\frac{1}{5} \cdot (-\frac{1}{4}) - (-1\frac{1}{3}) \cdot 2\frac{1}{2} = 3\frac{1}{30} \\
 D_x &= \begin{vmatrix} 5\frac{1}{3} & -1\frac{1}{3} \\ 12\frac{3}{8} & -\frac{1}{4} \end{vmatrix} = 5\frac{1}{3} \cdot (-\frac{1}{4}) - (-1\frac{1}{3}) \cdot 12\frac{3}{8} = 15\frac{1}{6} \\
 D_y &= \begin{vmatrix} 1\frac{1}{5} & 5\frac{1}{3} \\ 2\frac{1}{2} & 12\frac{3}{8} \end{vmatrix} = 1\frac{1}{5} \cdot 12\frac{3}{8} - 5\frac{1}{3} \cdot 2\frac{1}{2} = 1\frac{31}{60} \\
 x &= \frac{15\frac{1}{6}}{3\frac{1}{30}} \\
 x &= 5 \\
 y &= \frac{1\frac{31}{60}}{3\frac{1}{30}} \\
 y &= \frac{1}{2} \\
 L &= \{5 / \frac{1}{2}\}
 \end{aligned}$$

Aufgabe (12)

$$\begin{aligned}
 D_h &= \begin{vmatrix} \frac{2}{3} & -\frac{5}{7} \\ 1 & 1 \end{vmatrix} = \frac{2}{3} \cdot 1 - (-\frac{5}{7}) \cdot 1 = 1\frac{8}{21} \\
 D_x &= \begin{vmatrix} \frac{2}{3} & -\frac{5}{7} \\ 10\frac{2}{3} & 1 \end{vmatrix} = \frac{2}{3} \cdot 1 - (-\frac{5}{7}) \cdot 10\frac{2}{3} = 8\frac{2}{7} \\
 D_y &= \begin{vmatrix} \frac{2}{3} & \frac{2}{3} \\ 1 & 10\frac{2}{3} \end{vmatrix} = \frac{2}{3} \cdot 10\frac{2}{3} - \frac{2}{3} \cdot 1 = 6\frac{4}{9} \\
 x &= \frac{8\frac{2}{7}}{1\frac{8}{21}} \\
 x &= 6 \\
 y &= \frac{6\frac{4}{9}}{1\frac{8}{21}} \\
 y &= 4\frac{2}{3} \\
 L &= \{6 / 4\frac{2}{3}\}
 \end{aligned}$$

Aufgabe (13)

$$D_h = \begin{vmatrix} 1\frac{1}{2} & -2 \\ \frac{1}{5} & \frac{1}{3} \end{vmatrix} = 1\frac{1}{2} \cdot \frac{1}{3} - (-2) \cdot \frac{2}{5} = 1\frac{3}{10}$$

$$D_x = \begin{vmatrix} 9 & -2 \\ 5 & \frac{1}{3} \end{vmatrix} = 9 \cdot \frac{1}{3} - (-2) \cdot 5 = 13$$

$$D_y = \begin{vmatrix} 1\frac{1}{2} & 9 \\ \frac{2}{5} & 5 \end{vmatrix} = 1\frac{1}{2} \cdot 5 - 9 \cdot \frac{2}{5} = 3\frac{9}{10}$$

$$x = \frac{13}{1\frac{3}{10}}$$

$$x = 10\frac{9}{3}$$

$$y = \frac{3\frac{9}{10}}{1\frac{3}{10}}$$

$$y = 3$$

$$L = \{10/3\}$$

Aufgabe (14)

$$D_h = \begin{vmatrix} 2 & 3 \\ \frac{1}{3} & -\frac{1}{5} \end{vmatrix} = 2 \cdot \left(-\frac{1}{5}\right) - 3 \cdot \frac{1}{3} = -1\frac{2}{5}$$

$$D_x = \begin{vmatrix} 4 & 3 \\ 12 & -\frac{1}{5} \end{vmatrix} = 4 \cdot \left(-\frac{1}{5}\right) - 3 \cdot 12 = -36\frac{4}{5}$$

$$D_y = \begin{vmatrix} 2 & 4 \\ \frac{1}{3} & 12 \end{vmatrix} = 2 \cdot 12 - 4 \cdot \frac{1}{3} = 22\frac{2}{3}$$

$$x = \frac{-36\frac{4}{5}}{-1\frac{2}{5}}$$

$$x = 26\frac{2}{7}$$

$$y = \frac{22\frac{2}{3}}{-1\frac{2}{5}}$$

$$y = -16\frac{4}{21}$$

$$L = \{26\frac{2}{7} / -16\frac{4}{21}\}$$

Aufgabe (15)

$$D_h = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 1 \cdot 2 - 2 \cdot (-1) = 4$$

$$D_x = \begin{vmatrix} -4 & 2 \\ 5 & 2 \end{vmatrix} = -4 \cdot 2 - 2 \cdot 5 = -18$$

$$D_y = \begin{vmatrix} 1 & -4 \\ -1 & 5 \end{vmatrix} = 1 \cdot 5 - (-4) \cdot (-1) = 1$$

$$x = \frac{-18}{4}$$

$$x = -4\frac{1}{2}$$

$$y = \frac{1}{4}$$

$$y = \frac{1}{4}$$

$$L = \{-4\frac{1}{2} / \frac{1}{4}\}$$

Aufgabe (16)

$$D_h = \begin{vmatrix} 2 & 4 \\ -\frac{1}{2} & 3 \end{vmatrix} = 2 \cdot 3 - 4 \cdot \left(-\frac{1}{2}\right) = 8$$

$$D_x = \begin{vmatrix} 4 & 4 \\ 3 & 3 \end{vmatrix} = 4 \cdot 3 - 4 \cdot 3 = 0$$

$$D_y = \begin{vmatrix} 2 & 4 \\ -\frac{1}{2} & 3 \end{vmatrix} = 2 \cdot 3 - 4 \cdot \left(-\frac{1}{2}\right) = 8$$

$$x = \frac{0}{8}$$

$$\begin{aligned}
 x &= 0 \\
 y &= \frac{8}{8} \\
 y &= 1 \\
 L &= \{0/1\}
 \end{aligned}$$

Aufgabe (17)

$$\begin{aligned}
 D_h &= \begin{vmatrix} -\frac{1}{2} & 4 \\ -2 & -8 \end{vmatrix} = -\frac{1}{2} \cdot (-8) - 4 \cdot (-2) = 12 \\
 D_x &= \begin{vmatrix} 6 & 4 \\ 2 & -8 \end{vmatrix} = 6 \cdot (-8) - 4 \cdot 2 = -56 \\
 D_y &= \begin{vmatrix} -\frac{1}{2} & 6 \\ -2 & 2 \end{vmatrix} = -\frac{1}{2} \cdot 2 - 6 \cdot (-2) = 11 \\
 x &= \frac{-56}{12} \\
 x &= -4\frac{2}{3} \\
 y &= \frac{11}{12} \\
 y &= \frac{11}{12} \\
 L &= \{-4\frac{2}{3}/\frac{11}{12}\}
 \end{aligned}$$

Aufgabe (18)

$$\begin{aligned}
 D_h &= \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0 \\
 D_x &= \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0 \\
 D_y &= \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 1 \cdot 2 = 0 \\
 L &= \text{unendlich}
 \end{aligned}$$

Aufgabe (19)

$$\begin{aligned}
 D_h &= \begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} = 1 \cdot 3 - 1 \cdot 3 = 0 \\
 D_x &= \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 3 = 3 \\
 D_y &= \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 1 \cdot 3 - 2 \cdot 3 = -3 \\
 L &= \{\}
 \end{aligned}$$

Aufgabe (20)

$$\begin{aligned}
 D_h &= \begin{vmatrix} -1\frac{4}{5} & 1\frac{1}{3} \\ -\frac{2}{3} & \frac{1}{9} \end{vmatrix} = -1\frac{4}{5} \cdot \frac{1}{9} - 1\frac{1}{3} \cdot \left(-\frac{2}{3}\right) = \frac{31}{45} \\
 D_x &= \begin{vmatrix} -1 & 1\frac{1}{3} \\ 9 & \frac{1}{9} \end{vmatrix} = -1 \cdot \frac{1}{9} - 1\frac{1}{3} \cdot 9 = -12\frac{1}{9} \\
 D_y &= \begin{vmatrix} -1\frac{4}{5} & -1 \\ -\frac{2}{3} & 9 \end{vmatrix} = -1\frac{4}{5} \cdot 9 - (-1) \cdot \left(-\frac{2}{3}\right) = -16\frac{13}{15}
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{-12\frac{1}{9}}{\frac{31}{45}} \\
 x &= -17\frac{18}{31} \\
 y &= \frac{-16\frac{13}{15}}{\frac{31}{45}} \\
 y &= -24\frac{15}{31} \\
 L &= \{-17\frac{18}{31} / -24\frac{15}{31}\}
 \end{aligned}$$

Aufgabe (21)

$$\begin{aligned}
 D_h &= \begin{vmatrix} 2 & -7 \\ 7 & -1 \end{vmatrix} = 2 \cdot (-1) - (-7) \cdot 7 = 47 \\
 D_x &= \begin{vmatrix} -8 & -7 \\ -9 & -1 \end{vmatrix} = -8 \cdot (-1) - (-7) \cdot (-9) = -55 \\
 D_y &= \begin{vmatrix} 2 & -8 \\ 7 & -9 \end{vmatrix} = 2 \cdot (-9) - (-8) \cdot 7 = 38 \\
 x &= \frac{-55}{47} \\
 x &= -1\frac{8}{47} \\
 y &= \frac{38}{47} \\
 y &= \frac{38}{47} \\
 L &= \{-1\frac{8}{47} / \frac{38}{47}\}
 \end{aligned}$$

Aufgabe (22)

$$\begin{aligned}
 D_h &= \begin{vmatrix} -7 & 9 \\ 5 & -6 \end{vmatrix} = -7 \cdot (-6) - 9 \cdot 5 = -3 \\
 D_x &= \begin{vmatrix} -3 & 9 \\ -4 & -6 \end{vmatrix} = -3 \cdot (-6) - 9 \cdot (-4) = 54 \\
 D_y &= \begin{vmatrix} -7 & -3 \\ 5 & -4 \end{vmatrix} = -7 \cdot (-4) - (-3) \cdot 5 = 43 \\
 x &= \frac{54}{-3} \\
 x &= -18 \\
 y &= \frac{43}{-3} \\
 y &= -14\frac{1}{3} \\
 L &= \{-18 / -14\frac{1}{3}\}
 \end{aligned}$$

## 5 Determinantenverfahren (3)

### 5.1 Aufgaben

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

(1)  
 $11x + 13 + 4z = 37$   
 $12x + 14y + 5z = 40$   
 $9x + 3y + 3z = 15$

(2)  
 $9x + 5 + 4z = 13$   
 $6x + 3y + -5z = 17$   
 $3x - 10y + 6z = 23$

(3)  
 $4x - 3 + 2z = 10$   
 $5x + 6y + -7z = 4$   
 $10x + 2y + -3z = 7$

(4)  
 $2x + 3 + -4z = 16$   
 $4x + 9y + -1z = 58$   
 $1x + 6y + 2z = 34$

(5)

$$1x + 2 + 3z = 4$$

$$2x + 3y + 2z = 6$$

$$0x + 2y + 6z = 0$$

(6)  
 $-2x - 8 + 0z = 1$   
 $1x + 4y + 0z = -\frac{1}{2}$   
 $8x - 2y + -1z = 8$

(7)  
 $-2x + 2 + 4z = 0$   
 $4x - \frac{1}{2}y + 2z = 5$   
 $4x - 2y + -1z = 8$

(8)  
 $1x + 1 + 1z = 1$   
 $2x + 2y + 2z = 4$   
 $\frac{4}{5}x - 2y + -1z = 8$

### 5.2 Lösungen

Aufgabe (1)

$$11x + 13y + 4z = 37$$

$$12x + 14y + 5z = 40$$

$$9x + 3y + 3z = 15$$

$$D_h = \begin{vmatrix} 11 & 13 & 4 & | & 11 & 13 \\ 12 & 14 & 5 & | & 12 & 14 \\ 9 & 3 & 3 & | & 9 & 3 \end{vmatrix}$$

$$D_h = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3 - 4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$$

$$D_x = \begin{vmatrix} 37 & 13 & 4 & | & 37 & 13 \\ 40 & 14 & 5 & | & 40 & 14 \\ 15 & 3 & 3 & | & 15 & 3 \end{vmatrix}$$

$$D_x = 37 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 15 + 4 \cdot 40 \cdot 3 - 4 \cdot 14 \cdot 15 - 37 \cdot 5 \cdot 3 - 13 \cdot 40 \cdot 3 = 54$$

$$D_y = \begin{vmatrix} 11 & 37 & 4 & | & 11 & 37 \\ 12 & 40 & 5 & | & 12 & 40 \\ 9 & 15 & 3 & | & 9 & 15 \end{vmatrix}$$

$$D_y = 11 \cdot 40 \cdot 3 + 37 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 15 - 4 \cdot 40 \cdot 9 - 11 \cdot 5 \cdot 15 - 37 \cdot 12 \cdot 3 = 108$$

$$D_z = \begin{vmatrix} 11 & 13 & 37 & | & 11 & 13 \\ 12 & 14 & 40 & | & 12 & 14 \\ 9 & 3 & 15 & | & 9 & 3 \end{vmatrix}$$

$$D_z = 11 \cdot 14 \cdot 15 + 13 \cdot 40 \cdot 9 + 37 \cdot 12 \cdot 3 - 37 \cdot 14 \cdot 9 - 11 \cdot 40 \cdot 3 - 13 \cdot 12 \cdot 15 = 0$$

$$\begin{aligned}
 x &= \frac{54}{54} \\
 x &= 1 \\
 y &= \frac{108}{54} \\
 y &= 2 \\
 z &= \frac{0}{54} \\
 z &= 0 \\
 L &= \{1/2/0\}
 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned}
 9x + 5y + 4z &= 13 \\
 6x + 3y - 5z &= 17 \\
 3x - 10y + 6z &= 23
 \end{aligned}$$

$$D_h = \begin{vmatrix} 9 & 5 & 4 \\ 6 & 3 & -5 \\ 3 & -10 & 6 \end{vmatrix} \begin{vmatrix} 9 & 5 \\ 6 & 3 \\ 3 & -10 \end{vmatrix}$$

$$D_h = 9 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot (-10) - 4 \cdot 3 \cdot 3 - 9 \cdot (-5) \cdot (-10) - 5 \cdot 6 \cdot 6 = -819$$

$$D_x = \begin{vmatrix} 13 & 5 & 4 \\ 17 & 3 & -5 \\ 23 & -10 & 6 \end{vmatrix} \begin{vmatrix} 13 & 5 \\ 17 & 3 \\ 23 & -10 \end{vmatrix}$$

$$D_x = 13 \cdot 3 \cdot 6 + 5 \cdot (-5) \cdot 23 + 4 \cdot 17 \cdot (-10) - 4 \cdot 3 \cdot 23 - 13 \cdot (-5) \cdot (-10) - 5 \cdot 17 \cdot 6 = -2457$$

$$D_y = \begin{vmatrix} 9 & 13 & 4 \\ 6 & 17 & -5 \\ 3 & 23 & 6 \end{vmatrix} \begin{vmatrix} 9 & 13 \\ 6 & 17 \\ 3 & 23 \end{vmatrix}$$

$$D_y = 9 \cdot 17 \cdot 6 + 13 \cdot (-5) \cdot 3 + 4 \cdot 6 \cdot 23 - 4 \cdot 17 \cdot 3 - 9 \cdot (-5) \cdot 23 - 13 \cdot 6 \cdot 6 = 1638$$

$$D_z = \begin{vmatrix} 9 & 5 & 13 \\ 6 & 3 & 17 \\ 3 & -10 & 23 \end{vmatrix} \begin{vmatrix} 9 & 5 \\ 6 & 3 \\ 3 & -10 \end{vmatrix}$$

$$D_z = 9 \cdot 3 \cdot 23 + 5 \cdot 17 \cdot 3 + 13 \cdot 6 \cdot (-10) - 13 \cdot 3 \cdot 3 - 9 \cdot 17 \cdot (-10) - 5 \cdot 6 \cdot 23 = 819$$

$$x = \frac{-2457}{-819}$$

$$x = 3$$

$$y = \frac{1638}{-819}$$

$$y = -2$$

$$z = \frac{819}{-819}$$

$$z = -1$$

$$L = \{3/-2/-1\}$$

Aufgabe (3)

$$\begin{aligned}
 4x + -3y + 2z &= 10 \\
 5x + 6y - 7z &= 4 \\
 10x + 2y - 3z &= 7
 \end{aligned}$$

$$D_h = \begin{vmatrix} 4 & -3 & 2 \\ 5 & 6 & -7 \\ 10 & 2 & -3 \end{vmatrix} \begin{vmatrix} 4 & -3 \\ 5 & 6 \\ 10 & 2 \end{vmatrix}$$

$$D_h = 4 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot 2 - 2 \cdot 6 \cdot 10 - 4 \cdot (-7) \cdot 2 - (-3) \cdot 5 \cdot (-3) = 49$$

$$D_x = \begin{vmatrix} 10 & -3 & 2 \\ 4 & 6 & -7 \\ 7 & 2 & -3 \end{vmatrix} \begin{vmatrix} 10 & -3 \\ 4 & 6 \\ 7 & 2 \end{vmatrix}$$

$$D_x = 10 \cdot 6 \cdot (-3) + (-3) \cdot (-7) \cdot 7 + 2 \cdot 4 \cdot 2 - 2 \cdot 6 \cdot 7 - 10 \cdot (-7) \cdot 2 - (-3) \cdot 4 \cdot (-3) = 3$$

$$D_y = \begin{vmatrix} 4 & 10 & 2 \\ 5 & 4 & -7 \\ 10 & 7 & -3 \end{vmatrix} \begin{vmatrix} 4 & 10 \\ 5 & 4 \\ 10 & 7 \end{vmatrix}$$

$$D_y = 4 \cdot 4 \cdot (-3) + 10 \cdot (-7) \cdot 10 + 2 \cdot 5 \cdot 7 - 2 \cdot 4 \cdot 10 - 4 \cdot (-7) \cdot 7 - 10 \cdot 5 \cdot (-3) = -412$$

$$D_z = \begin{vmatrix} 4 & -3 & 10 \\ 5 & 6 & 4 \\ 10 & 2 & 7 \end{vmatrix} \begin{vmatrix} 4 & -3 \\ 5 & 6 \\ 10 & 2 \end{vmatrix}$$

$$D_z = 4 \cdot 6 \cdot 7 + (-3) \cdot 4 \cdot 10 + 10 \cdot 5 \cdot 2 - 10 \cdot 6 \cdot 10 - 4 \cdot 4 \cdot 2 - (-3) \cdot 5 \cdot 7 = -379$$

$$x = \frac{3}{49}$$

$$x = \frac{3}{49}$$

$$y = \frac{-412}{49}$$

$$y = -8\frac{20}{49}$$

$$z = \frac{-379}{49}$$

$$z = -7\frac{36}{49}$$

$$L = \left\{ \frac{3}{49} / -8\frac{20}{49} / -7\frac{36}{49} \right\}$$

Aufgabe (4)

$$2x + 3y + 4z = 16$$

$$4x + 9y - 1z = 58$$

$$1x + 6y + 2z = 34$$

$$D_h = \begin{vmatrix} 2 & 3 & -4 \\ 4 & 9 & -1 \\ 1 & 6 & 2 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 9 \\ 1 & 6 \end{vmatrix}$$

$$D_h = 2 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 6 - (-4) \cdot 9 \cdot 1 - 2 \cdot (-1) \cdot 6 - 3 \cdot 4 \cdot 2 = -39$$

$$D_x = \begin{vmatrix} 16 & 3 & -4 \\ 58 & 9 & -1 \\ 34 & 6 & 2 \end{vmatrix} \begin{vmatrix} 16 & 3 \\ 58 & 9 \\ 34 & 6 \end{vmatrix}$$

$$D_x = 16 \cdot 9 \cdot 2 + 3 \cdot (-1) \cdot 34 + (-4) \cdot 58 \cdot 6 - (-4) \cdot 9 \cdot 34 - 16 \cdot (-1) \cdot 6 - 3 \cdot 58 \cdot 2 = -234$$

$$D_y = \begin{vmatrix} 2 & 16 & -4 \\ 4 & 58 & -1 \\ 1 & 34 & 2 \end{vmatrix} \begin{vmatrix} 2 & 16 \\ 4 & 58 \\ 1 & 34 \end{vmatrix}$$

$$D_y = 2 \cdot 58 \cdot 2 + 16 \cdot (-1) \cdot 1 + (-4) \cdot 4 \cdot 34 - (-4) \cdot 58 \cdot 1 - 2 \cdot (-1) \cdot 34 - 16 \cdot 4 \cdot 2 = -156$$

$$D_z = \begin{vmatrix} 2 & 3 & 16 \\ 4 & 9 & 58 \\ 1 & 6 & 34 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 4 & 9 \\ 1 & 6 \end{vmatrix}$$

$$D_z = 2 \cdot 9 \cdot 34 + 3 \cdot 58 \cdot 1 + 16 \cdot 4 \cdot 6 - 16 \cdot 9 \cdot 1 - 2 \cdot 58 \cdot 6 - 3 \cdot 4 \cdot 34 = -78$$

$$x = \frac{-234}{-39}$$

$$x = 6$$

$$y = \frac{-156}{-39}$$

$$y = 4$$

$$z = \frac{-78}{-39}$$

$$z = 2$$

$$L = \{6/4/2\}$$

Aufgabe (5)

$$1x + 2y + 3z = 4$$

$$2x + 3y + 2z = 6$$

$$0x + 2y + 6z = 0$$

$$D_h = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 0 & 2 & 6 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 2 & 3 \\ 0 & 2 \end{vmatrix}$$

$$D_h = 1 \cdot 3 \cdot 6 + 2 \cdot 2 \cdot 0 + 3 \cdot 2 \cdot 2 - 3 \cdot 3 \cdot 0 - 1 \cdot 2 \cdot 2 - 2 \cdot 2 \cdot 6 = 2$$

$$D_x = \begin{vmatrix} 4 & 2 & 3 \\ 6 & 3 & 2 \\ 0 & 2 & 6 \end{vmatrix} \begin{vmatrix} 4 & 2 \\ 6 & 3 \\ 0 & 2 \end{vmatrix}$$

$$D_x = 4 \cdot 3 \cdot 6 + 2 \cdot 2 \cdot 0 + 3 \cdot 6 \cdot 2 - 3 \cdot 3 \cdot 0 - 4 \cdot 2 \cdot 2 - 2 \cdot 6 \cdot 6 = 20$$

$$D_y = \begin{vmatrix} 1 & 4 & 3 & | & 1 & 4 \\ 2 & 6 & 2 & | & 2 & 6 \\ 0 & 0 & 6 & | & 0 & 0 \end{vmatrix}$$

$$D_y = 1 \cdot 6 \cdot 6 + 4 \cdot 2 \cdot 0 + 3 \cdot 2 \cdot 0 - 3 \cdot 6 \cdot 0 - 1 \cdot 2 \cdot 0 - 4 \cdot 2 \cdot 6 = -12$$

$$D_z = \begin{vmatrix} 1 & 2 & 4 & | & 1 & 2 \\ 2 & 3 & 6 & | & 2 & 3 \\ 0 & 2 & 0 & | & 0 & 2 \end{vmatrix}$$

$$D_z = 1 \cdot 3 \cdot 0 + 2 \cdot 6 \cdot 0 + 4 \cdot 2 \cdot 2 - 4 \cdot 3 \cdot 0 - 1 \cdot 6 \cdot 2 - 2 \cdot 2 \cdot 0 = 4$$

$$x = \frac{20}{2}$$

$$x = 10$$

$$y = \frac{-12}{2}$$

$$y = -6$$

$$z = \frac{4}{2}$$

$$z = 2$$

$$L = \{10 / -6 / 2\}$$

Aufgabe (6)

$$-2x + -8y + 0z = 1$$

$$1x + 4y + 0z = -\frac{1}{2}$$

$$8x - 2y - 1z = 8$$

$$D_h = \begin{vmatrix} -2 & -8 & 0 & | & -2 & -8 \\ 1 & 4 & 0 & | & 1 & 4 \\ 8 & -2 & -1 & | & 8 & -2 \end{vmatrix}$$

$$D_h = (-2) \cdot 4 \cdot (-1) + (-8) \cdot 0 \cdot 8 + 0 \cdot 1 \cdot (-2) - 0 \cdot 4 \cdot 8 - (-2) \cdot 0 \cdot (-2) - (-8) \cdot 1 \cdot (-1) = 0$$

$$D_x = \begin{vmatrix} 1 & -8 & 0 & | & 1 & -8 \\ -\frac{1}{2} & 4 & 0 & | & -\frac{1}{2} & 4 \\ 8 & -2 & -1 & | & 8 & -2 \end{vmatrix}$$

$$D_x = 1 \cdot 4 \cdot (-1) + (-8) \cdot 0 \cdot 8 + 0 \cdot (-\frac{1}{2}) \cdot (-2) - 0 \cdot 4 \cdot 8 - 1 \cdot 0 \cdot (-2) - (-8) \cdot (-\frac{1}{2}) \cdot (-1) = 0$$

$$D_y = \begin{vmatrix} -2 & 1 & 0 & | & -2 & 1 \\ 1 & -\frac{1}{2} & 0 & | & 1 & -\frac{1}{2} \\ 8 & 8 & -1 & | & 8 & 8 \end{vmatrix}$$

$$D_y = (-2) \cdot (-\frac{1}{2}) \cdot (-1) + 1 \cdot 0 \cdot 8 + 0 \cdot 1 \cdot 8 - 0 \cdot (-\frac{1}{2}) \cdot 8 - (-2) \cdot 0 \cdot 8 - 1 \cdot 1 \cdot (-1) = 0$$

$$D_z = \begin{vmatrix} -2 & -8 & 1 & | & -2 & -8 \\ 1 & 4 & -\frac{1}{2} & | & 1 & 4 \\ 8 & -2 & 8 & | & 8 & -2 \end{vmatrix}$$

$$D_z = (-2) \cdot 4 \cdot 8 + (-8) \cdot (-\frac{1}{2}) \cdot 8 + 1 \cdot 1 \cdot (-2) - 1 \cdot 4 \cdot 8 - (-2) \cdot (-\frac{1}{2}) \cdot (-2) - (-8) \cdot 1 \cdot 8 = 0$$

$L = \text{unendlich}$

Aufgabe (7)

$$-2x + 2y + 4z = 0$$

$$4x - \frac{1}{2}y + 2z = 5$$

$$4x - 2y - 1z = 8$$

$$D_h = \begin{vmatrix} -2 & 2 & 4 & | & -2 & 2 \\ 4 & -\frac{1}{2} & 2 & | & 4 & -\frac{1}{2} \\ 4 & -2 & -1 & | & 4 & -2 \end{vmatrix}$$

$$D_h = (-2) \cdot (-\frac{1}{2}) \cdot (-1) + 2 \cdot 2 \cdot 4 + 4 \cdot 4 \cdot (-2) - 4 \cdot (-\frac{1}{2}) \cdot 4 - (-2) \cdot 2 \cdot (-2) - 2 \cdot 4 \cdot (-1) = -9$$

$$D_x = \begin{vmatrix} 0 & 2 & 4 & | & 0 & 2 \\ 5 & -\frac{1}{2} & 2 & | & 5 & -\frac{1}{2} \\ 8 & -2 & -1 & | & 8 & -2 \end{vmatrix}$$

$$D_x = 0 \cdot (-\frac{1}{2}) \cdot (-1) + 2 \cdot 2 \cdot 8 + 4 \cdot 5 \cdot (-2) - 4 \cdot (-\frac{1}{2}) \cdot 8 - 0 \cdot 2 \cdot (-2) - 2 \cdot 5 \cdot (-1) = 18$$

$$D_y = \begin{vmatrix} -2 & 0 & 4 \\ 4 & 5 & 2 \\ 4 & 8 & -1 \end{vmatrix} \begin{vmatrix} -2 & 0 \\ 4 & 5 \\ 4 & 8 \end{vmatrix}$$

$$D_y = (-2) \cdot 5 \cdot (-1) + 0 \cdot 2 \cdot 4 + 4 \cdot 4 \cdot 8 - 4 \cdot 5 \cdot 4 - (-2) \cdot 2 \cdot 8 - 0 \cdot 4 \cdot (-1) = 90$$

$$D_z = \begin{vmatrix} -2 & 2 & 0 \\ 4 & -\frac{1}{2} & 5 \\ 4 & -2 & 8 \end{vmatrix} \begin{vmatrix} -2 & 2 \\ 4 & -\frac{1}{2} \\ 4 & -2 \end{vmatrix}$$

$$D_z = (-2) \cdot \left(-\frac{1}{2}\right) \cdot 8 + 2 \cdot 5 \cdot 4 + 0 \cdot 4 \cdot (-2) - 0 \cdot \left(-\frac{1}{2}\right) \cdot 4 - (-2) \cdot 5 \cdot (-2) - 2 \cdot 4 \cdot 8 = -36$$

$$x = \frac{18}{-9}$$

$$x = -2$$

$$y = \frac{90}{-9}$$

$$y = -10$$

$$z = \frac{-36}{-9}$$

$$z = 4$$

$$L = \{-2/ -10/4\}$$

Aufgabe (8)

$$1x + 1y + 1z = 1$$

$$2x + 2y + 2z = 4$$

$$\frac{4}{5}x - 2y - 1z = 8$$

$$D_h = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ \frac{4}{5} & -2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 2 \\ \frac{4}{5} & -2 \end{vmatrix}$$

$$D_h = 1 \cdot 2 \cdot (-1) + 1 \cdot 2 \cdot \frac{4}{5} + 1 \cdot 2 \cdot (-2) - 1 \cdot 2 \cdot \frac{4}{5} - 1 \cdot 2 \cdot (-2) - 1 \cdot 2 \cdot (-1) = 0$$

$$D_x = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 2 \\ 8 & -2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$D_x = 1 \cdot 2 \cdot (-1) + 1 \cdot 2 \cdot 8 + 1 \cdot 4 \cdot (-2) - 1 \cdot 2 \cdot 8 - 1 \cdot 2 \cdot (-2) - 1 \cdot 4 \cdot (-1) = -2$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 2 \\ \frac{4}{5} & 8 & -1 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 4 \\ \frac{4}{5} & 8 \end{vmatrix}$$

$$D_y = 1 \cdot 4 \cdot (-1) + 1 \cdot 2 \cdot \frac{4}{5} + 1 \cdot 2 \cdot 8 - 1 \cdot 4 \cdot \frac{4}{5} - 1 \cdot 2 \cdot 8 - 1 \cdot 2 \cdot (-1) = -3\frac{3}{5}$$

$$D_z = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 4 \\ \frac{4}{5} & -2 & 8 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ 2 & 2 \\ \frac{4}{5} & -2 \end{vmatrix}$$

$$D_z = 1 \cdot 2 \cdot 8 + 1 \cdot 4 \cdot \frac{4}{5} + 1 \cdot 2 \cdot (-2) - 1 \cdot 2 \cdot \frac{4}{5} - 1 \cdot 4 \cdot (-2) - 1 \cdot 2 \cdot 8 = 5\frac{3}{5}$$

$$L = \{\}$$

## 6 Gaußsches Eliminationsverfahren

### 6.1 Aufgaben

- Berechnen Sie die Lösung des linearen Gleichungssystems.

Um eigene Aufgaben zu lösen, klicken Sie hier: [Gaußsches Eliminationsverfahren](#)

$$(1) \quad \begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2 \end{aligned}$$

$$(2) \quad \begin{aligned} 3x_1 + 3x_2 &= 3 \\ 6x_1 + 6x_2 &= 12 \end{aligned}$$

$$(3) \quad \begin{aligned} x_1 + x_2 &= 2 \\ 2x_1 - x_2 &= 1 \end{aligned}$$

$$(4) \quad \begin{aligned} x_1 + 2x_2 &= 3 \\ 4x_1 + 5x_2 &= 6 \end{aligned}$$

$$(5) \quad \begin{aligned} 3x_1 + 3x_2 &= 0 \\ 4x_1 + 2x_2 &= 0 \end{aligned}$$

$$(6) \quad \begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 5x_1 + 6x_2 + 7x_3 &= 8 \\ 9x_1 + 10x_2 + 11x_3 &= 12 \end{aligned}$$

$$(7) \quad \begin{aligned} -2x_1 - 8x_2 &= 1 \\ x_1 + 4x_2 &= -\frac{1}{2} \\ 8x_1 - 2x_2 - x_3 &= 8 \end{aligned}$$

$$(8) \quad \begin{aligned} 2x_1 + 4x_2 &= 4 \\ -\frac{1}{2}x_1 + 2x_2 + 5x_3 &= 4 \\ -2x_1 - x_2 + 8x_3 &= 0 \end{aligned}$$

$$(9) \quad \begin{aligned} x_1 + x_2 + x_3 &= 1 \\ 2x_1 + 2x_2 + 2x_3 &= 4 \\ \frac{4}{5}x_1 - 2x_2 - x_3 &= 8 \end{aligned}$$

$$(10) \quad \begin{aligned} 1\frac{1}{10}x_1 - 3\frac{2}{5}x_2 + 4\frac{1}{5}x_3 &= 4 \\ x_1 + x_2 + 4x_3 &= -2\frac{3}{10} \\ x_1 - 3x_2 + 4x_3 &= 4 \end{aligned}$$

$$(11) \quad \begin{aligned} 2x_1 - 4x_2 + x_3 - x_4 &= -8 \\ 3x_1 + 2x_2 - x_3 + 2x_4 &= 12 \\ x_1 - x_2 + 2x_3 - 2x_4 &= -5 \\ 4x_1 - 2x_2 - 3x_3 + x_4 &= 0 \end{aligned}$$

$$(12) \quad \begin{aligned} x_1 + x_2 - x_3 + 3x_4 - 3x_5 &= 0 \\ 2x_1 + x_2 + x_3 + 4x_4 - x_5 &= 0 \\ 2x_1 + x_2 + x_3 + 4x_4 - x_5 &= 0 \\ 2x_1 + 3x_2 - 5x_3 + 8x_4 - 11x_5 &= 0 \\ -x_1 + x_2 - 5x_3 + x_4 - 7x_5 &= 0 \end{aligned}$$

$$(13) \quad \begin{aligned} 3x_1 + 2x_4 &= 0 \\ x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 3x_1 + 2x_4 &= 0 \\ x_1 - x_2 + x_4 &= 6 \end{aligned}$$

$$(14) \quad \begin{aligned} 2x_1 + x_2 + x_3 + x_4 + x_5 &= 8 \\ 3x_1 - 2x_2 - 4x_3 + x_4 + 3x_5 &= 22 \\ 4x_1 + 3x_2 - 5x_3 + 2x_4 - x_5 &= 13 \\ 5x_1 - 4x_2 + 2x_3 - 3x_4 + 2x_5 &= 19 \\ 6x_1 + 8x_2 - 7x_3 + 9x_4 + 4x_5 &= 41 \end{aligned}$$

## 6.2 Lösungen

Aufgabe (1)

$$\begin{aligned}x_1 + x_2 &= 1 \\ 2x_1 + 2x_2 &= 2\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 1 & 1 \\ 2 & 2 & 2\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{1} \\ z2s1 &= 2 - 1 \cdot \frac{2}{1} = 0; z2s2 = 2 - 1 \cdot \frac{2}{1} = 0; z2s3 = 2 - 1 \cdot \frac{2}{1} = 0;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 1 & 1 \\ 0 & 0 & 0\end{array}$$

 $L = \text{unendlich}$ 

Aufgabe (2)

$$\begin{aligned}3x_1 + 3x_2 &= 3 \\ 6x_1 + 6x_2 &= 12\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 3 & 3 \\ 6 & 6 & 12\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{3} \\ z2s1 &= 6 - 3 \cdot \frac{6}{3} = 0; z2s2 = 6 - 3 \cdot \frac{6}{3} = 0; z2s3 = 12 - 3 \cdot \frac{6}{3} = 6;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 3 & 3 \\ 0 & 0 & 6\end{array}$$

 $L = \{\}$ 

Aufgabe (3)

$$\begin{aligned}x_1 + x_2 &= 2 \\ 2x_1 - x_2 &= 1\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 1 & 2 \\ 2 & -1 & 1\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{1} \\ z2s1 &= 2 - 1 \cdot \frac{2}{1} = 0; z2s2 = -1 - 1 \cdot \frac{2}{1} = -3; z2s3 = 1 - 2 \cdot \frac{2}{1} = -3;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 1 & 2 \\ 0 & -3 & -3\end{array}$$

$$\begin{aligned}\text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-3} \\ z1s2 &= 1 - (-3) \cdot \frac{1}{-3} = 0; z1s3 = 2 - (-3) \cdot \frac{1}{-3} = 1;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 0 & 1 \\ 0 & -3 & -3\end{array}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 0 & 1 \\ 0 & -3 & -3\end{array}$$

$$\begin{aligned}x_1 &= \frac{1}{1} = 1 \\ x_2 &= \frac{-3}{-3} = 1 \\ L &= \{1/1\}\end{aligned}$$

Aufgabe (4)

$$\begin{aligned}x_1 + 2x_2 &= 3 \\4x_1 + 5x_2 &= 6\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 2 & 3 \\ 4 & 5 & 6\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{1} \\z_2s_1 &= 4 - 1 \cdot \frac{4}{1} = 0; z_2s_2 = 5 - 2 \cdot \frac{4}{1} = -3; z_2s_3 = 6 - 3 \cdot \frac{4}{1} = -6;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 2 & 3 \\ 0 & -3 & -6\end{array}$$

$$\begin{aligned}\text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-3} \\z_1s_2 &= 2 - (-3) \cdot \frac{2}{-3} = 0; z_1s_3 = 3 - (-6) \cdot \frac{2}{-3} = -1;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 0 & -1 \\ 0 & -3 & -6\end{array}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 1 & 0 & -1 \\ 0 & -3 & -6\end{array}$$

$$\begin{aligned}x_1 &= \frac{-1}{1} = -1 \\x_2 &= \frac{-6}{-3} = 2 \\L &= \{-1/2\}\end{aligned}$$

Aufgabe (5)

$$\begin{aligned}3x_1 + 3x_2 &= 0 \\4x_1 + 2x_2 &= 0\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 3 & 0 \\ 4 & 2 & 0\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{3} \\z_2s_1 &= 4 - 3 \cdot \frac{4}{3} = 0; z_2s_2 = 2 - 3 \cdot \frac{4}{3} = -2; z_2s_3 = 0 - 0 \cdot \frac{4}{3} = 0;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 3 & 0 \\ 0 & -2 & 0\end{array}$$

$$\begin{aligned}\text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{-2} \\z_1s_2 &= 3 - (-2) \cdot \frac{3}{-2} = 0; z_1s_3 = 0 - 0 \cdot \frac{3}{-2} = 0;\end{aligned}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 0 & 0 \\ 0 & -2 & 0\end{array}$$

$$\begin{array}{cc|c}x_1 & x_2 & \\ \hline 3 & 0 & 0 \\ 0 & -2 & 0\end{array}$$

$$\begin{aligned}x_1 &= \frac{0}{3} = 0 \\x_2 &= \frac{0}{-2} = 0 \\L &= \{0/0\}\end{aligned}$$

Aufgabe (6)

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\5x_1 + 6x_2 + 7x_3 &= 8 \\9x_1 + 10x_2 + 11x_3 &= 12\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12\end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{1}$$

$$z2s1 = 5 - 1 \cdot \frac{5}{1} = 0; z2s2 = 6 - 2 \cdot \frac{5}{1} = -4; z2s3 = 7 - 3 \cdot \frac{5}{1} = -8; z2s4 = 8 - 4 \cdot \frac{5}{1} = -12;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{1}$$

$$z3s1 = 9 - 1 \cdot \frac{9}{1} = 0; z3s2 = 10 - 2 \cdot \frac{9}{1} = -8; z3s3 = 11 - 3 \cdot \frac{9}{1} = -16; z3s4 = 12 - 4 \cdot \frac{9}{1} = -24;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-4}$$

$$z1s2 = 2 - (-4) \cdot \frac{2}{-4} = 0; z1s3 = 3 - (-8) \cdot \frac{2}{-4} = -1; z1s4 = 4 - (-12) \cdot \frac{2}{-4} = -2;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \end{array}$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-8}{-4}$$

$$z3s2 = -8 - (-4) \cdot \frac{-8}{-4} = 0; z3s3 = -16 - (-8) \cdot \frac{-8}{-4} = 0; z3s4 = -24 - (-12) \cdot \frac{-8}{-4} = 0;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0 \end{array}$$

$L = \text{unendlich}$

### Aufgabe (7)

$$-2x_1 - 8x_2 = 1$$

$$x_1 + 4x_2 = -\frac{1}{2}$$

$$8x_1 - 2x_2 - x_3 = 8$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline -2 & -8 & 0 & 1 \\ 1 & 4 & 0 & -\frac{1}{2} \\ 8 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{-2}$$

$$z2s1 = 1 - (-2) \cdot \frac{1}{-2} = 0; z2s2 = 4 - (-8) \cdot \frac{1}{-2} = 0; z2s3 = 0 - 0 \cdot \frac{1}{-2} = 0; z2s4 = -\frac{1}{2} - 1 \cdot \frac{1}{-2} = 0;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline -2 & -8 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 8 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{8}{-2}$$

$$z3s1 = 8 - (-2) \cdot \frac{8}{-2} = 0; z3s2 = -2 - (-8) \cdot \frac{8}{-2} = -34; z3s3 = -1 - 0 \cdot \frac{8}{-2} = -1; z3s4 = 8 - 1 \cdot \frac{8}{-2} = 12;$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline -2 & -8 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -34 & -1 & 12 \end{array}$$

$$\begin{array}{ccc|c} x1 & x2 & x3 & \\ \hline -2 & -8 & 0 & 1 \\ 0 & -34 & -1 & 12 \\ 0 & 0 & 0 & 0 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-8}{-34}$$

$$z1s2 = -8 - (-34) \cdot \frac{-8}{-34} = 0; z1s3 = 0 - (-1) \cdot \frac{-8}{-34} = \frac{4}{17}; z1s4 = 1 - 12 \cdot \frac{-8}{-34} = -1\frac{14}{17};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline -2 & 0 & \frac{4}{17} & -1\frac{14}{17} \\ 0 & -34 & -1 & 12 \\ 0 & 0 & 0 & 0 \\ \hline x_1 & x_2 & x_3 & \\ \hline -2 & 0 & \frac{4}{17} & -1\frac{14}{17} \\ 0 & -34 & -1 & 12 \\ 0 & 0 & 0 & 0 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{0}{-34}$$

$$z3s2 = 0 - (-34) \cdot \frac{0}{-34} = 0; z3s3 = 0 - (-1) \cdot \frac{0}{-34} = 0; z3s4 = 0 - 12 \cdot \frac{0}{-34} = 0;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline -2 & 0 & \frac{4}{17} & -1\frac{14}{17} \\ 0 & -34 & -1 & 12 \\ 0 & 0 & 0 & 0 \end{array}$$

$L = \text{unendlich}$

### Aufgabe (8)

$$\begin{aligned} 2x_1 + 4x_2 &= 4 \\ -\frac{1}{2}x_1 + 2x_2 + 5x_3 &= 4 \\ -2x_1 - x_2 + 8x_3 &= 0 \end{aligned}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 4 & 0 & 4 \\ -\frac{1}{2} & 2 & 5 & 4 \\ -2 & -1 & 8 & 0 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{-\frac{1}{2}}{2}$$

$$z2s1 = -\frac{1}{2} - 2 \cdot \frac{-\frac{1}{2}}{2} = 0; z2s2 = 2 - 4 \cdot \frac{-\frac{1}{2}}{2} = 3; z2s3 = 5 - 0 \cdot \frac{-\frac{1}{2}}{2} = 5; z2s4 = 4 - 4 \cdot \frac{-\frac{1}{2}}{2} = 5;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 4 & 0 & 4 \\ 0 & 3 & 5 & 5 \\ -2 & -1 & 8 & 0 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{-2}{2}$$

$$z3s1 = -2 - 2 \cdot \frac{-2}{2} = 0; z3s2 = -1 - 4 \cdot \frac{-2}{2} = 3; z3s3 = 8 - 0 \cdot \frac{-2}{2} = 8; z3s4 = 0 - 4 \cdot \frac{-2}{2} = 4;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 4 & 0 & 4 \\ 0 & 3 & 5 & 5 \\ 0 & 3 & 8 & 4 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{\frac{4}{3}}{3}$$

$$z1s2 = 4 - 3 \cdot \frac{\frac{4}{3}}{3} = 0; z1s3 = 0 - 5 \cdot \frac{\frac{4}{3}}{3} = -6\frac{2}{3}; z1s4 = 4 - 5 \cdot \frac{\frac{4}{3}}{3} = -2\frac{2}{3};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & -6\frac{2}{3} & -2\frac{2}{3} \\ 0 & 3 & 5 & 5 \\ 0 & 3 & 8 & 4 \end{array}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & -6\frac{2}{3} & -2\frac{2}{3} \\ 0 & 3 & 5 & 5 \\ 0 & 3 & 8 & 4 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{\frac{3}{3}}{3}$$

$$z3s2 = 3 - 3 \cdot \frac{\frac{3}{3}}{3} = 0; z3s3 = 8 - 5 \cdot \frac{\frac{3}{3}}{3} = 3; z3s4 = 4 - 5 \cdot \frac{\frac{3}{3}}{3} = -1;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & -6\frac{2}{3} & -2\frac{2}{3} \\ 0 & 3 & 5 & 5 \\ 0 & 0 & 3 & -1 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-6\frac{2}{3}}{3}$$

$$z1s3 = -6\frac{2}{3} - 3 \cdot \frac{-6\frac{2}{3}}{3} = 0; z1s4 = -2\frac{2}{3} - (-1) \cdot \frac{-6\frac{2}{3}}{3} = -4\frac{8}{9};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & 0 & -4\frac{8}{9} \\ 0 & 3 & 5 & 5 \\ 0 & 0 & 3 & -1 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{5}{3}$$

$$z2s3 = 5 - 3 \cdot \frac{5}{3} = 0; z2s4 = 5 - (-1) \cdot \frac{5}{3} = 6\frac{2}{3};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & 0 & -4\frac{8}{9} \\ 0 & 3 & 0 & 6\frac{2}{3} \\ 0 & 0 & 3 & -1 \end{array}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 0 & 0 & -4\frac{8}{9} \\ 0 & 3 & 0 & 6\frac{2}{3} \\ 0 & 0 & 3 & -1 \end{array}$$

$$x_1 = \frac{-4\frac{8}{9}}{2} = -2\frac{4}{9}$$

$$x_2 = \frac{6\frac{2}{3}}{3} = 2\frac{2}{9}$$

$$x_3 = \frac{-1}{3} = -\frac{1}{3}$$

$$L = \{-2\frac{4}{9} / 2\frac{2}{9} / -\frac{1}{3}\}$$

Aufgabe (9)

$$x_1 + x_2 + x_3 = 1$$

$$2x_1 + 2x_2 + 2x_3 = 4$$

$$\frac{4}{5}x_1 - 2x_2 - x_3 = 8$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 4 \\ \frac{4}{5} & -2 & -1 & 8 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{1}$$

$$z2s1 = 2 - 1 \cdot \frac{2}{1} = 0; z2s2 = 2 - 1 \cdot \frac{2}{1} = 0; z2s3 = 2 - 1 \cdot \frac{2}{1} = 0; z2s4 = 4 - 1 \cdot \frac{2}{1} = 2;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ \frac{4}{5} & -2 & -1 & 8 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{4}{5}$$

$$z3s1 = \frac{4}{5} - 1 \cdot \frac{4}{5} = 0; z3s2 = -2 - 1 \cdot \frac{4}{5} = -2\frac{4}{5}; z3s3 = -1 - 1 \cdot \frac{4}{5} = -1\frac{4}{5}; z3s4 = 8 - 1 \cdot \frac{4}{5} = 7\frac{1}{5};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & -2\frac{4}{5} & -1\frac{4}{5} & 7\frac{1}{5} \end{array}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 1 & 1 & 1 \\ 0 & -2\frac{4}{5} & -1\frac{4}{5} & 7\frac{1}{5} \\ 0 & 0 & 0 & 2 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-2\frac{4}{5}}$$

$$z1s2 = 1 - (-2\frac{4}{5}) \cdot \frac{1}{-2\frac{4}{5}} = 0; z1s3 = 1 - (-1\frac{4}{5}) \cdot \frac{1}{-2\frac{4}{5}} = \frac{5}{14}; z1s4 = 1 - 7\frac{1}{5} \cdot \frac{1}{-2\frac{4}{5}} = 3\frac{4}{7};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 0 & \frac{5}{14} & 3\frac{4}{7} \\ 0 & -2\frac{4}{5} & -1\frac{4}{5} & 7\frac{1}{5} \\ 0 & 0 & 0 & 2 \end{array}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 0 & \frac{5}{14} & 3\frac{4}{7} \\ 0 & -2\frac{4}{5} & -1\frac{4}{5} & 7\frac{1}{5} \\ 0 & 0 & 0 & 2 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{0}{-2\frac{4}{5}}$$

$$z3s2 = 0 - (-2\frac{4}{5}) \cdot \frac{0}{-2\frac{4}{5}} = 0; z3s3 = 0 - (-1\frac{4}{5}) \cdot \frac{0}{-2\frac{4}{5}} = 0; z3s4 = 2 - 7\frac{1}{5} \cdot \frac{0}{-2\frac{4}{5}} = 2;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1 & 0 & \frac{5}{14} & 3\frac{4}{7} \\ 0 & -2\frac{4}{5} & -1\frac{4}{5} & 7\frac{1}{5} \\ 0 & 0 & 0 & 2 \end{array}$$

$$L = \{ \}$$

## Aufgabe (10)

$$\begin{aligned} 1\frac{1}{10}x_1 - 3\frac{2}{5}x_2 + 4\frac{1}{5}x_3 &= 4 \\ x_1 + x_2 + 4x_3 &= -2\frac{3}{10} \\ x_1 - 3x_2 + 4x_3 &= 4 \end{aligned}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & -3\frac{2}{5} & 4\frac{1}{5} & 4 \\ 1 & 1 & 4 & -2\frac{3}{10} \\ 1 & -3 & 4 & 4 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{1\frac{1}{10}}$$

$$z2s1 = 1 - 1\frac{1}{10} \cdot \frac{1}{1\frac{1}{10}} = 0; z2s2 = 1 - (-3\frac{2}{5}) \cdot \frac{1}{1\frac{1}{10}} = 4\frac{1}{11}; z2s3 = 4 - 4\frac{1}{5} \cdot \frac{1}{1\frac{1}{10}} = \frac{2}{11}; z2s4 = -2\frac{3}{10} - 4 \cdot \frac{1}{1\frac{1}{10}} = -5,936;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & -3\frac{2}{5} & 4\frac{1}{5} & 4 \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 1 & -3 & 4 & 4 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{1\frac{1}{10}}$$

$$z3s1 = 1 - 1\frac{1}{10} \cdot \frac{1}{1\frac{1}{10}} = 0; z3s2 = -3 - (-3\frac{2}{5}) \cdot \frac{1}{1\frac{1}{10}} = \frac{1}{11}; z3s3 = 4 - 4\frac{1}{5} \cdot \frac{1}{1\frac{1}{10}} = \frac{2}{11}; z3s4 = 4 - 4 \cdot \frac{1}{1\frac{1}{10}} = \frac{4}{11};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & -3\frac{2}{5} & 4\frac{1}{5} & 4 \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 0 & \frac{1}{11} & \frac{1}{11} & \frac{4}{11} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-3\frac{2}{5}}{4\frac{1}{11}}$$

$$z1s2 = -3\frac{2}{5} - 4\frac{1}{11} \cdot \frac{-3\frac{2}{5}}{4\frac{1}{11}} = 0; z1s3 = 4\frac{1}{5} - \frac{2}{11} \cdot \frac{-3\frac{2}{5}}{4\frac{1}{11}} = 4,351; z1s4 = 4 - (-5,936) \cdot \frac{-3\frac{2}{5}}{4\frac{1}{11}} = -0,934;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 4,351 & -0,934 \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 0 & \frac{1}{11} & \frac{1}{11} & \frac{4}{11} \end{array}$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 4,351 & -0,934 \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 0 & \frac{1}{11} & \frac{1}{11} & \frac{4}{11} \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{\frac{1}{11}}{4\frac{1}{11}}$$

$$z3s2 = \frac{1}{11} - 4\frac{1}{11} \cdot \frac{\frac{1}{11}}{4\frac{1}{11}} = 0; z3s3 = \frac{2}{11} - \frac{2}{11} \cdot \frac{\frac{1}{11}}{4\frac{1}{11}} = \frac{8}{45}; z3s4 = \frac{4}{11} - (-5,936) \cdot \frac{\frac{1}{11}}{4\frac{1}{11}} = 0,496;$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 4,351 & -0,934 \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 0 & 0 & \frac{8}{45} & 0,496 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{4,351}{\frac{8}{45}}$$

$$z1s3 = 4,351 - \frac{8}{45} \cdot \frac{4,351}{\frac{8}{45}} = 0; z1s4 = -0,934 - 0,496 \cdot \frac{4,351}{\frac{8}{45}} = -13\frac{1}{16};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 0 & -13\frac{1}{16} \\ 0 & 4\frac{1}{11} & \frac{2}{11} & -5,936 \\ 0 & 0 & \frac{8}{45} & 0,496 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{2}{11}}{\frac{8}{45}}$$

$$z2s3 = \frac{2}{11} - \frac{8}{45} \cdot \frac{\frac{2}{11}}{\frac{8}{45}} = 0; z2s4 = -5,936 - 0,496 \cdot \frac{\frac{2}{11}}{\frac{8}{45}} = -6\frac{39}{88};$$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 0 & -13\frac{1}{16} \\ 0 & 4\frac{1}{11} & 0 & -6\frac{39}{88} \\ 0 & 0 & \frac{8}{45} & 0,496 \\ \hline x_1 & x_2 & x_3 & \\ \hline 1\frac{1}{10} & 0 & 0 & -13\frac{1}{16} \\ 0 & 4\frac{1}{11} & 0 & -6\frac{39}{88} \\ 0 & 0 & \frac{8}{45} & 0,496 \end{array}$$

$$x_1 = \frac{-13\frac{1}{16}}{1\frac{1}{10}} = -11\frac{7}{8}$$

$$x_2 = \frac{-6\frac{39}{88}}{4\frac{1}{11}} = -1\frac{23}{40}$$

$$x_3 = \frac{0,496}{\frac{8}{45}} = 2\frac{63}{80}$$

$$L = \left\{ -11\frac{7}{8} / -1\frac{23}{40} / 2\frac{63}{80} \right\}$$

Aufgabe (11)

$$2x_1 - 4x_2 + x_3 - x_4 = -8$$

$$3x_1 + 2x_2 - x_3 + 2x_4 = 12$$

$$x_1 - x_2 + 2x_3 - 2x_4 = -5$$

$$4x_1 - 2x_2 - 3x_3 + x_4 = 0$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & -4 & 1 & -1 & -8 \\ 3 & 2 & -1 & 2 & 12 \\ 1 & -1 & 2 & -2 & -5 \\ 4 & -2 & -3 & 1 & 0 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2}$$

$$z2s1 = 3 - 2 \cdot \frac{3}{2} = 0; z2s2 = 2 - (-4) \cdot \frac{3}{2} = 8; z2s3 = -1 - 1 \cdot \frac{3}{2} = -2\frac{1}{2}; z2s4 = 2 - (-1) \cdot \frac{3}{2} = 3\frac{1}{2}; z2s5 = 12 - (-8) \cdot \frac{3}{2} = 24;$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & -4 & 1 & -1 & -8 \\ 0 & 8 & -2\frac{1}{2} & 3\frac{1}{2} & 24 \\ 1 & -1 & 2 & -2 & -5 \\ 4 & -2 & -3 & 1 & 0 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{2}$$

$$z3s1 = 1 - 2 \cdot \frac{1}{2} = 0; z3s2 = -1 - (-4) \cdot \frac{1}{2} = 1; z3s3 = 2 - 1 \cdot \frac{1}{2} = 1\frac{1}{2}; z3s4 = -2 - (-1) \cdot \frac{1}{2} = -1\frac{1}{2}; z3s5 = -5 - (-8) \cdot \frac{1}{2} = -1;$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & -4 & 1 & -1 & -8 \\ 0 & 8 & -2\frac{1}{2} & 3\frac{1}{2} & 24 \\ 0 & 1 & 1\frac{1}{2} & -1\frac{1}{2} & -1 \\ 4 & -2 & -3 & 1 & 0 \end{array}$$

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile1} \cdot \frac{4}{2}$$

$$z4s1 = 4 - 2 \cdot \frac{4}{2} = 0; z4s2 = -2 - (-4) \cdot \frac{4}{2} = 6; z4s3 = -3 - 1 \cdot \frac{4}{2} = -5; z4s4 = 1 - (-1) \cdot \frac{4}{2} = 3; z4s5 = 0 - (-8) \cdot \frac{4}{2} = 16;$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & -4 & 1 & -1 & -8 \\ 0 & 8 & -2\frac{1}{2} & 3\frac{1}{2} & 24 \\ 0 & 1 & 1\frac{1}{2} & -1\frac{1}{2} & -1 \\ 0 & 6 & -5 & 3 & 16 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-4}{8}$$

$$z1s2 = -4 - 8 \cdot \frac{-4}{8} = 0; z1s3 = 1 - (-2\frac{1}{2}) \cdot \frac{-4}{8} = -\frac{1}{4}; z1s4 = -1 - 3\frac{1}{2} \cdot \frac{-4}{8} = \frac{3}{4}; z1s5 = -8 - 24 \cdot \frac{-4}{8} = 4;$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & 0 & -\frac{1}{4} & \frac{3}{4} & 4 \\ 0 & 8 & -2\frac{1}{2} & 3\frac{1}{2} & 24 \\ 0 & 1 & 1\frac{1}{2} & -1\frac{1}{2} & -1 \\ 0 & 6 & -5 & 3 & 16 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 2 & 0 & -\frac{1}{4} & \frac{3}{4} & 4 \\ 0 & 8 & -2\frac{1}{2} & 3\frac{1}{2} & 24 \\ 0 & 1 & 1\frac{1}{2} & -1\frac{1}{2} & -1 \\ 0 & 6 & -5 & 3 & 16 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{8}$$

$$z3s2 = 1 - 8 \cdot \frac{1}{8} = 0; z3s3 = 1\frac{1}{2} - (-2\frac{1}{2}) \cdot \frac{1}{8} = 1\frac{13}{16}; z3s4 = -1\frac{1}{2} - 3\frac{1}{2} \cdot \frac{1}{8} = -1\frac{15}{16}; z3s5 = -1 - 24 \cdot \frac{1}{8} = -4;$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	$-\frac{1}{4}$	$\frac{3}{4}$	4
0	8	$-2\frac{1}{2}$	$3\frac{1}{2}$	24
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	6	-5	3	16

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile2} \cdot \frac{6}{8}$$

$$z4s2 = 6 - 8 \cdot \frac{6}{8} = 0; z4s3 = -5 - (-2\frac{1}{2}) \cdot \frac{6}{8} = -3\frac{1}{8}; z4s4 = 3 - 3\frac{1}{2} \cdot \frac{6}{8} = \frac{3}{8}; z4s5 = 16 - 24 \cdot \frac{6}{8} = -2;$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	$-\frac{1}{4}$	$\frac{3}{4}$	4
0	8	$-2\frac{1}{2}$	$3\frac{1}{2}$	24
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	$-3\frac{1}{8}$	$\frac{3}{8}$	-2

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-\frac{1}{4}}{1\frac{13}{16}}$$

$$z1s3 = -\frac{1}{4} - 1\frac{13}{16} \cdot \frac{-\frac{1}{4}}{1\frac{13}{16}} = 0; z1s4 = \frac{3}{4} - (-1\frac{15}{16}) \cdot \frac{-\frac{1}{4}}{1\frac{13}{16}} = \frac{14}{29}; z1s5 = 4 - (-4) \cdot \frac{-\frac{1}{4}}{1\frac{13}{16}} = 3\frac{13}{29};$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	$\frac{14}{29}$	$3\frac{13}{29}$
0	8	$-2\frac{1}{2}$	$3\frac{1}{2}$	24
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	$-3\frac{1}{8}$	$\frac{3}{8}$	-2

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-2\frac{1}{2}}{1\frac{13}{16}}$$

$$z2s3 = -2\frac{1}{2} - 1\frac{13}{16} \cdot \frac{-2\frac{1}{2}}{1\frac{13}{16}} = 0; z2s4 = 3\frac{1}{2} - (-1\frac{15}{16}) \cdot \frac{-2\frac{1}{2}}{1\frac{13}{16}} = \frac{24}{29}; z2s5 = 24 - (-4) \cdot \frac{-2\frac{1}{2}}{1\frac{13}{16}} = 18\frac{14}{29};$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	$\frac{14}{29}$	$3\frac{13}{29}$
0	8	0	$\frac{24}{29}$	$18\frac{14}{29}$
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	$-3\frac{1}{8}$	$\frac{3}{8}$	-2

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	$\frac{14}{29}$	$3\frac{13}{29}$
0	8	0	$\frac{24}{29}$	$18\frac{14}{29}$
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	$-3\frac{1}{8}$	$\frac{3}{8}$	-2

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile3} \cdot \frac{-3\frac{1}{8}}{1\frac{13}{16}}$$

$$z4s3 = -3\frac{1}{8} - 1\frac{13}{16} \cdot \frac{-3\frac{1}{8}}{1\frac{13}{16}} = 0; z4s4 = \frac{3}{8} - (-1\frac{15}{16}) \cdot \frac{-3\frac{1}{8}}{1\frac{13}{16}} = -2\frac{28}{29}; z4s5 = -2 - (-4) \cdot \frac{-3\frac{1}{8}}{1\frac{13}{16}} = -8\frac{26}{29};$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	$\frac{14}{29}$	$3\frac{13}{29}$
0	8	0	$\frac{24}{29}$	$18\frac{14}{29}$
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	0	$-2\frac{28}{29}$	$-8\frac{26}{29}$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile4} \cdot \frac{\frac{14}{29}}{-2\frac{28}{29}}$$

$$z1s4 = \frac{14}{29} - (-2\frac{28}{29}) \cdot \frac{\frac{14}{29}}{-2\frac{28}{29}} = 0; z1s5 = 3\frac{13}{29} - (-8\frac{26}{29}) \cdot \frac{\frac{14}{29}}{-2\frac{28}{29}} = 2;$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	0	2
0	8	0	$\frac{24}{29}$	$18\frac{14}{29}$
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	0	$-2\frac{28}{29}$	$-8\frac{26}{29}$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile4} \cdot \frac{\frac{24}{29}}{-2\frac{28}{29}}$$

$$z2s4 = \frac{24}{29} - (-2\frac{28}{29}) \cdot \frac{\frac{24}{29}}{-2\frac{28}{29}} = 0; z2s5 = 18\frac{14}{29} - (-8\frac{26}{29}) \cdot \frac{\frac{24}{29}}{-2\frac{28}{29}} = 16;$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	0	2
0	8	0	0	16
0	0	$1\frac{13}{16}$	$-1\frac{15}{16}$	-4
0	0	0	$-2\frac{28}{29}$	$-8\frac{26}{29}$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile4} \cdot \frac{-1 \frac{15}{16}}{-2 \frac{28}{29}} = 0; z3s4 = -1 \frac{15}{16} - (-2 \frac{28}{29}) \cdot \frac{-1 \frac{15}{16}}{-2 \frac{28}{29}} = 0; z3s5 = -4 - (-8 \frac{26}{29}) \cdot \frac{-1 \frac{15}{16}}{-2 \frac{28}{29}} = 1 \frac{13}{16};$$

$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	0	2
0	8	0	0	16
0	0	$1 \frac{13}{16}$	0	$1 \frac{13}{16}$
0	0	0	$-2 \frac{28}{29}$	$-8 \frac{26}{29}$
$x_1$	$x_2$	$x_3$	$x_4$	
2	0	0	0	2
0	8	0	0	16
0	0	$1 \frac{13}{16}$	0	$1 \frac{13}{16}$
0	0	0	$-2 \frac{28}{29}$	$-8 \frac{26}{29}$

$$x_1 = \frac{2}{2} = 1$$

$$x_2 = \frac{16}{8} = 2$$

$$x_3 = \frac{1 \frac{13}{16}}{1 \frac{13}{16}} = 1$$

$$x_4 = \frac{-8 \frac{26}{29}}{-2 \frac{28}{29}} = 3$$

$$L = \{1/2/1/3\}$$

Aufgabe (12)

$$x_1 + x_2 - x_3 + 3x_4 - 3x_5 = 0$$

$$2x_1 + x_2 + x_3 + 4x_4 - x_5 = 0$$

$$2x_1 + x_2 + x_3 + 4x_4 - x_5 = 0$$

$$2x_1 + 3x_2 - 5x_3 + 8x_4 - 11x_5 = 0$$

$$-x_1 + x_2 - 5x_3 + x_4 - 7x_5 = 0$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	1	-1	3	-3	0
2	1	1	4	-1	0
2	1	1	4	-1	0
2	3	-5	8	-11	0
-1	1	-5	1	-7	0

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{1}$$

$$z2s1 = 2 - 1 \cdot \frac{2}{1} = 0; z2s2 = 1 - 1 \cdot \frac{2}{1} = -1; z2s3 = 1 - (-1) \cdot \frac{2}{1} = 3; z2s4 = 4 - 3 \cdot \frac{2}{1} = -2; z2s5 = -1 - (-3) \cdot \frac{2}{1} = 5; z2s6 = 0 - 0 \cdot \frac{2}{1} = 0;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	1	-1	3	-3	0
0	-1	3	-2	5	0
2	1	1	4	-1	0
2	3	-5	8	-11	0
-1	1	-5	1	-7	0

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{2}{1}$$

$$z3s1 = 2 - 1 \cdot \frac{2}{1} = 0; z3s2 = 1 - 1 \cdot \frac{2}{1} = -1; z3s3 = 1 - (-1) \cdot \frac{2}{1} = 3; z3s4 = 4 - 3 \cdot \frac{2}{1} = -2; z3s5 = -1 - (-3) \cdot \frac{2}{1} = 5; z3s6 = 0 - 0 \cdot \frac{2}{1} = 0;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	1	-1	3	-3	0
0	-1	3	-2	5	0
0	-1	3	-2	5	0
2	3	-5	8	-11	0
-1	1	-5	1	-7	0

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile1} \cdot \frac{2}{1}$$

$$z4s1 = 2 - 1 \cdot \frac{2}{1} = 0; z4s2 = 3 - 1 \cdot \frac{2}{1} = 1; z4s3 = -5 - (-1) \cdot \frac{2}{1} = -3; z4s4 = 8 - 3 \cdot \frac{2}{1} = 2; z4s5 = -11 - (-3) \cdot \frac{2}{1} = -5; z4s6 = 0 - 0 \cdot \frac{2}{1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 1 & -1 & 3 & -3 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 1 & -3 & 2 & -5 & 0 \\
 -1 & 1 & -5 & 1 & -7 & 0
 \end{array}$$

$$\text{Zeile5} = \text{Zeile5} - \text{Zeile1} \cdot \frac{-1}{1}$$

$$z_5s_1 = -1 - 1 \cdot \frac{-1}{1} = 0; z_5s_2 = 1 - 1 \cdot \frac{-1}{1} = 2; z_5s_3 = -5 - (-1) \cdot \frac{-1}{1} = -6; z_5s_4 = 1 - 3 \cdot \frac{-1}{1} = 4; z_5s_5 = -7 - (-3) \cdot \frac{-1}{1} = -10; z_5s_6 = 0 - 0 \cdot \frac{-1}{1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 1 & -1 & 3 & -3 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 1 & -3 & 2 & -5 & 0 \\
 0 & 2 & -6 & 4 & -10 & 0
 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-1}$$

$$z_1s_2 = 1 - (-1) \cdot \frac{1}{-1} = 0; z_1s_3 = -1 - 3 \cdot \frac{1}{-1} = 2; z_1s_4 = 3 - (-2) \cdot \frac{1}{-1} = 1; z_1s_5 = -3 - 5 \cdot \frac{1}{-1} = 2; z_1s_6 = 0 - 0 \cdot \frac{1}{-1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 1 & -3 & 2 & -5 & 0 \\
 0 & 2 & -6 & 4 & -10 & 0
 \end{array}$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 1 & -3 & 2 & -5 & 0 \\
 0 & 2 & -6 & 4 & -10 & 0
 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-1}{-1}$$

$$z_3s_2 = -1 - (-1) \cdot \frac{-1}{-1} = 0; z_3s_3 = 3 - 3 \cdot \frac{-1}{-1} = 0; z_3s_4 = -2 - (-2) \cdot \frac{-1}{-1} = 0; z_3s_5 = 5 - 5 \cdot \frac{-1}{-1} = 0; z_3s_6 = 0 - 0 \cdot \frac{-1}{-1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & -3 & 2 & -5 & 0 \\
 0 & 2 & -6 & 4 & -10 & 0
 \end{array}$$

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile2} \cdot \frac{1}{-1}$$

$$z_4s_2 = 1 - (-1) \cdot \frac{1}{-1} = 0; z_4s_3 = -3 - 3 \cdot \frac{1}{-1} = 0; z_4s_4 = 2 - (-2) \cdot \frac{1}{-1} = 0; z_4s_5 = -5 - 5 \cdot \frac{1}{-1} = 0; z_4s_6 = 0 - 0 \cdot \frac{1}{-1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 2 & -6 & 4 & -10 & 0
 \end{array}$$

$$\text{Zeile5} = \text{Zeile5} - \text{Zeile2} \cdot \frac{2}{-1}$$

$$z_5s_2 = 2 - (-1) \cdot \frac{2}{-1} = 0; z_5s_3 = -6 - 3 \cdot \frac{2}{-1} = 0; z_5s_4 = 4 - (-2) \cdot \frac{2}{-1} = 0; z_5s_5 = -10 - 5 \cdot \frac{2}{-1} = 0; z_5s_6 = 0 - 0 \cdot \frac{2}{-1} = 0;$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{array}$$

$$\begin{array}{ccccc|c}
 x_1 & x_2 & x_3 & x_4 & x_5 & \\
 \hline
 1 & 0 & 2 & 1 & 2 & 0 \\
 0 & -1 & 3 & -2 & 5 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{array}$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
1	0	2	1	2	0
0	-1	3	-2	5	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

$L = \text{unendlich}$

Aufgabe (13)

$$3x_1 + 2x_4 = 0$$

$$x_1 + x_2 + 2x_3 + x_4 = 6$$

$$3x_1 + 2x_4 = 0$$

$$x_1 - x_2 + x_4 = 6$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
1	1	2	1	6
3	0	0	2	0
1	-1	0	1	6

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{3}$$

$$z2s1 = 1 - 3 \cdot \frac{1}{3} = 0; z2s2 = 1 - 0 \cdot \frac{1}{3} = 1; z2s3 = 2 - 0 \cdot \frac{1}{3} = 2; z2s4 = 1 - 2 \cdot \frac{1}{3} = \frac{1}{3}; z2s5 = 6 - 0 \cdot \frac{1}{3} = 6;$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
3	0	0	2	0
1	-1	0	1	6

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{3}$$

$$z3s1 = 3 - 3 \cdot \frac{3}{3} = 0; z3s2 = 0 - 0 \cdot \frac{3}{3} = 0; z3s3 = 0 - 0 \cdot \frac{3}{3} = 0; z3s4 = 2 - 2 \cdot \frac{3}{3} = 0; z3s5 = 0 - 0 \cdot \frac{3}{3} = 0;$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
1	-1	0	1	6

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile1} \cdot \frac{1}{3}$$

$$z4s1 = 1 - 3 \cdot \frac{1}{3} = 0; z4s2 = -1 - 0 \cdot \frac{1}{3} = -1; z4s3 = 0 - 0 \cdot \frac{1}{3} = 0; z4s4 = 1 - 2 \cdot \frac{1}{3} = \frac{1}{3}; z4s5 = 6 - 0 \cdot \frac{1}{3} = 6;$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
0	-1	0	$\frac{1}{3}$	6

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{0}{1}$$

$$z1s2 = 0 - 1 \cdot \frac{0}{1} = 0; z1s3 = 0 - 2 \cdot \frac{0}{1} = 0; z1s4 = 2 - \frac{1}{3} \cdot \frac{0}{1} = 2; z1s5 = 0 - 6 \cdot \frac{0}{1} = 0;$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
0	-1	0	$\frac{1}{3}$	6

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
0	-1	0	$\frac{1}{3}$	6

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{0}{1}$$

$$z3s2 = 0 - 1 \cdot \frac{0}{1} = 0; z3s3 = 0 - 2 \cdot \frac{0}{1} = 0; z3s4 = 0 - \frac{1}{3} \cdot \frac{0}{1} = 0; z3s5 = 0 - 6 \cdot \frac{0}{1} = 0;$$

$x_1$	$x_2$	$x_3$	$x_4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
0	-1	0	$\frac{1}{3}$	6

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile2} \cdot \frac{-1}{1}$$

$$z4s2 = -1 - 1 \cdot \frac{-1}{1} = 0; z4s3 = 0 - 2 \cdot \frac{-1}{1} = 2; z4s4 = \frac{1}{3} - \frac{1}{3} \cdot \frac{-1}{1} = \frac{2}{3}; z4s5 = 6 - 6 \cdot \frac{-1}{1} = 12;$$

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	0	0	0
0	0	2	$\frac{2}{3}$	12

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	2	$\frac{2}{3}$	12
0	0	0	0	0

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{0}{2}$$

$$z1s3 = 0 - 2 \cdot \frac{0}{2} = 0; z1s4 = 2 - \frac{2}{3} \cdot \frac{0}{2} = 2; z1s5 = 0 - 12 \cdot \frac{0}{2} = 0;$$

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	2	$\frac{1}{3}$	6
0	0	2	$\frac{2}{3}$	12
0	0	0	0	0

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{2}{2}$$

$$z2s3 = 2 - 2 \cdot \frac{2}{2} = 0; z2s4 = \frac{1}{3} - \frac{2}{3} \cdot \frac{2}{2} = -\frac{1}{3}; z2s5 = 6 - 12 \cdot \frac{2}{2} = -6;$$

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	0	$-\frac{1}{3}$	-6
0	0	2	$\frac{2}{3}$	12
0	0	0	0	0

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	0	$-\frac{1}{3}$	-6
0	0	2	$\frac{2}{3}$	12
0	0	0	0	0

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile3} \cdot \frac{0}{2}$$

$$z4s3 = 0 - 2 \cdot \frac{0}{2} = 0; z4s4 = 0 - \frac{2}{3} \cdot \frac{0}{2} = 0; z4s5 = 0 - 12 \cdot \frac{0}{2} = 0;$$

$x1$	$x2$	$x3$	$x4$	
3	0	0	2	0
0	1	0	$-\frac{1}{3}$	-6
0	0	2	$\frac{2}{3}$	12
0	0	0	0	0

$L = \text{unendlich}$

Aufgabe (14)

$$2x_1 + x_2 + x_3 + x_4 + x_5 = 8$$

$$3x_1 - 2x_2 - 4x_3 + x_4 + 3x_5 = 22$$

$$4x_1 + 3x_2 - 5x_3 + 2x_4 - x_5 = 13$$

$$5x_1 - 4x_2 + 2x_3 - 3x_4 + 2x_5 = 19$$

$$6x_1 + 8x_2 - 7x_3 + 9x_4 + 4x_5 = 41$$

$x1$	$x2$	$x3$	$x4$	$x5$	
2	1	1	1	1	8
3	-2	-4	1	3	22
4	3	-5	2	-1	13
5	-4	2	-3	2	19
6	8	-7	9	4	41

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2}$$

$$z2s1 = 3 - 2 \cdot \frac{3}{2} = 0; z2s2 = -2 - 1 \cdot \frac{3}{2} = -\frac{7}{2}; z2s3 = -4 - 1 \cdot \frac{3}{2} = -\frac{11}{2}; z2s4 = 1 - 1 \cdot \frac{3}{2} = -\frac{1}{2}; z2s5 = 3 - 1 \cdot \frac{3}{2} = \frac{3}{2}; z2s6 = 22 - 8 \cdot \frac{3}{2} = 10;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	1	1	1	1	8
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
4	3	-5	2	-1	13
5	-4	2	-3	2	19
6	8	-7	9	4	41

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{4}{2}$$

$$z_{3s1} = 4 - 2 \cdot \frac{4}{2} = 0; z_{3s2} = 3 - 1 \cdot \frac{4}{2} = 1; z_{3s3} = -5 - 1 \cdot \frac{4}{2} = -7; z_{3s4} = 2 - 1 \cdot \frac{4}{2} = 0; z_{3s5} = -1 - 1 \cdot \frac{4}{2} = -3; z_{3s6} = 13 - 8 \cdot \frac{4}{2} = -3;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	1	1	1	1	8
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	1	-7	0	-3	-3
5	-4	2	-3	2	19
6	8	-7	9	4	41

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile1} \cdot \frac{5}{2}$$

$$z_{4s1} = 5 - 2 \cdot \frac{5}{2} = 0; z_{4s2} = -4 - 1 \cdot \frac{5}{2} = -6\frac{1}{2}; z_{4s3} = 2 - 1 \cdot \frac{5}{2} = -\frac{1}{2}; z_{4s4} = -3 - 1 \cdot \frac{5}{2} = -5\frac{1}{2}; z_{4s5} = 2 - 1 \cdot \frac{5}{2} = -\frac{1}{2}; z_{4s6} = 19 - 8 \cdot \frac{5}{2} = -1;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	1	1	1	1	8
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	1	-7	0	-3	-3
0	$-6\frac{1}{2}$	$-\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	-1
6	8	-7	9	4	41

$$\text{Zeile5} = \text{Zeile5} - \text{Zeile1} \cdot \frac{6}{2}$$

$$z_{5s1} = 6 - 2 \cdot \frac{6}{2} = 0; z_{5s2} = 8 - 1 \cdot \frac{6}{2} = 5; z_{5s3} = -7 - 1 \cdot \frac{6}{2} = -10; z_{5s4} = 9 - 1 \cdot \frac{6}{2} = 6; z_{5s5} = 4 - 1 \cdot \frac{6}{2} = 1; z_{5s6} = 41 - 8 \cdot \frac{6}{2} = 17;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	1	1	1	1	8
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	1	-7	0	-3	-3
0	$-6\frac{1}{2}$	$-\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	-1
0	5	-10	6	1	17

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-3\frac{1}{2}}$$

$$z_{1s2} = 1 - (-3\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = 0; z_{1s3} = 1 - (-5\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = -\frac{4}{7}; z_{1s4} = 1 - (-\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = \frac{6}{7}; z_{1s5} = 1 - 1\frac{1}{2} \cdot \frac{1}{-3\frac{1}{2}} = 1\frac{3}{7}; z_{1s6} = 8 - 10 \cdot \frac{1}{-3\frac{1}{2}} = 10\frac{6}{7};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	$-\frac{4}{7}$	$\frac{6}{7}$	$1\frac{3}{7}$	$10\frac{6}{7}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	1	-7	0	-3	-3
0	$-6\frac{1}{2}$	$-\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	-1
0	5	-10	6	1	17

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	$-\frac{4}{7}$	$\frac{6}{7}$	$1\frac{3}{7}$	$10\frac{6}{7}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	1	-7	0	-3	-3
0	$-6\frac{1}{2}$	$-\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	-1
0	5	-10	6	1	17

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{-3\frac{1}{2}}$$

$$z_{3s2} = 1 - (-3\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = 0; z_{3s3} = -7 - (-5\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = -8\frac{4}{7}; z_{3s4} = 0 - (-\frac{1}{2}) \cdot \frac{1}{-3\frac{1}{2}} = -\frac{1}{7}; z_{3s5} = -3 - 1\frac{1}{2} \cdot \frac{1}{-3\frac{1}{2}} = -2\frac{4}{7}; z_{3s6} = -3 - 10 \cdot \frac{1}{-3\frac{1}{2}} = -\frac{1}{7};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	$-\frac{4}{7}$	$\frac{6}{7}$	$1\frac{3}{7}$	$10\frac{6}{7}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	$-6\frac{1}{2}$	$-\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	-1
0	5	-10	6	1	17

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile2} \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}}$$

$$z4s2 = -6\frac{1}{2} - (-3\frac{1}{2}) \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}} = 0; z4s3 = -\frac{1}{2} - (-5\frac{1}{2}) \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}} = 9\frac{5}{7}; z4s4 = -5\frac{1}{2} - (-\frac{1}{2}) \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}} = -4\frac{4}{7}; z4s5 = -\frac{1}{2} - 1\frac{1}{2} \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}} = -3\frac{2}{7}; z4s6 = -1 - 10 \cdot \frac{-6\frac{1}{2}}{-3\frac{1}{2}} = -19\frac{4}{7};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	$-\frac{4}{7}$	$\frac{6}{7}$	$1\frac{3}{7}$	$10\frac{6}{7}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	$9\frac{5}{7}$	$-4\frac{4}{7}$	$-3\frac{2}{7}$	$-19\frac{4}{7}$
0	5	-10	6	1	17

$$\text{Zeile5} = \text{Zeile5} - \text{Zeile2} \cdot \frac{5}{-3\frac{1}{2}}$$

$$z5s2 = 5 - (-3\frac{1}{2}) \cdot \frac{5}{-3\frac{1}{2}} = 0; z5s3 = -10 - (-5\frac{1}{2}) \cdot \frac{5}{-3\frac{1}{2}} = -17\frac{6}{7}; z5s4 = 6 - (-\frac{1}{2}) \cdot \frac{5}{-3\frac{1}{2}} = 5\frac{2}{7}; z5s5 = 1 - 1\frac{1}{2} \cdot \frac{5}{-3\frac{1}{2}} = 3\frac{1}{7}; z5s6 = 17 - 10 \cdot \frac{5}{-3\frac{1}{2}} = 31\frac{2}{7};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	$-\frac{4}{7}$	$\frac{6}{7}$	$1\frac{3}{7}$	$10\frac{6}{7}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	$9\frac{5}{7}$	$-4\frac{4}{7}$	$-3\frac{2}{7}$	$-19\frac{4}{7}$
0	0	$-17\frac{6}{7}$	$5\frac{2}{7}$	$3\frac{1}{7}$	$31\frac{2}{7}$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-\frac{4}{7}}{-8\frac{4}{7}}$$

$$z1s3 = -\frac{4}{7} - (-8\frac{4}{7}) \cdot \frac{-\frac{4}{7}}{-8\frac{4}{7}} = 0; z1s4 = \frac{6}{7} - (-\frac{1}{7}) \cdot \frac{-\frac{4}{7}}{-8\frac{4}{7}} = \frac{13}{15}; z1s5 = 1\frac{3}{7} - (-2\frac{4}{7}) \cdot \frac{-\frac{4}{7}}{-8\frac{4}{7}} = 1\frac{3}{5}; z1s6 = 10\frac{6}{7} - (-\frac{1}{7}) \cdot \frac{-\frac{4}{7}}{-8\frac{4}{7}} = 10\frac{13}{15};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	$\frac{13}{15}$	$1\frac{3}{5}$	$10\frac{13}{15}$
0	$-3\frac{1}{2}$	$-5\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	10
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	$9\frac{5}{7}$	$-4\frac{4}{7}$	$-3\frac{2}{7}$	$-19\frac{4}{7}$
0	0	$-17\frac{6}{7}$	$5\frac{2}{7}$	$3\frac{1}{7}$	$31\frac{2}{7}$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-5\frac{1}{2}}{-8\frac{4}{7}}$$

$$z2s3 = -5\frac{1}{2} - (-8\frac{4}{7}) \cdot \frac{-5\frac{1}{2}}{-8\frac{4}{7}} = 0; z2s4 = -\frac{1}{2} - (-\frac{1}{7}) \cdot \frac{-5\frac{1}{2}}{-8\frac{4}{7}} = -0,408; z2s5 = 1\frac{1}{2} - (-2\frac{4}{7}) \cdot \frac{-5\frac{1}{2}}{-8\frac{4}{7}} = 3\frac{3}{20}; z2s6 = 10 - (-\frac{1}{7}) \cdot \frac{-5\frac{1}{2}}{-8\frac{4}{7}} = 10,092;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	$\frac{13}{15}$	$1\frac{3}{5}$	$10\frac{13}{15}$
0	$-3\frac{1}{2}$	0	-0,408	$3\frac{3}{20}$	10,092
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	$9\frac{5}{7}$	$-4\frac{4}{7}$	$-3\frac{2}{7}$	$-19\frac{4}{7}$
0	0	$-17\frac{6}{7}$	$5\frac{2}{7}$	$3\frac{1}{7}$	$31\frac{2}{7}$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	$\frac{13}{15}$	$1\frac{3}{5}$	$10\frac{13}{15}$
0	$-3\frac{1}{2}$	0	-0,408	$3\frac{3}{20}$	10,092
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	$9\frac{5}{7}$	$-4\frac{4}{7}$	$-3\frac{2}{7}$	$-19\frac{4}{7}$
0	0	$-17\frac{6}{7}$	$5\frac{2}{7}$	$3\frac{1}{7}$	$31\frac{2}{7}$

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile3} \cdot \frac{9\frac{5}{7}}{-8\frac{4}{7}}$$

$$z4s3 = 9\frac{5}{7} - (-8\frac{4}{7}) \cdot \frac{9\frac{5}{7}}{-8\frac{4}{7}} = 0; z4s4 = -4\frac{4}{7} - (-\frac{1}{7}) \cdot \frac{9\frac{5}{7}}{-8\frac{4}{7}} = -4\frac{11}{15}; z4s5 = -3\frac{2}{7} - (-2\frac{4}{7}) \cdot \frac{9\frac{5}{7}}{-8\frac{4}{7}} = -6\frac{1}{5}; z4s6 = -19\frac{4}{7} - (-\frac{1}{7}) \cdot \frac{9\frac{5}{7}}{-8\frac{4}{7}} = -19\frac{11}{15};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	$\frac{13}{15}$	$1\frac{3}{5}$	$10\frac{13}{15}$
0	$-3\frac{1}{2}$	0	-0,408	$3\frac{3}{20}$	10,092
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	$-17\frac{6}{7}$	$5\frac{2}{7}$	$3\frac{1}{7}$	$31\frac{2}{7}$

$$\text{Zeile5} = \text{Zeile5} - \text{Zeile3} \cdot \frac{-17\frac{6}{7}}{-8\frac{4}{7}}$$

$$z5s3 = -17\frac{6}{7} - (-8\frac{4}{7}) \cdot \frac{-17\frac{6}{7}}{-8\frac{4}{7}} = 0; z5s4 = 5\frac{2}{7} - (-\frac{1}{7}) \cdot \frac{-17\frac{6}{7}}{-8\frac{4}{7}} = 5\frac{7}{12}; z5s5 = 3\frac{1}{7} - (-2\frac{4}{7}) \cdot \frac{-17\frac{6}{7}}{-8\frac{4}{7}} = 8\frac{1}{2}; z5s6 = 31\frac{2}{7} - (-\frac{1}{7}) \cdot \frac{-17\frac{6}{7}}{-8\frac{4}{7}} = 31\frac{7}{12};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	$\frac{13}{15}$	$1\frac{3}{5}$	$10\frac{13}{15}$
0	$-3\frac{1}{2}$	0	-0,408	$3\frac{3}{20}$	10,092
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	$5\frac{7}{12}$	$8\frac{1}{2}$	$31\frac{7}{12}$

$$\text{Zeile1} = \text{Zeile1-Zeile4} \cdot \frac{\frac{13}{15}}{-4\frac{11}{15}}$$

$$z1s4 = \frac{13}{15} - (-4\frac{11}{15}) \cdot \frac{\frac{13}{15}}{-4\frac{11}{15}} = 0; z1s5 = 1\frac{3}{5} - (-6\frac{1}{5}) \cdot \frac{\frac{13}{15}}{-4\frac{11}{15}} = \frac{33}{71}; z1s6 = 10\frac{13}{15} - (-19\frac{11}{15}) \cdot \frac{\frac{13}{15}}{-4\frac{11}{15}} = 7\frac{18}{71};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	$\frac{33}{71}$	$7\frac{18}{71}$
0	$-3\frac{1}{2}$	0	-0,408	$3\frac{3}{20}$	10,092
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	$5\frac{7}{12}$	$8\frac{1}{2}$	$31\frac{7}{12}$

$$\text{Zeile2} = \text{Zeile2-Zeile4} \cdot \frac{-0,408}{-4\frac{11}{15}}$$

$$z2s4 = -0,408 - (-4\frac{11}{15}) \cdot \frac{-0,408}{-4\frac{11}{15}} = 0; z2s5 = 3\frac{3}{20} - (-6\frac{1}{5}) \cdot \frac{-0,408}{-4\frac{11}{15}} = 3,685; z2s6 = 10,092 - (-19\frac{11}{15}) \cdot \frac{-0,408}{-4\frac{11}{15}} = 11,794;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	$\frac{33}{71}$	$7\frac{18}{71}$
0	$-3\frac{1}{2}$	0	0	3,685	11,794
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	$-\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	$5\frac{7}{12}$	$8\frac{1}{2}$	$31\frac{7}{12}$

$$\text{Zeile3} = \text{Zeile3-Zeile4} \cdot \frac{-\frac{1}{7}}{-4\frac{11}{15}}$$

$$z3s4 = -\frac{1}{7} - (-4\frac{11}{15}) \cdot \frac{-\frac{1}{7}}{-4\frac{11}{15}} = 0; z3s5 = -2\frac{4}{7} - (-6\frac{1}{5}) \cdot \frac{-\frac{1}{7}}{-4\frac{11}{15}} = -2,384; z3s6 = -\frac{1}{7} - (-19\frac{11}{15}) \cdot \frac{-\frac{1}{7}}{-4\frac{11}{15}} = 0,453;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	$\frac{33}{71}$	$7\frac{18}{71}$
0	$-3\frac{1}{2}$	0	0	3,685	11,794
0	0	$-8\frac{4}{7}$	$-\frac{1}{7}$	$-2\frac{4}{7}$	0,453
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	$5\frac{7}{12}$	$8\frac{1}{2}$	$31\frac{7}{12}$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	$\frac{33}{71}$	$7\frac{18}{71}$
0	$-3\frac{1}{2}$	0	0	3,685	11,794
0	0	$-8\frac{4}{7}$	0	-2,384	0,453
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	$5\frac{7}{12}$	$8\frac{1}{2}$	$31\frac{7}{12}$

$$\text{Zeile5} = \text{Zeile5-Zeile4} \cdot \frac{5\frac{7}{12}}{-4\frac{11}{15}}$$

$$z5s4 = 5\frac{7}{12} - (-4\frac{11}{15}) \cdot \frac{5\frac{7}{12}}{-4\frac{11}{15}} = 0; z5s5 = 8\frac{1}{2} - (-6\frac{1}{5}) \cdot \frac{5\frac{7}{12}}{-4\frac{11}{15}} = 1,187; z5s6 = 31\frac{7}{12} - (-19\frac{11}{15}) \cdot \frac{5\frac{7}{12}}{-4\frac{11}{15}} = 8,306;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	$\frac{33}{71}$	$7\frac{18}{71}$
0	$-3\frac{1}{2}$	0	0	3,685	11,794
0	0	$-8\frac{4}{7}$	0	-2,384	0,453
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	0	1,187	8,306

$$\text{Zeile1} = \text{Zeile1-Zeile5} \cdot \frac{\frac{33}{71}}{1,187}$$

$$z1s5 = \frac{33}{71} - 1,187 \cdot \frac{\frac{33}{71}}{1,187} = 0; z1s6 = 7\frac{18}{71} - 8,306 \cdot \frac{\frac{33}{71}}{1,187} = 4;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	0	4
0	$-3\frac{1}{2}$	0	0	3,685	11,794
0	0	$-8\frac{4}{7}$	0	-2,384	0,453
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	0	1,187	8,306

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile5} \cdot \frac{3,685}{1,187}$$

$$z2s5 = 3,685 - 1,187 \cdot \frac{3,685}{1,187} = 0; z2s6 = 11,794 - 8,306 \cdot \frac{3,685}{1,187} = -14;$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	0	4
0	$-3\frac{1}{2}$	0	0	0	-14
0	0	$-8\frac{4}{7}$	0	-2,384	0,453
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	0	1,187	8,306

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile5} \cdot \frac{-2,384}{1,187}$$

$$z3s5 = -2,384 - 1,187 \cdot \frac{-2,384}{1,187} = 0; z3s6 = 0,453 - 8,306 \cdot \frac{-2,384}{1,187} = 17\frac{1}{7};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	0	4
0	$-3\frac{1}{2}$	0	0	0	-14
0	0	$-8\frac{4}{7}$	0	0	$17\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	$-6\frac{1}{5}$	$-19\frac{11}{15}$
0	0	0	0	1,187	8,306

$$\text{Zeile4} = \text{Zeile4} - \text{Zeile5} \cdot \frac{-6\frac{1}{5}}{1,187}$$

$$z4s5 = -6\frac{1}{5} - 1,187 \cdot \frac{-6\frac{1}{5}}{1,187} = 0; z4s6 = -19\frac{11}{15} - 8,306 \cdot \frac{-6\frac{1}{5}}{1,187} = 23\frac{2}{3};$$

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	0	4
0	$-3\frac{1}{2}$	0	0	0	-14
0	0	$-8\frac{4}{7}$	0	0	$17\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	0	$23\frac{2}{3}$
0	0	0	0	1,187	8,306

$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	
2	0	0	0	0	4
0	$-3\frac{1}{2}$	0	0	0	-14
0	0	$-8\frac{4}{7}$	0	0	$17\frac{1}{7}$
0	0	0	$-4\frac{11}{15}$	0	$23\frac{2}{3}$
0	0	0	0	1,187	8,306

$$x_1 = \frac{4}{2} = 2$$

$$x_2 = \frac{-14}{-3\frac{1}{2}} = 4$$

$$x_3 = \frac{17\frac{1}{7}}{-8\frac{4}{7}} = -2$$

$$x_4 = \frac{23\frac{2}{3}}{-4\frac{11}{15}} = -5$$

$$x_5 = \frac{8,306}{1,187} = 7$$

$$L = \{2/4/ -2/ -5/7\}$$