

Gleichungen Aufgaben und Lösungen

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1 Lineare Gleichung

- Klammern auflösen
- Terme zusammenfassen
- Äquivalenzumformung: Alle Terme mit der Variablen auf die eine Seite und alle Terme ohne Variable auf die andere Seite.
- durch die Zahl vor der Variablen dividieren

$$2\frac{1}{2}x + 5 = 4(x - 2) - 2x + 12$$

Klammern auflösen

$$2\frac{1}{2}x + 5 = 4x - 8 - 2x + 12$$

Terme zusammenfassen

$$2\frac{1}{2}x + 5 = 2x + 4$$

Äquivalenzumformung:

$$2\frac{1}{2}x + 5 = 2x + 4 \quad / - 5 \quad / - 2x$$

$$2\frac{1}{2}x - 2x = 4 - 5$$

durch die Zahl vor der Variablen dividieren

$$\frac{1}{2}x = -1 \quad / : \frac{1}{2}$$

$$x = \frac{-1}{\frac{1}{2}}$$

$$x = -2$$

$$a \cdot x = b$$

$$a \cdot x = b \quad / : a$$

$$x = \frac{b}{a}$$

$$5 \cdot x = 45 \quad / : 5 \quad -2 \cdot x = -6 \quad / : (-2)$$

$$x = \frac{45}{5} \quad x = \frac{-6}{-2}$$

$$x = 9 \quad x = 3$$

$$x + a = b$$

$$x + a = b \quad / - a$$

$$x = b - a$$

$$x + 2 = 5 \quad / - 2 \quad x + 5 = -7 \quad / - 5$$

$$x = 5 - 2 \quad x = -7 - 5$$

$$x = 3 \quad x = -12$$

$$a \cdot x + b = c$$

$$a \cdot x + b = c \quad / - b$$

$$a \cdot x = c - b \quad / : a$$

$$x = \frac{c - b}{a}$$

$$5 \cdot x - 4 = 6 \quad / + 4 \quad -2 \cdot x + 4 = -6 \quad / - 4$$

$$5 \cdot x = 10 \quad / : 5 \quad -2 \cdot x = -10 \quad / : (-2)$$

$$x = \frac{10}{5} \quad x = \frac{-10}{-2}$$

$$x = 2 \quad x = 5$$

$$\frac{x}{a} = b$$

$$\frac{x}{a} = b \quad / \cdot a$$

$$x = b \cdot a$$

$$\frac{x}{2} = 5 \quad / \cdot 2$$

$$x = 5 \cdot 2$$

$$x = 10$$

$$\frac{x}{5} = -7 \quad / \cdot 5$$

$$x = -7 \cdot 5$$

$$x = -35$$

$$a - x = b$$

$$a - x = b \quad / - a$$

$$-x = b - a \quad / : (-1)$$

$$x = a - b$$

$$2 - x = 5 \quad / - 2 \quad x - 5 = -7 \quad / + 5$$

$$-x = 5 - 2 \quad x = -7 + 5$$

$$-x = 3 / : (-1) \quad x = -2$$

$$x = -3$$

$$x - a = b$$

$$\begin{aligned} x - a &= b & / + a \\ x &= b + a \end{aligned}$$

$$\begin{aligned} x - 2 &= 5 & / + 2 & & x - 5 &= -7 & / + 5 \\ x &= 5 + 2 & & & x &= -7 + 5 \\ x &= 7 & & & x &= -2 \end{aligned}$$

$$\begin{aligned} ax + b &= cx + d & / - cx \\ ax - cx + b &= d & / - b \\ (a - c)x &= d - b & / : (a - c) \\ a - c &\neq 0 \\ x &= \frac{d-b}{a-c} \end{aligned}$$

$$\begin{aligned} 2x + 4 &= 6x + 7 & / - 6x \\ -4x + 4 &= 7 & / - 4 \\ -4x &= 3 & / : (-4) \\ x &= -\frac{3}{4} \end{aligned}$$

1.1 $a \cdot x + b = c$

1.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = c$

Koeffizienten: a, b, c

Gesucht: x

(1) $a = 9$ $b = 7$ $c = 2$

(2) $a = 5$ $b = 6$ $c = 8$

(3) $a = 7$ $b = 7$ $c = 5$

(4) $a = 1\frac{7}{12}$ $b = \frac{12}{19}$ $c = 6$

(5) $a = \frac{2}{3}$ $b = \frac{5}{7}$ $c = \frac{13}{16}$

(6) $a = \frac{16}{19}$ $b = 1\frac{6}{7}$ $c = 1\frac{1}{6}$

(7) $a = -2$ $b = 3$ $c = 4$

(8) $a = 4$ $b = 5$ $c = 6$

(9) $a = 4$ $b = \frac{1}{6}$ $c = -3$

(10) $a = \frac{1}{4}$ $b = 6$ $c = 7$

(11) $a = -\frac{1}{3}$ $b = 4$ $c = -\frac{1}{5}$

(12) $a = 1\frac{2}{3}$ $b = -\frac{1}{4}$ $c = 5$

(13) $a = -\frac{2}{5}$ $b = 3$ $c = \frac{3}{4}$

(14) $a = \frac{1}{3}$ $b = \frac{1}{3}$ $c = -\frac{4}{7}$

(15) $a = 5$ $b = 6$ $c = 7$

(16) $a = -5$ $b = 6$ $c = 7$

1.1.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 9x + 7 &= 2 && / - 7 \\ 9x &= -5 && / : 9 \\ x &= -\frac{5}{9} \end{aligned}$$

$$\begin{aligned} -2x + 3 &= 4 && / - 3 \\ -2x &= 1 && / : (-2) \\ x &= -\frac{1}{2} \end{aligned}$$

Aufgabe (8)

Aufgabe (2)

$$\begin{aligned} 5x + 6 &= 8 && / - 6 \\ 5x &= 2 && / : 5 \\ x &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} 4x + 5 &= 6 && / - 5 \\ 4x &= 1 && / : 4 \\ x &= \frac{1}{4} \end{aligned}$$

Aufgabe (9)

Aufgabe (3)

$$\begin{aligned} 7x + 7 &= 5 && / - 7 \\ 7x &= -2 && / : 7 \\ x &= -\frac{2}{7} \end{aligned}$$

$$\begin{aligned} 4x + \frac{1}{6} &= -3 && / - \frac{1}{6} \\ 4x &= -3\frac{1}{6} && / : 4 \\ x &= -\frac{19}{24} \end{aligned}$$

Aufgabe (10)

Aufgabe (4)

$$\begin{aligned} 1\frac{7}{12}x + \frac{12}{19} &= 6 && / - \frac{12}{19} \\ 1\frac{7}{12}x &= 5\frac{7}{19} && / : 1\frac{7}{12} \\ x &= 3,39 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}x + 6 &= 7 && / - 6 \\ \frac{1}{4}x &= 1 && / : \frac{1}{4} \\ x &= 4 \end{aligned}$$

Aufgabe (11)

Aufgabe (5)

$$\begin{aligned} \frac{2}{3}x + \frac{5}{7} &= \frac{13}{16} && / - \frac{5}{7} \\ \frac{2}{3}x &= 0,0982 && / : \frac{2}{3} \\ x &= 0,147 \end{aligned}$$

$$\begin{aligned} -\frac{1}{3}x + 4 &= -\frac{1}{5} && / - 4 \\ -\frac{1}{3}x &= -4\frac{1}{5} && / : (-\frac{1}{3}) \\ x &= 12\frac{3}{5} \end{aligned}$$

Aufgabe (12)

Aufgabe (6)

$$\begin{aligned} \frac{16}{19}x + 1\frac{6}{7} &= 1\frac{1}{6} && / - 1\frac{6}{7} \\ \frac{16}{19}x &= -\frac{29}{42} && / : \frac{16}{19} \\ x &= -0,82 \end{aligned}$$

$$\begin{aligned} 1\frac{2}{3}x - \frac{1}{4} &= 5 && / + \frac{1}{4} \\ 1\frac{2}{3}x &= 5\frac{1}{4} && / : 1\frac{2}{3} \\ x &= 3\frac{3}{20} \end{aligned}$$

Aufgabe (13)

Aufgabe (7)

$$\begin{aligned} -\frac{2}{5}x + 3 &= \frac{3}{4} && / - 3 \\ -\frac{2}{5}x &= -2\frac{1}{4} && / : (-\frac{2}{5}) \\ x &= 5\frac{5}{8} \end{aligned}$$

Aufgabe (14)

$$x = \frac{1}{5}$$

$$\begin{aligned}\frac{1}{3}x + \frac{1}{3} &= -\frac{4}{7} && / -\frac{1}{3} \\ \frac{1}{3}x &= -\frac{19}{21} && / : \frac{1}{3} \\ x &= -2\frac{2}{7}\end{aligned}$$

Aufgabe (16)

Aufgabe (15)

$$\begin{aligned}-5x + 6 &= 7 && / -6 \\ -5x &= 1 && / : (-5) \\ x &= -\frac{1}{5}\end{aligned}$$

$$\begin{aligned}5x + 6 &= 7 && / -6 \\ 5x &= 1 && / : 5\end{aligned}$$

1.2 $a \cdot x + b = c \cdot x + d$

1.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = c \cdot x + d$

Koeffizienten: a, b, c, d

Gesucht: x

- (1) $a = 9$ $b = 7$ $c = 2$ $d = 4$
- (2) $a = 5$ $b = 6$ $c = 8$ $d = 1$
- (3) $a = 7$ $b = 7$ $c = 5$ $d = 2$
- (4) $a = 1\frac{7}{12}$ $b = \frac{12}{19}$ $c = 6$ $d = -3$
- (5) $a = \frac{2}{3}$ $b = \frac{5}{7}$ $c = \frac{13}{16}$ $d = 1$
- (6) $a = 4$ $b = 5$ $c = 6$ $d = -2$
- (7) $a = 1$ $b = 3$ $c = 2$ $d = 5$

- (8) $a = 1$ $b = 3$ $c = 2$ $d = 3$
- (9) $a = 4$ $b = 5$ $c = 0$ $d = 7$
- (10) $a = \frac{4}{5}$ $b = 5$ $c = \frac{3}{5}$ $d = 7$
- (11) $a = -\frac{4}{9}$ $b = -\frac{5}{6}$ $c = 3$ $d = 7$
- (12) $a = -\frac{4}{5}$ $b = -1\frac{1}{2}$ $c = -3$ $d = 2$
- (13) $a = -\frac{3}{8}$ $b = 1\frac{1}{3}$ $c = 5$ $d = \frac{2}{3}$

1.2.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 9x + 7 &= 2x + 4 & / - 2x \\ 7x + 7 &= 4 & / - 7 \\ 7x &= -3 & / : 7 \\ x &= -\frac{3}{7} \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} x + 3 &= 2x + 5 & / - 2x \\ -1x + 3 &= 5 & / - 3 \\ -1x &= 2 & / : (-1) \\ x &= -2 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} 5x + 6 &= 8x + 1 & / - 8x \\ -3x + 6 &= 1 & / - 6 \\ -3x &= -5 & / : (-3) \\ x &= 1\frac{2}{3} \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} x + 3 &= 2x + 3 & / - 2x \\ -1x + 3 &= 3 & / - 3 \\ -1x &= 0 & / : (-1) \\ x &= 0 \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} 7x + 7 &= 5x + 2 & / - 5x \\ 2x + 7 &= 2 & / - 7 \\ 2x &= -5 & / : 2 \\ x &= -2\frac{1}{2} \end{aligned}$$

Aufgabe (9)

$$\begin{aligned} 4x + 5 &= 7 & / - 5 \\ 4x &= 2 & / : 4 \\ x &= \frac{1}{2} \end{aligned}$$

Aufgabe (4)

$$\begin{aligned} 1\frac{7}{12}x + \frac{12}{19} &= 6x - 3 & / - 6x \\ -4\frac{5}{12}x + \frac{12}{19} &= -3 & / - \frac{12}{19} \\ -4\frac{5}{12}x &= -3\frac{12}{19} & / : (-4\frac{5}{12}) \\ x &= 0,822 \end{aligned}$$

Aufgabe (10)

$$\begin{aligned} \frac{4}{5}x + 5 &= \frac{3}{5}x + 7 & / - \frac{3}{5}x \\ \frac{1}{5}x + 5 &= 7 & / - 5 \\ \frac{1}{5}x &= 2 & / : \frac{1}{5} \\ x &= 10 \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} \frac{2}{3}x + \frac{5}{7} &= \frac{13}{16}x + 1 & / - \frac{13}{16}x \\ -\frac{7}{48}x + \frac{5}{7} &= 1 & / - \frac{5}{7} \\ -\frac{7}{48}x &= \frac{2}{7} & / : (-\frac{7}{48}) \\ x &= -1\frac{47}{49} \end{aligned}$$

Aufgabe (11)

$$\begin{aligned} -\frac{4}{9}x - \frac{5}{6} &= 3x + 7 & / - 3x \\ -3\frac{4}{9}x - \frac{5}{6} &= 7 & / + \frac{5}{6} \\ -3\frac{4}{9}x &= 7\frac{5}{6} & / : (-3\frac{4}{9}) \\ x &= -2\frac{17}{62} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} 4x + 5 &= 6x - 2 & / - 6x \\ -2x + 5 &= -2 & / - 5 \\ -2x &= -7 & / : (-2) \\ x &= 3\frac{1}{2} \end{aligned}$$

Aufgabe (12)

$$\begin{aligned} -\frac{4}{5}x - 1\frac{1}{2} &= -3x + 2 & / + 3x \\ 2\frac{1}{5}x - 1\frac{1}{2} &= 2 & / + 1\frac{1}{2} \\ 2\frac{1}{5}x &= 3\frac{1}{2} & / : 2\frac{1}{5} \\ x &= 1\frac{13}{22} \end{aligned}$$

Aufgabe (13)

$$\begin{aligned} -5\frac{3}{8}x &= -\frac{2}{3} & / : (-5\frac{3}{8}) \\ x &= 0,124 \end{aligned}$$

$$\begin{aligned} -\frac{3}{8}x + 1\frac{1}{3} &= 5x + \frac{2}{3} & / -5x \\ -5\frac{3}{8}x + 1\frac{1}{3} &= \frac{2}{3} & / -1\frac{1}{3} \end{aligned}$$

1.3 $a \cdot x + b = 0$

1.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x + b = 0$

Koeffizienten: a, b

Gesucht: x

- (1) $a = 3$ $b = 9$
- (2) $a = 8$ $b = 1$
- (3) $a = 2$ $b = 3$
- (4) $a = 3$ $b = 5$
- (5) $a = 7$ $b = 7$
- (6) $a = 5$ $b = 5$
- (7) $a = 6$ $b = 6$
- (8) $a = 8$ $b = 6$
- (9) $a = 6$ $b = 4$
- (10) $a = 1$ $b = 2$

- (11) $a = 4$ $b = 7$
- (12) $a = 2$ $b = 0$
- (13) $a = -\frac{1}{2}$ $b = 0$
- (14) $a = 6$ $b = -36$
- (15) $a = 3$ $b = 3$
- (16) $a = -\frac{1}{2}$ $b = 4\frac{1}{2}$
- (17) $a = -\frac{2}{3}$ $b = \frac{1}{6}$
- (18) $a = \frac{1}{4}$ $b = -2$
- (19) $a = \frac{1}{4}$ $b = -3$

1.3.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 3x + 9 &= 0 & / -9 \\ 3x &= -9 & / : 3 \\ x &= -3 \end{aligned}$$

$$\begin{aligned} 6x + 6 &= 0 & / -6 \\ 6x &= -6 & / : 6 \\ x &= -1 \end{aligned}$$

Aufgabe (8)

Aufgabe (2)

$$\begin{aligned} 8x + 1 &= 0 & / -1 \\ 8x &= -1 & / : 8 \\ x &= -\frac{1}{8} \end{aligned}$$

$$\begin{aligned} 8x + 6 &= 0 & / -6 \\ 8x &= -6 & / : 8 \\ x &= -\frac{3}{4} \end{aligned}$$

Aufgabe (9)

Aufgabe (3)

$$\begin{aligned} 2x + 3 &= 0 & / -3 \\ 2x &= -3 & / : 2 \\ x &= -1\frac{1}{2} \end{aligned}$$

$$\begin{aligned} 6x + 4 &= 0 & / -4 \\ 6x &= -4 & / : 6 \\ x &= -\frac{2}{3} \end{aligned}$$

Aufgabe (10)

Aufgabe (4)

$$\begin{aligned} 3x + 5 &= 0 & / -5 \\ 3x &= -5 & / : 3 \\ x &= -1\frac{2}{3} \end{aligned}$$

$$\begin{aligned} x + 2 &= 0 & / -2 \\ x &= -2 \end{aligned}$$

Aufgabe (11)

Aufgabe (5)

$$\begin{aligned} 7x + 7 &= 0 & / -7 \\ 7x &= -7 & / : 7 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} 4x + 7 &= 0 & / -7 \\ 4x &= -7 & / : 4 \\ x &= -1\frac{3}{4} \end{aligned}$$

Aufgabe (12)

Aufgabe (6)

$$\begin{aligned} 5x + 5 &= 0 & / -5 \\ 5x &= -5 & / : 5 \\ x &= -1 \end{aligned}$$

$$\begin{aligned} 2x &= 0 & / : 2 \\ x &= 0 \end{aligned}$$

Aufgabe (13)

Aufgabe (7)

Aufgabe (14)

$$\begin{aligned} -\frac{1}{2}x &= 0 & / : (-\frac{1}{2}) \\ x &= 0 \end{aligned}$$

$$\begin{aligned}6x - 36 &= 0 && / + 36 \\6x &= 36 && / : 6 \\x &= 6\end{aligned}$$

Aufgabe (15)

$$\begin{aligned}-\frac{2}{3}x + \frac{1}{6} &= 0 && / -\frac{1}{6} \\-\frac{2}{3}x &= -\frac{1}{6} && / : (-\frac{2}{3}) \\x &= \frac{1}{4}\end{aligned}$$

Aufgabe (18)

$$\begin{aligned}3x + 3 &= 0 && / - 3 \\3x &= -3 && / : 3 \\x &= -1\end{aligned}$$

Aufgabe (16)

$$\begin{aligned}\frac{1}{4}x - 2 &= 0 && / + 2 \\ \frac{1}{4}x &= 2 && / : \frac{1}{4} \\x &= 8\end{aligned}$$

Aufgabe (19)

$$\begin{aligned}-\frac{1}{2}x + 4\frac{1}{2} &= 0 && / - 4\frac{1}{2} \\-\frac{1}{2}x &= -4\frac{1}{2} && / : (-\frac{1}{2}) \\x &= 9\end{aligned}$$

Aufgabe (17)

$$\begin{aligned}\frac{1}{4}x - 3 &= 0 && / + 3 \\ \frac{1}{4}x &= 3 && / : \frac{1}{4} \\x &= 12\end{aligned}$$

1.4 $a \cdot x = d$

1.4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \cdot x = d$

Koeffizienten: a, d

Gesucht: x

(1) $a = 3$ $d = 9$

(2) $a = 8$ $d = 1$

(3) $a = 2$ $d = 3$

(4) $a = 3$ $d = 5$

(5) $a = 7$ $d = 7$

(6) $a = 5$ $d = 5$

(7) $a = 6$ $d = 6$

(8) $a = 8$ $d = 6$

(9) $a = 6$ $d = 4$

(10) $a = 1$ $d = 2$

(11) $a = 4$ $d = 7$

(12) $a = 2$ $d = 0$

(13) $a = -\frac{1}{2}$ $d = 0$

(14) $a = 6$ $d = -36$

(15) $a = 3$ $d = 3$

(16) $a = -\frac{1}{2}$ $d = 4\frac{1}{2}$

(17) $a = -\frac{2}{3}$ $d = \frac{1}{6}$

(18) $a = \frac{1}{4}$ $d = -2$

(19) $a = \frac{1}{4}$ $d = -3$

(20) $a = -2$ $d = 4$

(21) $a = 1$ $d = -2$

(22) $a = -1\frac{1}{4}$ $d = -10$

(23) $a = 4$ $d = -8$

(24) $a = -\frac{24}{49}$ $d = 2\frac{22}{49}$

(25) $a = \frac{8}{27}$ $d = 2\frac{2}{3}$

(26) $a = \frac{20}{81}$ $d = 2\frac{2}{9}$

1.4.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 3x &= 9 & / : 3 \\ x &= 3 \end{aligned}$$

$$\begin{aligned} 8x &= 6 & / : 8 \\ x &= \frac{3}{4} \end{aligned}$$

Aufgabe (9)

Aufgabe (2)

$$\begin{aligned} 8x &= 1 & / : 8 \\ x &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} 6x &= 4 & / : 6 \\ x &= \frac{2}{3} \end{aligned}$$

Aufgabe (10)

Aufgabe (3)

$$\begin{aligned} 2x &= 3 & / : 2 \\ x &= 1\frac{1}{2} \end{aligned}$$

$$x = 2$$

Aufgabe (11)

Aufgabe (4)

$$\begin{aligned} 3x &= 5 & / : 3 \\ x &= 1\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 4x &= 7 & / : 4 \\ x &= 1\frac{3}{4} \end{aligned}$$

Aufgabe (12)

Aufgabe (5)

$$\begin{aligned} 7x &= 7 & / : 7 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 2x &= 0 & / : 2 \\ x &= 0 \end{aligned}$$

Aufgabe (13)

Aufgabe (6)

$$\begin{aligned} -\frac{1}{2}x &= 0 & / : (-\frac{1}{2}) \\ x &= 0 \end{aligned}$$

Aufgabe (14)

$$\begin{aligned} 5x &= 5 & / : 5 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 6x &= -36 & / : 6 \\ x &= -6 \end{aligned}$$

Aufgabe (7)

Aufgabe (15)

$$\begin{aligned} 6x &= 6 & / : 6 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} 3x &= 3 & / : 3 \\ x &= 1 \end{aligned}$$

Aufgabe (8)

Aufgabe (16)

$$\begin{aligned} -\frac{1}{2}x &= 4\frac{1}{2} & / : (-\frac{1}{2}) \\ x &= -9 \end{aligned}$$

Aufgabe (22)

Aufgabe (17)

$$\begin{aligned} -1\frac{1}{4}x &= -10 & / : (-1\frac{1}{4}) \\ x &= 8 \end{aligned}$$

$$\begin{aligned} -\frac{2}{3}x &= \frac{1}{6} & / : (-\frac{2}{3}) \\ x &= -\frac{1}{4} \end{aligned}$$

Aufgabe (23)

Aufgabe (18)

$$\begin{aligned} 4x &= -8 & / : 4 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}x &= -2 & / : \frac{1}{4} \\ x &= -8 \end{aligned}$$

Aufgabe (24)

Aufgabe (19)

$$\begin{aligned} -\frac{24}{49}x &= 2\frac{22}{49} & / : (-\frac{24}{49}) \\ x &= -5 \end{aligned}$$

$$\begin{aligned} \frac{1}{4}x &= -3 & / : \frac{1}{4} \\ x &= -12 \end{aligned}$$

Aufgabe (25)

Aufgabe (20)

$$\begin{aligned} \frac{8}{27}x &= 2\frac{2}{3} & / : \frac{8}{27} \\ x &= 9 \end{aligned}$$

$$\begin{aligned} -2x &= 4 & / : (-2) \\ x &= -2 \end{aligned}$$

Aufgabe (26)

Aufgabe (21)

$$\begin{aligned} \frac{20}{81}x &= 2\frac{2}{9} & / : \frac{20}{81} \\ x &= 9 \end{aligned}$$

$$x = -2$$

2 Quadratische Gleichung

Umformen: $ax^2 + c = 0$

$$ax^2 + c = 0 \quad / -c$$

$$ax^2 = -c \quad / : a$$

$$x_{1/2} = \pm \sqrt{\frac{-c}{a}}$$

Diskriminante:

$$D = \frac{-c}{a}$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$\begin{aligned} -\frac{2}{3}x^2 + \frac{1}{6} &= 0 & / -\frac{1}{6} \\ -\frac{2}{3}x^2 &= -\frac{1}{6} & / : \left(-\frac{2}{3}\right) \end{aligned}$$

$$x^2 = \frac{-\frac{1}{6}}{-\frac{2}{3}}$$

$$x = \pm \sqrt{\frac{1}{4}}$$

$$x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$$

Faktorisieren: $ax^2 + bx = 0$

$$ax^2 + bx = 0$$

$$x(ax + b) = 0$$

$$x_1 = 0 \quad \vee \quad x_2 = \frac{-b}{a}$$

$$-2x^2 - 8x = 0$$

$$x(-2x - 8) = 0$$

$$x_1 = 0$$

$$-2x - 8 = 0 \quad / +8$$

$$-2x = 8 \quad / : (-2)$$

$$x = \frac{8}{-2}$$

$$x_2 = -4$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x_1 = 0$$

$$x - 1 = 0 \quad / +1$$

$$x = 1$$

$$x_2 = 1$$

Lösungsformel (Mitternachtsformel): $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Diskriminante:

$$D = b^2 - 4 \cdot a \cdot c$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{49}}{2}$$

$$x_{1/2} = \frac{-3 \pm 7}{2}$$

$$x_1 = \frac{-3 + 7}{2} \quad x_2 = \frac{-3 - 7}{2}$$

$$x_1 = 2 \quad x_2 = -5$$

p-q Formel: $x^2 + px + q = 0$

$$x^2 + px + q = 0$$

$$x_{1/2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Diskriminante:

$$D = \left(\frac{p}{2}\right)^2 - q$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$$

$$x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = 2 \quad x_2 = -5$$

Satz von Vieta: $x^2 + px + q = 0$

$$x^2 + px + q = 0$$

x_1, x_2 sind die Lösungen der Gleichung

$$(x - x_1) \cdot (x - x_2) = 0$$

$$x^2 - x_2 \cdot x - x_1 \cdot x + x_1 \cdot x_2 = 0$$

$$x^2 - (x_1 + x_2)x + x_1 \cdot x_2 = 0$$

$$x_1 + x_2 = -p$$

$$x_1 \cdot x_2 = q$$

$$x^2 + 3x - 10 = 0$$

$$p = 3 \quad q = -10$$

$$x_1 + x_2 = -3$$

$$x_1 \cdot x_2 = 10$$

$$2 - 5 = -3$$

$$2 \cdot (-5) = -10$$

$$x_1 = 2 \quad x_2 = -5$$

$$(x - 2) \cdot (x + 5) = 0$$

2.1 $ax^2 + bx + c = 0$

2.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ax^2 + bx + c = 0$

Gesucht:

Lösung der Gleichung

(1) $3x^2 + 3 = 0$

(2) $-\frac{1}{2}x^2 + 4\frac{1}{2} = 0$

(3) $-\frac{2}{3}x^2 + \frac{1}{6} = 0$

(4) $\frac{1}{4}x^2 - 2 = 0$

(5) $\frac{1}{4}x^2 - 3 = 0$

(6) $-2x^2 + 4 = 0$

(7) $x^2 - 2 = 0$

(8) $-\frac{1}{3}x^2 + 2x = 0$

(9) $-2x^2 - 8x = 0$

(10) $x^2 - 1x = 0$

(11) $\frac{1}{2}x^2 - \frac{2}{3}x = 0$

(12) $2x^2 - 5x = 0$

(13) $x^2 + 2x - 24 = 0$

(14) $x^2 + 3x - 10 = 0$

(15) $x^2 - 1x = 0$

(16) $x^2 - 8x - 20 = 0$

(17) $x^2 - 8x + 15 = 0$

(18) $-\frac{1}{3}x^2 - 2x + 3 = 0$

(19) $x^2 - 4x + 7 = 0$

(20) $-1x^2 + 4x - 7 = 0$

(21) $2x^2 + 4x = 0$

(22) $-\frac{1}{2}x^2 + 2x + 5 = 0$

(23) $-2x^2 + 3x + 4 = 0$

(24) $x^2 + 6x - 2 = 0$

(25) $-\frac{1}{3}x^2 + 2x + 5 = 0$

(26) $\frac{1}{2}x^2 - 1x + 4 = 0$

(27) $-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$

(28) $-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$

(29) $-1\frac{1}{4}x^2 + 5x = 0$

(30) $-\frac{3}{4}x^2 - 3x = 0$

(31) $\frac{5}{9}x^2 - 5 = 0$

(32) $12x^2 + 12x = 0$

(33) $-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$

(34) $-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$

(35) $-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$

(36) $\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$

(37) $-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$

(38) $-2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} = 0$

(39) $-\frac{7}{9}x^2 + 4\frac{2}{3}x = 0$

(40) $\frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} = 0$

(41) $\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$

(42) $-1\frac{1}{4}x^2 - 10x - 15 = 0$

(43) $4x^2 - 8x = 0$

(44) $-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$

(45) $\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$

(46) $\frac{20}{81}x^2 + 2\frac{2}{9}x = 0$

(47) $1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$

2.1.2 Lösungen

Aufgabe (1)

<p>Umformen</p> $3x^2 + 3 = 0 \quad / -3$ $3x^2 = -3 \quad / : 3$ $x^2 = \frac{-3}{3}$ <p>keine Lösung</p>	<p>a-b-c Formel</p> $3x^2 + 0x + 3 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 3 \cdot 3}}{2 \cdot 3}$ $x_{1/2} = \frac{-0 \pm \sqrt{-36}}{6}$ <p>Diskriminante negativ keine Lösung</p>	<p>p-q Formel</p> $3x^2 + 0x + 3 = 0 \quad / : 3$ $x^2 + 0x + 1 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - 1}$ $x_{1/2} = 0 \pm \sqrt{-1}$ <p>Diskriminante negativ keine Lösung</p>
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Aufgabe (2)

<p>Umformen</p> $-\frac{1}{2}x^2 + 4\frac{1}{2} = 0 \quad / -4\frac{1}{2}$ $-\frac{1}{2}x^2 = -4\frac{1}{2} \quad / : (-\frac{1}{2})$ $x^2 = \frac{-4\frac{1}{2}}{-\frac{1}{2}}$ $x = \pm\sqrt{9}$ $x_1 = 3 \quad x_2 = -3$	<p>a-b-c Formel</p> $-\frac{1}{2}x^2 + 0x + 4\frac{1}{2} = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-\frac{1}{2}) \cdot 4\frac{1}{2}}}{2 \cdot (-\frac{1}{2})}$ $x_{1/2} = \frac{-0 \pm \sqrt{9}}{-1}$ $x_{1/2} = \frac{0 \pm 3}{-1}$ $x_1 = \frac{0+3}{-1} \quad x_2 = \frac{0-3}{-1}$ $x_1 = -3 \quad x_2 = 3$	<p>p-q Formel</p> $-\frac{1}{2}x^2 + 0x + 4\frac{1}{2} = 0 \quad / : -\frac{1}{2}$ $x^2 + 0x - 9 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)}$ $x_{1/2} = 0 \pm \sqrt{9}$ $x_{1/2} = 0 \pm 3$ $x_1 = 3 \quad x_2 = -3$
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Aufgabe (3)

<p>Umformen</p> $-\frac{2}{3}x^2 + \frac{1}{6} = 0 \quad / -\frac{1}{6}$ $-\frac{2}{3}x^2 = -\frac{1}{6} \quad / : (-\frac{2}{3})$ $x^2 = \frac{-\frac{1}{6}}{-\frac{2}{3}}$ $x = \pm\sqrt{\frac{1}{4}}$ $x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$	<p>a-b-c Formel</p> $-\frac{2}{3}x^2 + 0x + \frac{1}{6} = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-\frac{2}{3}) \cdot \frac{1}{6}}}{2 \cdot (-\frac{2}{3})}$ $x_{1/2} = \frac{-0 \pm \sqrt{\frac{4}{9}}}{-1\frac{1}{3}}$ $x_{1/2} = \frac{0 \pm \frac{2}{3}}{-1\frac{1}{3}}$ $x_1 = \frac{0 + \frac{2}{3}}{-1\frac{1}{3}} \quad x_2 = \frac{0 - \frac{2}{3}}{-1\frac{1}{3}}$ $x_1 = -\frac{1}{2} \quad x_2 = \frac{1}{2}$	<p>p-q Formel</p> $-\frac{2}{3}x^2 + 0x + \frac{1}{6} = 0 \quad / : -\frac{2}{3}$ $x^2 + 0x - \frac{1}{4} = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - \left(-\frac{1}{4}\right)}$ $x_{1/2} = 0 \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = 0 \pm \frac{1}{2}$ $x_1 = \frac{1}{2} \quad x_2 = -\frac{1}{2}$
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Aufgabe (4)

<p>Umformen</p> <hr/> $\frac{1}{4}x^2 - 2 = 0 \quad / + 2$ $\frac{1}{4}x^2 = 2 \quad / : \frac{1}{4}$ $x^2 = \frac{2}{\frac{1}{4}}$ $x = \pm\sqrt{8}$ $x_1 = 2,83 \quad x_2 = -2,83$	<p>a-b-c Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 2 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-2)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{2}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm 1,41}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,41}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,41}{\frac{1}{2}}$ $x_1 = 2,83 \quad x_2 = -2,83$	<p>p-q Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 2 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 8 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-8)}$ $x_{1/2} = 0 \pm \sqrt{8}$ $x_{1/2} = 0 \pm 2,83$ $x_1 = 2,83 \quad x_2 = -2,83$
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Aufgabe (5)

<p>Umformen</p> <hr/> $\frac{1}{4}x^2 - 3 = 0 \quad / + 3$ $\frac{1}{4}x^2 = 3 \quad / : \frac{1}{4}$ $x^2 = \frac{3}{\frac{1}{4}}$ $x = \pm\sqrt{12}$ $x_1 = 3,46 \quad x_2 = -3,46$	<p>a-b-c Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 3 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{1}{4} \cdot (-3)}}{2 \cdot \frac{1}{4}}$ $x_{1/2} = \frac{-0 \pm \sqrt{3}}{\frac{1}{2}}$ $x_{1/2} = \frac{0 \pm 1,73}{\frac{1}{2}}$ $x_1 = \frac{0 + 1,73}{\frac{1}{2}} \quad x_2 = \frac{0 - 1,73}{\frac{1}{2}}$ $x_1 = 3,46 \quad x_2 = -3,46$	<p>p-q Formel</p> <hr/> $\frac{1}{4}x^2 + 0x - 3 = 0 \quad / : \frac{1}{4}$ $x^2 + 0x - 12 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-12)}$ $x_{1/2} = 0 \pm \sqrt{12}$ $x_{1/2} = 0 \pm 3,46$ $x_1 = 3,46 \quad x_2 = -3,46$
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Aufgabe (6)

<p>Umformen</p> <hr/> $-2x^2 + 4 = 0 \quad / - 4$ $-2x^2 = -4 \quad / : (-2)$ $x^2 = \frac{-4}{-2}$ $x = \pm\sqrt{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>a-b-c Formel</p> <hr/> $-2x^2 + 0x + 4 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)}$ $x_{1/2} = \frac{-0 \pm \sqrt{32}}{-4}$ $x_{1/2} = \frac{0 \pm 5,66}{-4}$ $x_1 = \frac{0 + 5,66}{-4} \quad x_2 = \frac{0 - 5,66}{-4}$ $x_1 = -1,41 \quad x_2 = 1,41$	<p>p-q Formel</p> <hr/> $-2x^2 + 0x + 4 = 0 \quad / : -2$ $x^2 + 0x - 2 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)}$ $x_{1/2} = 0 \pm \sqrt{2}$ $x_{1/2} = 0 \pm 1,41$ $x_1 = 1,41 \quad x_2 = -1,41$
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Aufgabe (7)

<p>Umformen</p> <hr/> $1x^2 - 2 = 0 \quad / + 2$ $1x^2 = 2 \quad / : 1$ $x^2 = \frac{2}{1}$ $x = \pm\sqrt{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>a-b-c Formel</p> <hr/> $1x^2 + 0x - 2 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$ $x_{1/2} = \frac{-0 \pm \sqrt{8}}{2}$ $x_{1/2} = \frac{0 \pm 2,83}{2}$ $x_1 = \frac{0 + 2,83}{2} \quad x_2 = \frac{0 - 2,83}{2}$ $x_1 = 1,41 \quad x_2 = -1,41$	<p>p-q Formel</p> <hr/> $x^2 + 0x - 2 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-2)}$ $x_{1/2} = 0 \pm \sqrt{2}$ $x_{1/2} = 0 \pm 1,41$ $x_1 = 1,41 \quad x_2 = -1,41$
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Aufgabe (8)

<p>x-Ausklammern</p> $-\frac{1}{3}x^2 + 2x = 0$ $x(-\frac{1}{3}x + 2) = 0$ $-\frac{1}{3}x + 2 = 0 \quad / -2$ $-\frac{1}{3}x = -2 \quad / : (-\frac{1}{3})$ $x = \frac{-2}{-\frac{1}{3}}$ $x_1 = 0$ $x_2 = 6$	<p>a-b-c Formel</p> $-\frac{1}{3}x^2 + 2x + 0 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{3}) \cdot 0}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{-2 \pm \sqrt{4}}{-\frac{2}{3}}$ $x_{1/2} = \frac{-2 \pm 2}{-\frac{2}{3}}$ $x_1 = \frac{-2 + 2}{-\frac{2}{3}} \quad x_2 = \frac{-2 - 2}{-\frac{2}{3}}$ $x_1 = 0 \quad x_2 = 6$	<p>p-q Formel</p> $-\frac{1}{3}x^2 + 2x + 0 = 0 \quad / : -\frac{1}{3}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$
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Aufgabe (9)

<p>x-Ausklammern</p> $-2x^2 - 8x = 0$ $x(-2x - 8) = 0$ $-2x - 8 = 0 \quad / +8$ $-2x = 8 \quad / : (-2)$ $x = \frac{8}{-2}$ $x_1 = 0$ $x_2 = -4$	<p>a-b-c Formel</p> $-2x^2 - 8x + 0 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot (-2) \cdot 0}}{2 \cdot (-2)}$ $x_{1/2} = \frac{+8 \pm \sqrt{64}}{-4}$ $x_{1/2} = \frac{8 \pm 8}{-4}$ $x_1 = \frac{8 + 8}{-4} \quad x_2 = \frac{8 - 8}{-4}$ $x_1 = -4 \quad x_2 = 0$	<p>p-q Formel</p> $-2x^2 - 8x + 0 = 0 \quad / : -2$ $x^2 + 4x + 0 = 0$ $x_{1/2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0}$ $x_{1/2} = -2 \pm \sqrt{4}$ $x_{1/2} = -2 \pm 2$ $x_1 = 0 \quad x_2 = -4$
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Aufgabe (10)

<p>x-Ausklammern</p> $1x^2 - 1x = 0$ $x(x - 1) = 0$ $1x - 1 = 0 \quad / +1$ $1x = 1 \quad / : 1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	<p>a-b-c Formel</p> $1x^2 - 1x + 0 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1 + 1}{2} \quad x_2 = \frac{1 - 1}{2}$ $x_1 = 1 \quad x_2 = 0$	<p>p-q Formel</p> $x^2 - 1x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$
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Aufgabe (11)

x-Ausklammern	a-b-c Formel	p-q Formel
$\frac{1}{2}x^2 - \frac{2}{3}x = 0$ $x(\frac{1}{2}x - \frac{2}{3}) = 0$ $\frac{1}{2}x - \frac{2}{3} = 0 \quad / + \frac{2}{3}$ $\frac{1}{2}x = \frac{2}{3} \quad / : \frac{1}{2}$ $x = \frac{2}{3} \cdot 2$ $x_1 = 0$ $x_2 = 1\frac{1}{3}$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0$ $+ \frac{2}{3} \pm \sqrt{(-\frac{2}{3})^2 - 4 \cdot \frac{1}{2} \cdot 0}$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9}}}{2 \cdot \frac{1}{2}}$ $x_{1/2} = \frac{+\frac{2}{3} \pm \sqrt{\frac{4}{9}}}{1}$ $x_{1/2} = \frac{\frac{2}{3} \pm \frac{1}{3}}{1}$ $x_1 = \frac{\frac{2}{3} + \frac{1}{3}}{1} \quad x_2 = \frac{\frac{2}{3} - \frac{1}{3}}{1}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$	$\frac{1}{2}x^2 - \frac{2}{3}x + 0 = 0 \quad / : \frac{1}{2}$ $x^2 - 1\frac{1}{3}x + 0 = 0$ $x_{1/2} = -\frac{-1\frac{1}{3}}{2} \pm \sqrt{\left(\frac{(-1\frac{1}{3})}{2}\right)^2 - 0}$ $x_{1/2} = \frac{2}{3} \pm \sqrt{\frac{4}{9}}$ $x_{1/2} = \frac{2}{3} \pm \frac{2}{3}$ $x_1 = 1\frac{1}{3} \quad x_2 = 0$

Aufgabe (12)

x-Ausklammern	a-b-c Formel	p-q Formel
$2x^2 - 5x = 0$ $x(2x - 5) = 0$ $2x - 5 = 0 \quad / + 5$ $2x = 5 \quad / : 2$ $x = \frac{5}{2}$ $x_1 = 0$ $x_2 = 2\frac{1}{2}$	$2x^2 - 5x + 0 = 0$ $+ 5 \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 0}$ $x_{1/2} = \frac{+5 \pm \sqrt{25}}{2 \cdot 2}$ $x_{1/2} = \frac{+5 \pm 5}{4}$ $x_{1/2} = \frac{5 \pm 5}{4}$ $x_1 = \frac{5 + 5}{4} \quad x_2 = \frac{5 - 5}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$	$2x^2 - 5x + 0 = 0 \quad / : 2$ $x^2 - 2\frac{1}{2}x + 0 = 0$ $x_{1/2} = -\frac{-2\frac{1}{2}}{2} \pm \sqrt{\left(\frac{(-2\frac{1}{2})}{2}\right)^2 - 0}$ $x_{1/2} = 1\frac{1}{4} \pm \sqrt{1\frac{9}{16}}$ $x_{1/2} = 1\frac{1}{4} \pm 1\frac{1}{4}$ $x_1 = 2\frac{1}{2} \quad x_2 = 0$

Aufgabe (13)

a-b-c Formel	p-q Formel
$1x^2 + 2x - 24 = 0$ $x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-24)}}{2 \cdot 1}$ $x_{1/2} = \frac{-2 \pm \sqrt{100}}{2}$ $x_{1/2} = \frac{-2 \pm 10}{2}$ $x_1 = \frac{-2 + 10}{2} \quad x_2 = \frac{-2 - 10}{2}$ $x_1 = 4 \quad x_2 = -6$	$x^2 + 2x - 24 = 0$ $x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - (-24)}$ $x_{1/2} = -1 \pm \sqrt{25}$ $x_{1/2} = -1 \pm 5$ $x_1 = 4 \quad x_2 = -6$

Aufgabe (14)

a-b-c Formel	p-q Formel
$1x^2 + 3x - 10 = 0$ $x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1}$ $x_{1/2} = \frac{-3 \pm \sqrt{49}}{2}$ $x_{1/2} = \frac{-3 \pm 7}{2}$ $x_1 = \frac{-3 + 7}{2} \quad x_2 = \frac{-3 - 7}{2}$ $x_1 = 2 \quad x_2 = -5$	$x^2 + 3x - 10 = 0$ $x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$ $x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$ $x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$ $x_1 = 2 \quad x_2 = -5$

Aufgabe (15)

x-Ausklammern	a-b-c Formel	p-q Formel
$1x^2 - 1x = 0$ $x(1x - 1) = 0$ $1x - 1 = 0 \quad / + 1$ $1x = 1 \quad / : 1$ $x = \frac{1}{1}$ $x_1 = 0$ $x_2 = 1$	$1x^2 - 1x + 0 = 0$ $x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 0}}{2 \cdot 1}$ $x_{1/2} = \frac{+1 \pm \sqrt{1}}{2}$ $x_{1/2} = \frac{1 \pm 1}{2}$ $x_1 = \frac{1+1}{2} \quad x_2 = \frac{1-1}{2}$ $x_1 = 1 \quad x_2 = 0$	$x^2 - 1x + 0 = 0$ $x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{(-1)}{2}\right)^2 - 0}$ $x_{1/2} = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = \frac{1}{2} \pm \frac{1}{2}$ $x_1 = 1 \quad x_2 = 0$

Aufgabe (16)

a-b-c Formel	p-q Formel
$1x^2 - 8x - 20 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (-20)}}{2 \cdot 1}$ $x_{1/2} = \frac{+8 \pm \sqrt{144}}{2}$ $x_{1/2} = \frac{8 \pm 12}{2}$ $x_1 = \frac{8+12}{2} \quad x_2 = \frac{8-12}{2}$ $x_1 = 10 \quad x_2 = -2$	$x^2 - 8x - 20 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{(-8)}{2}\right)^2 - (-20)}$ $x_{1/2} = 4 \pm \sqrt{36}$ $x_{1/2} = 4 \pm 6$ $x_1 = 10 \quad x_2 = -2$

Aufgabe (17)

a-b-c Formel	p-q Formel
$1x^2 - 8x + 15 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 15}}{2 \cdot 1}$ $x_{1/2} = \frac{+8 \pm \sqrt{4}}{2}$ $x_{1/2} = \frac{8 \pm 2}{2}$ $x_1 = \frac{8+2}{2} \quad x_2 = \frac{8-2}{2}$ $x_1 = 5 \quad x_2 = 3$	$x^2 - 8x + 15 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{(-8)}{2}\right)^2 - 15}$ $x_{1/2} = 4 \pm \sqrt{1}$ $x_{1/2} = 4 \pm 1$ $x_1 = 5 \quad x_2 = 3$

Aufgabe (18)

a-b-c Formel	p-q Formel
$-\frac{1}{3}x^2 - 2x + 3 = 0$ $x_{1/2} = \frac{+2 \pm \sqrt{(-2)^2 - 4 \cdot (-\frac{1}{3}) \cdot 3}}{2 \cdot (-\frac{1}{3})}$ $x_{1/2} = \frac{+2 \pm \sqrt{8}}{-\frac{2}{3}}$ $x_{1/2} = \frac{2 \pm 2,83}{-\frac{2}{3}}$ $x_1 = \frac{2 + 2,83}{-\frac{2}{3}} \quad x_2 = \frac{2 - 2,83}{-\frac{2}{3}}$ $x_1 = -7,24 \quad x_2 = 1,24$	$-\frac{1}{3}x^2 - 2x + 3 = 0 \quad / : -\frac{1}{3}$ $x^2 + 6x - 9 = 0$ $x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-9)}$ $x_{1/2} = -3 \pm \sqrt{18}$ $x_{1/2} = -3 \pm 4,24$ $x_1 = 1,24 \quad x_2 = -7,24$

Aufgabe (19)

a-b-c Formel	p-q Formel
$1x^2 - 4x + 7 = 0$ $x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$ $x_{1/2} = \frac{+4 \pm \sqrt{-12}}{2}$ <p>Diskriminante negativ keine Lösung</p>	$x^2 - 4x + 7 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$ $x_{1/2} = 2 \pm \sqrt{-3}$ <p>Diskriminante negativ keine Lösung</p>

Aufgabe (20)

a-b-c Formel	p-q Formel
$-1x^2 + 4x - 7 = 0$ $x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-7)}}{2 \cdot (-1)}$ $x_{1/2} = \frac{-4 \pm \sqrt{-12}}{-2}$ <p>Diskriminante negativ keine Lösung</p>	$-1x^2 + 4x - 7 = 0 \quad / : -1$ $x^2 - 4x + 7 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 7}$ $x_{1/2} = 2 \pm \sqrt{-3}$ <p>Diskriminante negativ keine Lösung</p>

Aufgabe (21)

x-Ausklammern	a-b-c Formel	p-q Formel
$2x^2 + 4x = 0$ $x(2x + 4) = 0$ $2x + 4 = 0 \quad / -4$ $2x = -4 \quad / : 2$ $x = \frac{-4}{2}$ $x_1 = 0$ $x_2 = -2$	$2x^2 + 4x + 0 = 0$ $x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 2 \cdot 0}}{2 \cdot 2}$ $x_{1/2} = \frac{-4 \pm \sqrt{16}}{4}$ $x_{1/2} = \frac{-4 \pm 4}{4}$ $x_1 = \frac{-4 + 4}{4} \quad x_2 = \frac{-4 - 4}{4}$ $x_1 = 0 \quad x_2 = -2$	$2x^2 + 4x + 0 = 0 \quad / : 2$ $x^2 + 2x + 0 = 0$ $x_{1/2} = -\frac{2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 0}$ $x_{1/2} = -1 \pm \sqrt{1}$ $x_{1/2} = -1 \pm 1$ $x_1 = 0 \quad x_2 = -2$

Aufgabe (22)

a-b-c Formel

$$-\frac{1}{2}x^2 + 2x + 5 = 0$$

$$x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{2}) \cdot 5}}{2 \cdot (-\frac{1}{2})}$$

$$x_{1/2} = \frac{-2 \pm \sqrt{14}}{-1}$$

$$x_{1/2} = \frac{-2 \pm 3,74}{-1}$$

$$x_1 = \frac{-2 + 3,74}{-1} \quad x_2 = \frac{-2 - 3,74}{-1}$$

$$x_1 = -1,74 \quad x_2 = 5,74$$

p-q Formel

$$-\frac{1}{2}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{2}$$

$$x^2 - 4x - 10 = 0$$

$$x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - (-10)}$$

$$x_{1/2} = 2 \pm \sqrt{14}$$

$$x_{1/2} = 2 \pm 3,74$$

$$x_1 = 5,74 \quad x_2 = -1,74$$

Aufgabe (23)

a-b-c Formel

$$-2x^2 + 3x + 4 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot (-2) \cdot 4}}{2 \cdot (-2)}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{41}}{-4}$$

$$x_{1/2} = \frac{-3 \pm 6,4}{-4}$$

$$x_1 = \frac{-3 + 6,4}{-4} \quad x_2 = \frac{-3 - 6,4}{-4}$$

$$x_1 = -0,851 \quad x_2 = 2,35$$

p-q Formel

$$-2x^2 + 3x + 4 = 0 \quad / : -2$$

$$x^2 - 1\frac{1}{2}x - 2 = 0$$

$$x_{1/2} = -\frac{-1\frac{1}{2}}{2} \pm \sqrt{\left(\frac{(-1\frac{1}{2})}{2}\right)^2 - (-2)}$$

$$x_{1/2} = \frac{3}{4} \pm \sqrt{2\frac{9}{16}}$$

$$x_{1/2} = \frac{3}{4} \pm 1,6$$

$$x_1 = 2,35 \quad x_2 = -0,851$$

Aufgabe (24)

a-b-c Formel

$$1x^2 + 6x - 2 = 0$$

$$x_{1/2} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-6 \pm \sqrt{44}}{2}$$

$$x_{1/2} = \frac{-6 \pm 6,63}{2}$$

$$x_1 = \frac{-6 + 6,63}{2} \quad x_2 = \frac{-6 - 6,63}{2}$$

$$x_1 = 0,317 \quad x_2 = -6,32$$

p-q Formel

$$x^2 + 6x - 2 = 0$$

$$x_{1/2} = -\frac{6}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 - (-2)}$$

$$x_{1/2} = -3 \pm \sqrt{11}$$

$$x_{1/2} = -3 \pm 3,32$$

$$x_1 = 0,317 \quad x_2 = -6,32$$

Aufgabe (25)

a-b-c Formel

$$-\frac{1}{3}x^2 + 2x + 5 = 0$$

$$x_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-\frac{1}{3}) \cdot 5}}{2 \cdot (-\frac{1}{3})}$$

$$x_{1/2} = \frac{-2 \pm \sqrt{10\frac{2}{3}}}{-\frac{2}{3}}$$

$$x_{1/2} = \frac{-2 \pm 3,27}{-\frac{2}{3}}$$

$$x_1 = \frac{-2 + 3,27}{-\frac{2}{3}} \quad x_2 = \frac{-2 - 3,27}{-\frac{2}{3}}$$

$$x_1 = -1,9 \quad x_2 = 7,9$$

p-q Formel

$$-\frac{1}{3}x^2 + 2x + 5 = 0 \quad / : -\frac{1}{3}$$

$$x^2 - 6x - 15 = 0$$

$$x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{(-6)}{2}\right)^2 - (-15)}$$

$$x_{1/2} = 3 \pm \sqrt{24}$$

$$x_{1/2} = 3 \pm 4,9$$

$$x_1 = 7,9 \quad x_2 = -1,9$$

Aufgabe (26)

a-b-c Formel

$$\frac{1}{2}x^2 - 1x + 4 = 0$$

$$x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{2} \cdot 4}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+1 \pm \sqrt{-7}}{1}$$

Diskriminante negativ keine Lösung

p-q Formel

$$\frac{1}{2}x^2 - 1x + 4 = 0 \quad / : \frac{1}{2}$$

$$x^2 - 2x + 8 = 0$$

$$x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - 8}$$

$$x_{1/2} = 1 \pm \sqrt{-7}$$

Diskriminante negativ keine Lösung

Aufgabe (27)

a-b-c Formel

$$-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0$$

$$x_{1/2} = \frac{+\frac{24}{49} \pm \sqrt{\left(-\frac{24}{49}\right)^2 - 4 \cdot \left(-\frac{8}{49}\right) \cdot 1\frac{31}{49}}}{2 \cdot \left(-\frac{8}{49}\right)}$$

$$x_{1/2} = \frac{+\frac{24}{49} \pm \sqrt{1\frac{15}{49}}}{-\frac{16}{49}}$$

$$x_{1/2} = \frac{\frac{24}{49} \pm 1\frac{1}{7}}{-\frac{16}{49}}$$

$$x_1 = \frac{\frac{24}{49} + 1\frac{1}{7}}{-\frac{16}{49}} \quad x_2 = \frac{\frac{24}{49} - 1\frac{1}{7}}{-\frac{16}{49}}$$

$$x_1 = -5 \quad x_2 = 2$$

p-q Formel

$$-\frac{8}{49}x^2 - \frac{24}{49}x + 1\frac{31}{49} = 0 \quad / : -\frac{8}{49}$$

$$x^2 + 3x - 10 = 0$$

$$x_{1/2} = -\frac{3}{2} \pm \sqrt{\left(\frac{3}{2}\right)^2 - (-10)}$$

$$x_{1/2} = -1\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = -1\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = 2 \quad x_2 = -5$$

Aufgabe (28)

a-b-c Formel

$$-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0$$

$$x_{1/2} = \frac{+\frac{32}{81} \pm \sqrt{\left(-\frac{32}{81}\right)^2 - 4 \cdot \left(-\frac{32}{81}\right) \cdot 7\frac{73}{81}}}{2 \cdot \left(-\frac{32}{81}\right)}$$

$$x_{1/2} = \frac{+\frac{32}{81} \pm \sqrt{12\frac{52}{81}}}{-\frac{64}{81}}$$

$$x_{1/2} = \frac{\frac{32}{81} \pm 3\frac{5}{9}}{-\frac{64}{81}}$$

$$x_1 = \frac{\frac{32}{81} + 3\frac{5}{9}}{-\frac{64}{81}} \quad x_2 = \frac{\frac{32}{81} - 3\frac{5}{9}}{-\frac{64}{81}}$$

$$x_1 = -5 \quad x_2 = 4$$

p-q Formel

$$-\frac{32}{81}x^2 - \frac{32}{81}x + 7\frac{73}{81} = 0 \quad / : -\frac{32}{81}$$

$$x^2 + 1x - 20 = 0$$

$$x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-20)}$$

$$x_{1/2} = -\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$$

$$x_{1/2} = -\frac{1}{2} \pm 4\frac{1}{2}$$

$$x_1 = 4 \quad x_2 = -5$$

Aufgabe (29)

<p><u>x-Ausklammern</u></p> $-1\frac{1}{4}x^2 + 5x = 0$ $x(-1\frac{1}{4}x + 5) = 0$ $-1\frac{1}{4}x + 5 = 0 \quad / -5$ $-1\frac{1}{4}x = -5 \quad / : (-1\frac{1}{4})$ $x = \frac{-5}{-1\frac{1}{4}}$ $x_1 = 0$ $x_2 = 4$	<p><u>a-b-c Formel</u></p> $-1\frac{1}{4}x^2 + 5x + 0 = 0$ $x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot (-1\frac{1}{4}) \cdot 0}}{2 \cdot (-1\frac{1}{4})}$ $x_{1/2} = \frac{-5 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{-5 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{-5 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{-5 - 5}{-2\frac{1}{2}}$ $x_1 = 0 \quad x_2 = 4$	<p><u>p-q Formel</u></p> $-1\frac{1}{4}x^2 + 5x + 0 = 0 \quad / : -1\frac{1}{4}$ $x^2 - 4x + 0 = 0$ $x_{1/2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{(-4)}{2}\right)^2 - 0}$ $x_{1/2} = 2 \pm \sqrt{4}$ $x_{1/2} = 2 \pm 2$ $x_1 = 4 \quad x_2 = 0$
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Aufgabe (30)

<p><u>x-Ausklammern</u></p> $-\frac{3}{4}x^2 - 3x = 0$ $x(-\frac{3}{4}x - 3) = 0$ $-\frac{3}{4}x - 3 = 0 \quad / +3$ $-\frac{3}{4}x = 3 \quad / : (-\frac{3}{4})$ $x = \frac{3}{-\frac{3}{4}}$ $x_1 = 0$ $x_2 = -4$	<p><u>a-b-c Formel</u></p> $-\frac{3}{4}x^2 - 3x + 0 = 0$ $x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot (-\frac{3}{4}) \cdot 0}}{2 \cdot (-\frac{3}{4})}$ $x_{1/2} = \frac{+3 \pm \sqrt{9}}{-1\frac{1}{2}}$ $x_{1/2} = \frac{3 \pm 3}{-1\frac{1}{2}}$ $x_1 = \frac{3 + 3}{-1\frac{1}{2}} \quad x_2 = \frac{3 - 3}{-1\frac{1}{2}}$ $x_1 = -4 \quad x_2 = 0$	<p><u>p-q Formel</u></p> $-\frac{3}{4}x^2 - 3x + 0 = 0 \quad / : -\frac{3}{4}$ $x^2 + 4x + 0 = 0$ $x_{1/2} = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 0}$ $x_{1/2} = -2 \pm \sqrt{4}$ $x_{1/2} = -2 \pm 2$ $x_1 = 0 \quad x_2 = -4$
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Aufgabe (31)

<p><u>Umformen</u></p> $\frac{5}{9}x^2 - 5 = 0 \quad / +5$ $\frac{5}{9}x^2 = 5 \quad / : \frac{5}{9}$ $x^2 = \frac{5}{\frac{5}{9}}$ $x = \pm \sqrt{9}$ $x_1 = 3 \quad x_2 = -3$	<p><u>a-b-c Formel</u></p> $\frac{5}{9}x^2 + 0x - 5 = 0$ $x_{1/2} = \frac{-0 \pm \sqrt{0^2 - 4 \cdot \frac{5}{9} \cdot (-5)}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{-0 \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{0 \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{0 + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{0 - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 3 \quad x_2 = -3$	<p><u>p-q Formel</u></p> $\frac{5}{9}x^2 + 0x - 5 = 0 \quad / : \frac{5}{9}$ $x^2 + 0x - 9 = 0$ $x_{1/2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-9)}$ $x_{1/2} = 0 \pm \sqrt{9}$ $x_{1/2} = 0 \pm 3$ $x_1 = 3 \quad x_2 = -3$
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Aufgabe (32)

x-Ausklammern	a-b-c Formel	p-q Formel
$12x^2 + 12x = 0$ $x(12x + 12) = 0$ $12x + 12 = 0 \quad / -12$ $12x = -12 \quad / :12$ $x = \frac{-12}{12}$ $x_1 = 0$ $x_2 = -1$	$12x^2 + 12x + 0 = 0$ $x_{1/2} = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 12 \cdot 0}}{2 \cdot 12}$ $x_{1/2} = \frac{-12 \pm \sqrt{144}}{24}$ $x_{1/2} = \frac{-12 \pm 12}{24}$ $x_1 = \frac{-12 + 12}{24} \quad x_2 = \frac{-12 - 12}{24}$ $x_1 = 0 \quad x_2 = -1$	$12x^2 + 12x + 0 = 0 \quad / :12$ $x^2 + 1x + 0 = 0$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - 0}$ $x_{1/2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}}$ $x_{1/2} = -\frac{1}{2} \pm \frac{1}{2}$ $x_1 = 0 \quad x_2 = -1$

Aufgabe (33)

a-b-c Formel	p-q Formel
$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{\left(1\frac{23}{25}\right)^2 - 4 \cdot \left(-\frac{6}{25}\right) \cdot 2\frac{4}{25}}}{2 \cdot \left(-\frac{6}{25}\right)}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm \sqrt{5\frac{19}{25}}}{-\frac{12}{25}}$ $x_{1/2} = \frac{-1\frac{23}{25} \pm 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = \frac{-1\frac{23}{25} + 2\frac{2}{5}}{-\frac{12}{25}} \quad x_2 = \frac{-1\frac{23}{25} - 2\frac{2}{5}}{-\frac{12}{25}}$ $x_1 = -1 \quad x_2 = 9$	$-\frac{6}{25}x^2 + 1\frac{23}{25}x + 2\frac{4}{25} = 0 \quad / : -\frac{6}{25}$ $x^2 - 8x - 9 = 0$ $x_{1/2} = -\frac{-8}{2} \pm \sqrt{\left(\frac{-8}{2}\right)^2 - (-9)}$ $x_{1/2} = 4 \pm \sqrt{25}$ $x_{1/2} = 4 \pm 5$ $x_1 = 9 \quad x_2 = -1$

Aufgabe (34)

a-b-c Formel	p-q Formel
$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{\left(-2\frac{22}{25}\right)^2 - 4 \cdot \left(-\frac{9}{25}\right) \cdot 3\frac{6}{25}}}{2 \cdot \left(-\frac{9}{25}\right)}$ $x_{1/2} = \frac{+2\frac{22}{25} \pm \sqrt{12\frac{24}{25}}}{-\frac{18}{25}}$ $x_{1/2} = \frac{2\frac{22}{25} \pm 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = \frac{2\frac{22}{25} + 3\frac{3}{5}}{-\frac{18}{25}} \quad x_2 = \frac{2\frac{22}{25} - 3\frac{3}{5}}{-\frac{18}{25}}$ $x_1 = -9 \quad x_2 = 1$	$-\frac{9}{25}x^2 - 2\frac{22}{25}x + 3\frac{6}{25} = 0 \quad / : -\frac{9}{25}$ $x^2 + 8x - 9 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - (-9)}$ $x_{1/2} = -4 \pm \sqrt{25}$ $x_{1/2} = -4 \pm 5$ $x_1 = 1 \quad x_2 = -9$

Aufgabe (35)

a-b-c Formel

$$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{\left(\frac{1}{4}\right)^2 - 4 \cdot \left(-\frac{1}{8}\right) \cdot 7\frac{7}{8}}}{2 \cdot \left(-\frac{1}{8}\right)}$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm \sqrt{4}}{-\frac{1}{4}}$$

$$x_{1/2} = \frac{-\frac{1}{4} \pm 2}{-\frac{1}{4}}$$

$$x_1 = \frac{-\frac{1}{4} + 2}{-\frac{1}{4}} \quad x_2 = \frac{-\frac{1}{4} - 2}{-\frac{1}{4}}$$

$$x_1 = -7 \quad x_2 = 9$$

p-q Formel

$$-\frac{1}{8}x^2 + \frac{1}{4}x + 7\frac{7}{8} = 0 \quad / : -\frac{1}{8}$$

$$x^2 - 2x - 63 = 0$$

$$x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-63)}$$

$$x_{1/2} = 1 \pm \sqrt{64}$$

$$x_{1/2} = 1 \pm 8$$

$$x_1 = 9 \quad x_2 = -7$$

Aufgabe (36)

a-b-c Formel

$$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{\left(3\frac{33}{49}\right)^2 - 4 \cdot \frac{20}{49} \cdot 3\frac{13}{49}}}{2 \cdot \frac{20}{49}}$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm \sqrt{8\frac{8}{49}}}{\frac{40}{49}}$$

$$x_{1/2} = \frac{-3\frac{33}{49} \pm 2\frac{6}{7}}{\frac{40}{49}}$$

$$x_1 = \frac{-3\frac{33}{49} + 2\frac{6}{7}}{\frac{40}{49}} \quad x_2 = \frac{-3\frac{33}{49} - 2\frac{6}{7}}{\frac{40}{49}}$$

$$x_1 = -1 \quad x_2 = -8$$

p-q Formel

$$\frac{20}{49}x^2 + 3\frac{33}{49}x + 3\frac{13}{49} = 0 \quad / : \frac{20}{49}$$

$$x^2 + 9x + 8 = 0$$

$$x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 8}$$

$$x_{1/2} = -4\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = -4\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = -1 \quad x_2 = -8$$

Aufgabe (37)

a-b-c Formel

$$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{\left(\frac{4}{9}\right)^2 - 4 \cdot \left(-\frac{4}{9}\right) \cdot \frac{8}{9}}}{2 \cdot \left(-\frac{4}{9}\right)}$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm \sqrt{1\frac{7}{9}}}{-\frac{8}{9}}$$

$$x_{1/2} = \frac{-\frac{4}{9} \pm 1\frac{1}{3}}{-\frac{8}{9}}$$

$$x_1 = \frac{-\frac{4}{9} + 1\frac{1}{3}}{-\frac{8}{9}} \quad x_2 = \frac{-\frac{4}{9} - 1\frac{1}{3}}{-\frac{8}{9}}$$

$$x_1 = -1 \quad x_2 = 2$$

p-q Formel

$$-\frac{4}{9}x^2 + \frac{4}{9}x + \frac{8}{9} = 0 \quad / : -\frac{4}{9}$$

$$x^2 - 1x - 2 = 0$$

$$x_{1/2} = -\frac{-1}{2} \pm \sqrt{\left(\frac{-1}{2}\right)^2 - (-2)}$$

$$x_{1/2} = \frac{1}{2} \pm \sqrt{2\frac{1}{4}}$$

$$x_{1/2} = \frac{1}{2} \pm 1\frac{1}{2}$$

$$x_1 = 2 \quad x_2 = -1$$

Aufgabe (38)

a-b-c Formel

$$\begin{aligned}
 -2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} &= 0 \\
 x_{1/2} &= \frac{+2\frac{2}{9} \pm \sqrt{(-2\frac{2}{9})^2 - 4 \cdot (-2\frac{2}{9}) \cdot 4\frac{4}{9}}}{2 \cdot (-2\frac{2}{9})} \\
 x_{1/2} &= \frac{+2\frac{2}{9} \pm \sqrt{44\frac{4}{9}}}{-4\frac{4}{9}} \\
 x_{1/2} &= \frac{2\frac{2}{9} \pm 6\frac{2}{3}}{-4\frac{4}{9}} \\
 x_1 &= \frac{2\frac{2}{9} + 6\frac{2}{3}}{-4\frac{4}{9}} & x_2 &= \frac{2\frac{2}{9} - 6\frac{2}{3}}{-4\frac{4}{9}} \\
 x_1 &= -2 & x_2 &= 1
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 -2\frac{2}{9}x^2 - 2\frac{2}{9}x + 4\frac{4}{9} &= 0 & /: -2\frac{2}{9} \\
 x^2 + 1x - 2 &= 0 \\
 x_{1/2} &= -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 - (-2)} \\
 x_{1/2} &= -\frac{1}{2} \pm \sqrt{2\frac{1}{4}} \\
 x_{1/2} &= -\frac{1}{2} \pm 1\frac{1}{2} \\
 x_1 &= 1 & x_2 &= -2
 \end{aligned}$$

Aufgabe (39)

x-Ausklammern

$$\begin{aligned}
 -\frac{7}{9}x^2 + 4\frac{2}{3}x &= 0 \\
 x(-\frac{7}{9}x + 4\frac{2}{3}) &= 0 \\
 -\frac{7}{9}x + 4\frac{2}{3} &= 0 & / -4\frac{2}{3} \\
 -\frac{7}{9}x &= -4\frac{2}{3} & /: (-\frac{7}{9}) \\
 x &= \frac{-4\frac{2}{3}}{-\frac{7}{9}} \\
 x_1 &= 0 \\
 x_2 &= 6
 \end{aligned}$$

a-b-c Formel

$$\begin{aligned}
 -\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 &= 0 \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm \sqrt{(4\frac{2}{3})^2 - 4 \cdot (-\frac{7}{9}) \cdot 0}}{2 \cdot (-\frac{7}{9})} \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm \sqrt{21\frac{7}{9}}}{-1\frac{5}{9}} \\
 x_{1/2} &= \frac{-4\frac{2}{3} \pm 4\frac{2}{3}}{-1\frac{5}{9}} \\
 x_1 &= \frac{-4\frac{2}{3} + 4\frac{2}{3}}{-1\frac{5}{9}} & x_2 &= \frac{-4\frac{2}{3} - 4\frac{2}{3}}{-1\frac{5}{9}} \\
 x_1 &= 0 & x_2 &= 6
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 -\frac{7}{9}x^2 + 4\frac{2}{3}x + 0 &= 0 & /: -\frac{7}{9} \\
 x^2 - 6x + 0 &= 0 \\
 x_{1/2} &= -\frac{-6}{2} \pm \sqrt{\left(\frac{-6}{2}\right)^2 - 0} \\
 x_{1/2} &= 3 \pm \sqrt{9} \\
 x_{1/2} &= 3 \pm 3 \\
 x_1 &= 6 & x_2 &= 0
 \end{aligned}$$

Aufgabe (40)

a-b-c Formel

$$\begin{aligned}
 \frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} &= 0 \\
 x_{1/2} &= \frac{+\frac{6}{49} \pm \sqrt{(-\frac{6}{49})^2 - 4 \cdot \frac{3}{49} \cdot (-2\frac{46}{49})}}{2 \cdot \frac{3}{49}} \\
 x_{1/2} &= \frac{+\frac{6}{49} \pm \sqrt{\frac{36}{49}}}{\frac{6}{49}} \\
 x_{1/2} &= \frac{\frac{6}{49} \pm \frac{6}{7}}{\frac{6}{49}} \\
 x_1 &= \frac{\frac{6}{49} + \frac{6}{7}}{\frac{6}{49}} & x_2 &= \frac{\frac{6}{49} - \frac{6}{7}}{\frac{6}{49}} \\
 x_1 &= 8 & x_2 &= -6
 \end{aligned}$$

p-q Formel

$$\begin{aligned}
 \frac{3}{49}x^2 - \frac{6}{49}x - 2\frac{46}{49} &= 0 & /: \frac{3}{49} \\
 x^2 - 2x - 48 &= 0 \\
 x_{1/2} &= -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-48)} \\
 x_{1/2} &= 1 \pm \sqrt{49} \\
 x_{1/2} &= 1 \pm 7 \\
 x_1 &= 8 & x_2 &= -6
 \end{aligned}$$

Aufgabe (41)

<p>x-Ausklammern</p> $\frac{5}{9}x^2 - 3\frac{1}{3}x = 0$ $x(\frac{5}{9}x - 3\frac{1}{3}) = 0$ $\frac{5}{9}x - 3\frac{1}{3} = 0 \quad / + 3\frac{1}{3}$ $\frac{5}{9}x = 3\frac{1}{3} \quad / : \frac{5}{9}$ $x = \frac{3\frac{1}{3}}{\frac{5}{9}}$ $x_1 = 0$ $x_2 = 6$	<p>a-b-c Formel</p> $\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{(-3\frac{1}{3})^2 - 4 \cdot \frac{5}{9} \cdot 0}}{2 \cdot \frac{5}{9}}$ $x_{1/2} = \frac{+3\frac{1}{3} \pm \sqrt{11\frac{1}{9}}}{1\frac{1}{9}}$ $x_{1/2} = \frac{3\frac{1}{3} \pm 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = \frac{3\frac{1}{3} + 3\frac{1}{3}}{1\frac{1}{9}} \quad x_2 = \frac{3\frac{1}{3} - 3\frac{1}{3}}{1\frac{1}{9}}$ $x_1 = 6 \quad x_2 = 0$	<p>p-q Formel</p> $\frac{5}{9}x^2 - 3\frac{1}{3}x + 0 = 0 \quad / : \frac{5}{9}$ $x^2 - 6x + 0 = 0$ $x_{1/2} = -\frac{-6}{2} \pm \sqrt{\left(\frac{(-6)}{2}\right)^2 - 0}$ $x_{1/2} = 3 \pm \sqrt{9}$ $x_{1/2} = 3 \pm 3$ $x_1 = 6 \quad x_2 = 0$
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Aufgabe (42)

<p>a-b-c Formel</p> $-1\frac{1}{4}x^2 - 10x - 15 = 0$ $x_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot (-1\frac{1}{4}) \cdot (-15)}}{2 \cdot (-1\frac{1}{4})}$ $x_{1/2} = \frac{+10 \pm \sqrt{25}}{-2\frac{1}{2}}$ $x_{1/2} = \frac{10 \pm 5}{-2\frac{1}{2}}$ $x_1 = \frac{10 + 5}{-2\frac{1}{2}} \quad x_2 = \frac{10 - 5}{-2\frac{1}{2}}$ $x_1 = -6 \quad x_2 = -2$	<p>p-q Formel</p> $-1\frac{1}{4}x^2 - 10x - 15 = 0 \quad / : -1\frac{1}{4}$ $x^2 + 8x + 12 = 0$ $x_{1/2} = -\frac{8}{2} \pm \sqrt{\left(\frac{8}{2}\right)^2 - 12}$ $x_{1/2} = -4 \pm \sqrt{4}$ $x_{1/2} = -4 \pm 2$ $x_1 = -2 \quad x_2 = -6$
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Aufgabe (43)

<p>x-Ausklammern</p> $4x^2 - 8x = 0$ $x(4x - 8) = 0$ $4x - 8 = 0 \quad / + 8$ $4x = 8 \quad / : 4$ $x = \frac{8}{4}$ $x_1 = 0$ $x_2 = 2$	<p>a-b-c Formel</p> $4x^2 - 8x + 0 = 0$ $x_{1/2} = \frac{+8 \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 0}}{2 \cdot 4}$ $x_{1/2} = \frac{+8 \pm \sqrt{64}}{8}$ $x_{1/2} = \frac{8 \pm 8}{8}$ $x_1 = \frac{8 + 8}{8} \quad x_2 = \frac{8 - 8}{8}$ $x_1 = 2 \quad x_2 = 0$	<p>p-q Formel</p> $4x^2 - 8x + 0 = 0 \quad / : 4$ $x^2 - 2x + 0 = 0$ $x_{1/2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{(-2)}{2}\right)^2 - 0}$ $x_{1/2} = 1 \pm \sqrt{1}$ $x_{1/2} = 1 \pm 1$ $x_1 = 2 \quad x_2 = 0$
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Aufgabe (44)

a-b-c Formel

$$-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0$$

$$x_{1/2} = \frac{-2\frac{22}{49} \pm \sqrt{\left(2\frac{22}{49}\right)^2 - 4 \cdot \left(-\frac{24}{49}\right) \cdot 2\frac{46}{49}}}{2 \cdot \left(-\frac{24}{49}\right)}$$

$$x_{1/2} = \frac{-2\frac{22}{49} \pm \sqrt{11\frac{37}{49}}}{-\frac{48}{49}}$$

$$x_{1/2} = \frac{-2\frac{22}{49} \pm 3\frac{3}{7}}{-\frac{48}{49}}$$

$$x_1 = \frac{-2\frac{22}{49} + 3\frac{3}{7}}{-\frac{48}{49}} \quad x_2 = \frac{-2\frac{22}{49} - 3\frac{3}{7}}{-\frac{48}{49}}$$

$$x_1 = -1 \quad x_2 = 6$$

p-q Formel

$$-\frac{24}{49}x^2 + 2\frac{22}{49}x + 2\frac{46}{49} = 0 \quad /: -\frac{24}{49}$$

$$x^2 - 5x - 6 = 0$$

$$x_{1/2} = -\frac{-5}{2} \pm \sqrt{\left(\frac{-5}{2}\right)^2 - (-6)}$$

$$x_{1/2} = 2\frac{1}{2} \pm \sqrt{12\frac{1}{4}}$$

$$x_{1/2} = 2\frac{1}{2} \pm 3\frac{1}{2}$$

$$x_1 = 6 \quad x_2 = -1$$

Aufgabe (45)

x-Ausklammern

$$\frac{8}{27}x^2 + 2\frac{2}{3}x = 0$$

$$x\left(\frac{8}{27}x + 2\frac{2}{3}\right) = 0$$

$$\frac{8}{27}x + 2\frac{2}{3} = 0 \quad / - 2\frac{2}{3}$$

$$\frac{8}{27}x = -2\frac{2}{3} \quad /: \frac{8}{27}$$

$$x = \frac{-2\frac{2}{3}}{\frac{8}{27}}$$

$$x_1 = 0$$

$$x_2 = -9$$

a-b-c Formel

$$\frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0$$

$$x_{1/2} = \frac{-2\frac{2}{3} \pm \sqrt{\left(2\frac{2}{3}\right)^2 - 4 \cdot \frac{8}{27} \cdot 0}}{2 \cdot \frac{8}{27}}$$

$$x_{1/2} = \frac{-2\frac{2}{3} \pm \sqrt{7\frac{1}{9}}}{\frac{16}{27}}$$

$$x_{1/2} = \frac{-2\frac{2}{3} \pm 2\frac{2}{3}}{\frac{16}{27}}$$

$$x_1 = \frac{-2\frac{2}{3} + 2\frac{2}{3}}{\frac{16}{27}} \quad x_2 = \frac{-2\frac{2}{3} - 2\frac{2}{3}}{\frac{16}{27}}$$

$$x_1 = 0 \quad x_2 = -9$$

p-q Formel

$$\frac{8}{27}x^2 + 2\frac{2}{3}x + 0 = 0 \quad /: \frac{8}{27}$$

$$x^2 + 9x + 0 = 0$$

$$x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0}$$

$$x_{1/2} = -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$$

$$x_{1/2} = -4\frac{1}{2} \pm 4\frac{1}{2}$$

$$x_1 = 0 \quad x_2 = -9$$

Aufgabe (46)

x-Ausklammern

$$\frac{20}{81}x^2 + 2\frac{2}{9}x = 0$$

$$x\left(\frac{20}{81}x + 2\frac{2}{9}\right) = 0$$

$$\frac{20}{81}x + 2\frac{2}{9} = 0 \quad / - 2\frac{2}{9}$$

$$\frac{20}{81}x = -2\frac{2}{9} \quad /: \frac{20}{81}$$

$$x = \frac{-2\frac{2}{9}}{\frac{20}{81}}$$

$$x_1 = 0$$

$$x_2 = -9$$

a-b-c Formel

$$\frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0$$

$$x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{\left(2\frac{2}{9}\right)^2 - 4 \cdot \frac{20}{81} \cdot 0}}{2 \cdot \frac{20}{81}}$$

$$x_{1/2} = \frac{-2\frac{2}{9} \pm \sqrt{4\frac{76}{81}}}{\frac{40}{81}}$$

$$x_{1/2} = \frac{-2\frac{2}{9} \pm 2\frac{2}{9}}{\frac{40}{81}}$$

$$x_1 = \frac{-2\frac{2}{9} + 2\frac{2}{9}}{\frac{40}{81}} \quad x_2 = \frac{-2\frac{2}{9} - 2\frac{2}{9}}{\frac{40}{81}}$$

$$x_1 = 0 \quad x_2 = -9$$

p-q Formel

$$\frac{20}{81}x^2 + 2\frac{2}{9}x + 0 = 0 \quad /: \frac{20}{81}$$

$$x^2 + 9x + 0 = 0$$

$$x_{1/2} = -\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 0}$$

$$x_{1/2} = -4\frac{1}{2} \pm \sqrt{20\frac{1}{4}}$$

$$x_{1/2} = -4\frac{1}{2} \pm 4\frac{1}{2}$$

$$x_1 = 0 \quad x_2 = -9$$

Aufgabe (47)

a-b-c Formel

$$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{\left(10\frac{2}{25}\right)^2 - 4 \cdot 1\frac{11}{25} \cdot 8\frac{16}{25}}}{2 \cdot 1\frac{11}{25}}$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm \sqrt{51\frac{21}{25}}}{2\frac{22}{25}}$$

$$x_{1/2} = \frac{-10\frac{2}{25} \pm 7\frac{1}{5}}{2\frac{22}{25}}$$

$$x_1 = \frac{-10\frac{2}{25} + 7\frac{1}{5}}{2\frac{22}{25}} \quad x_2 = \frac{-10\frac{2}{25} - 7\frac{1}{5}}{2\frac{22}{25}}$$

$$x_1 = -1 \quad x_2 = -6$$

p-q Formel

$$1\frac{11}{25}x^2 + 10\frac{2}{25}x + 8\frac{16}{25} = 0 \quad / : 1\frac{11}{25}$$

$$x^2 + 7x + 6 = 0$$

$$x_{1/2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 6}$$

$$x_{1/2} = -3\frac{1}{2} \pm \sqrt{6\frac{1}{4}}$$

$$x_{1/2} = -3\frac{1}{2} \pm 2\frac{1}{2}$$

$$x_1 = -1 \quad x_2 = -6$$

3 Kubische Gleichungen

Umformen: $ax^3 + b = 0$

$$ax^3 + b = 0$$

$$ax^3 + b = 0 \quad / -b$$

$$ax^3 = -b \quad / : a$$

$$x^3 = \frac{-b}{a}$$

$$x = \sqrt[3]{\frac{-b}{a}}$$

$$\frac{-b}{a} > 0 \quad x = \sqrt[3]{\frac{-b}{a}}$$

$$\frac{-b}{a} < 0 \quad x = -\sqrt[3]{\left|\frac{-b}{a}\right|}$$

$$3x^3 + 24 = 0$$

$$3x^3 + 24 = 0 \quad / -24$$

$$3x^3 = -24 \quad / : 3$$

$$x^3 = \frac{-24}{3}$$

$$x = \sqrt[3]{-8}$$

$$x = -2$$

$$-3x^3 + 24 = 0$$

$$-3x^3 + 24 = 0 \quad / -24$$

$$-3x^3 = -24 \quad / : (-3)$$

$$x^3 = \frac{-24}{-3}$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Faktorisieren: $ax^3 + bx = 0$

$$ax^3 + bx = 0$$

$$x(ax^2 + b) = 0$$

$$x_1 = 0 \quad \vee \quad (ax^2 + b) = 0$$

$$-9x^3 + 25x = 0$$

$$x(-9x^2 + 25) = 0$$

$$\Rightarrow x_1 = 0 \quad \vee \quad -9x^2 + 25 = 0$$

$$-9x^2 + 25 = 0 \quad / -25$$

$$-9x^2 = -25 \quad / : (-9)$$

$$x^2 = \frac{-25}{-9}$$

$$x = \pm \sqrt{2\frac{7}{9}}$$

$$x_2 = 1\frac{2}{3} \quad x_3 = -1\frac{2}{3}$$

Faktorisieren: $ax^3 + bx^2 = 0$

$$ax^3 + bx^2 = 0$$

$$x^2(ax + b) = 0$$

$$x_{1/2} = 0 \quad \vee \quad (ax + b) = 0$$

$$-6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$$

$$x^2(-6\frac{3}{4}x - 13\frac{1}{2}) = 0$$

$$\Rightarrow x_{1/2} = 0 \quad \vee \quad -6\frac{3}{4}x - 13\frac{1}{2} = 0$$

$$-6\frac{3}{4}x - 13\frac{1}{2} = 0 \quad / + 13\frac{1}{2}$$

$$-6\frac{3}{4}x = 13\frac{1}{2} \quad / : (-6\frac{3}{4})$$

$$x = \frac{13\frac{1}{2}}{-6\frac{3}{4}}$$

$$x_3 = -2$$

Polynomdivision

$$ax^3 + bx^2 + d = 0$$

$$ax^3 + cx + d = 0$$

$$ax^3 + bx^2 + cx + d = 0$$

• Die ganzzahligen Faktoren von d in die Funktion einsetzen. Wird bei einem Faktor der Funktionswert Null, hat man eine Nullstelle x_0 gefunden.

• Wenn x_0 ein Nullstelle von $f(x)$ ist, so ist $f(x)$ durch $(x - x_0)$ ohne Rest teilbar.

• Mit dem Linearfaktor $(x - x_0)$ wird die Polynomdivision durchführen.

$$(ax^3 + bx^2 + cx + d) : (x - x_0) = fx^2 + dx + e$$

$$f(x) = (ax^3 + bx^2 + cx + d) = (x - x_0) \cdot (fx^2 + dx + e)$$

$$x^3 + 3x^2 - 4 = 0$$

$$x^3 + 3x^2 - 4 = 0$$

$d = 4$ Ganzzahlige Faktoren: $\pm 1, \pm 2, \pm 4$

$$f(1) = 0$$

Nullstelle gefunden: $x_1 = 1$

$$\begin{array}{r} (x^3 + 3x^2 - 4) : (x - 1) = x^2 + 4x + 4 \\ -(x^3 - x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 - 4x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$1x^2 + 4x + 4 = 0$$

$$x_{2/3} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$x_{2/3} = \frac{-4 \pm \sqrt{0}}{2}$$

$$x_{2/3} = \frac{-4 \pm 0}{2}$$

$$x_2 = \frac{-4 + 0}{2} \quad x_3 = \frac{-4 - 0}{2}$$

$$x_2 = -2 \quad x_3 = -2$$

3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ax^3 + bx^2 + cx + d = 0$

Gesucht:

Lösung der Gleichung

$$(1) \quad -2x^3 = 0$$

$$(2) \quad 3x^3 + 24 = 0$$

$$(3) \quad -3x^3 + 24 = 0$$

$$(4) \quad -8x^3 + 27 = 0$$

$$(5) \quad -1x^3 + 4x = 0$$

$$(6) \quad -9x^3 + 25x = 0$$

$$(7) \quad -\frac{1}{4}x^3 + \frac{2}{3}x^2 = 0$$

$$(8) \quad x^3 - 3x^2 = 0$$

$$(9) \quad \frac{1}{2}x^3 + 4 = 0$$

$$(10) \quad -\frac{1}{6}x^3 + 2x = 0$$

$$(11) \quad \frac{1}{2}x^3 - 3x^2 + 5x = 0$$

$$(12) \quad -1x^3 + 3x + 2 = 0$$

$$(13) \quad -1x^3 + 3x^2 - 4 = 0$$

$$(14) \quad 4x^3 + 5x^2 - 6x = 0$$

$$(15) \quad -\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

$$(16) \quad x^3 - 4x^2 + 3x = 0$$

$$(17) \quad -\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

$$(18) \quad \frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

$$(19) \quad -5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

$$(20) \quad -6\frac{3}{4}x^3 - 13\frac{1}{2}x^2 = 0$$

$$(21) \quad \frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

$$(22) \quad -\frac{27}{28}x^3 - \frac{27}{28}x^2 + 5\frac{11}{14}x = 0$$

$$(23) \quad x^3 + 3x^2 - 4 = 0$$

$$(24) \quad -5\frac{1}{16}x^3 + 10\frac{1}{8}x^2 = 0$$

$$(25) \quad \frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

$$(26) \quad -2x^3 + 12x^2 - 18x = 0$$

$$(27) \quad 40\frac{1}{2}x^3 + 81x^2 + 40\frac{1}{2}x = 0$$

$$(28) \quad 54x^3 - 270x^2 + 432x - 216 = 0$$

$$(29) \quad 1\frac{19}{35}x^3 - 10\frac{4}{5}x^2 + 18\frac{18}{35}x = 0$$

$$(30) \quad -2x^3 + 6x^2 = 0$$

$$(31) \quad -2x^3 + 6x^2 = 0$$

$$(32) \quad 5\frac{2}{5}x^3 + 27x^2 + 32\frac{2}{5}x = 0$$

$$(33) \quad \frac{1}{3}x^3 - 1x^2 - 1\frac{1}{3}x = 0$$

$$(34) \quad -0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

$$(35) \quad -\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

$$(36) \quad -13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

$$(37) \quad x^3 - 2x^2 + 2x - 1 = 0$$

3.2 Lösungen

Aufgabe (1)

$$x^3 = 0 \Rightarrow x = 0$$

$$x_1 = 0; \quad \underline{\text{3-fache Nullstelle}}$$

Aufgabe (2)

$$3x^3 + 24 = 0$$

$$3x^3 + 24 = 0 \quad / -24$$

$$3x^3 = -24 \quad / :3$$

$$x^3 = \frac{-24}{3}$$

$$x = \sqrt[3]{-8}$$

$$x = -2$$

Polynomdivision: (-2)

$$\begin{array}{r} (3x^3 + 24) : (x+2) = 3x^2 - 6x + 12 \\ -(3x^3 + 6x^2) \\ \hline -6x^2 + 24 \\ -(-6x^2 - 12x) \\ \hline 12x + 24 \\ -(12x + 24) \\ \hline 0 \end{array}$$

$$3x^2 - 6x + 12 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot 3 \cdot 12}}{2 \cdot 3}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{-108}}{6}$$

Diskriminante negativ keine Lösung

$$x_1 = -2; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (3)

$$-3x^3 + 24 = 0$$

$$-3x^3 + 24 = 0 \quad / -24$$

$$-3x^3 = -24 \quad / :(-3)$$

$$x^3 = \frac{-24}{-3}$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Polynomdivision: 2

$$\begin{array}{r} (-3x^3 + 24) : (x-2) = -3x^2 - 6x - 12 \\ -(-3x^3 + 6x^2) \\ \hline -6x^2 + 24 \\ -(-6x^2 + 12x) \\ \hline -12x + 24 \\ -(-12x + 24) \\ \hline 0 \end{array}$$

$$-3x^2 - 6x - 12 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot (-3) \cdot (-12)}}{2 \cdot (-3)}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{-108}}{-6}$$

Diskriminante negativ keine Lösung

$$x_1 = 2; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (4)

$$\begin{aligned} -8x^3 + 27 &= 0 \\ -8x^3 + 27 &= 0 \quad / -27 \\ -8x^3 &= -27 \quad / : (-8) \end{aligned}$$

$$x^3 = \frac{-27}{-8}$$

$$x = \sqrt[3]{3\frac{3}{8}}$$

$$x = 1\frac{1}{2}$$

Polynomdivision: $1\frac{1}{2}$

$$\begin{array}{r} (-8x^3 \quad \quad \quad +27) : (x - 1\frac{1}{2}) = -8x^2 - 12x - 18 \\ \underline{-(-8x^3 \quad +12x^2)} \\ -12x^2 \\ \underline{ -(-12x^2 \quad +18x)} \\ -18x \\ \underline{ -(-18x \quad +27)} \\ 0 \end{array}$$

$$\begin{aligned} -8x^2 - 12x - 18 &= 0 \\ x_{1/2} &= \frac{+12 \pm \sqrt{(-12)^2 - 4 \cdot (-8) \cdot (-18)}}{2 \cdot (-8)} \end{aligned}$$

$$x_{1/2} = \frac{+12 \pm \sqrt{-432}}{-16}$$

Diskriminante negativ keine Lösung

$$x_1 = 1\frac{1}{2}; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (5)

$$\begin{aligned} x(-1x^2 + 4) = 0 &\Rightarrow x = 0 \quad \vee \quad -1x^2 + 4 = 0 \\ -1x^2 + 4 &= 0 \quad / -4 \\ -1x^2 &= -4 \quad / : (-1) \end{aligned}$$

$$x^2 = \frac{-4}{-1}$$

$$x = \pm\sqrt{4}$$

$$x_1 = 2 \quad x_2 = -2$$

$$x_1 = -2; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_3 = 2; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (6)

$$\begin{aligned} x(-9x^2 + 25) = 0 &\Rightarrow x = 0 \quad \vee \quad -9x^2 + 25 = 0 \\ -9x^2 + 25 &= 0 \quad / -25 \end{aligned}$$

$$\begin{aligned}
 -9x^2 &= -25 & / : (-9) \\
 x^2 &= \frac{-25}{-9} \\
 x &= \pm \sqrt{2\frac{7}{9}} \\
 x_1 &= 1\frac{2}{3} & x_2 &= -1\frac{2}{3} \\
 x_1 &= -1\frac{2}{3}; & \text{1-fache Nullstelle} \\
 \hline
 x_2 &= 0; & \text{1-fache Nullstelle} \\
 \hline
 x_3 &= 1\frac{2}{3}; & \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (7)

$$\begin{aligned}
 x^2(-\frac{1}{4}x + \frac{2}{3}) &= 0 \Rightarrow x = 0 \quad \vee \quad -\frac{1}{4}x + \frac{2}{3} = 0 \\
 -\frac{1}{4}x + \frac{2}{3} &= 0 & / -\frac{2}{3} \\
 -\frac{1}{4}x &= -\frac{2}{3} & / : (-\frac{1}{4}) \\
 x &= \frac{-\frac{2}{3}}{-\frac{1}{4}} \\
 x &= 2\frac{2}{3} \\
 x_1 &= 0; & \text{2-fache Nullstelle} \\
 \hline
 x_2 &= 2\frac{2}{3}; & \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (8)

$$\begin{aligned}
 x^2(x-3) &= 0 \Rightarrow x = 0 \quad \vee \quad x-3 = 0 \\
 x-3 &= 0 & / +3 \\
 x &= 3 \\
 x_1 &= 0; & \text{2-fache Nullstelle} \\
 \hline
 x_2 &= 3; & \text{1-fache Nullstelle} \\
 \hline
 \end{aligned}$$

Aufgabe (9)

$$\begin{aligned}
 \frac{1}{2}x^3 + 4 &= 0 \\
 \frac{1}{2}x^3 + 4 &= 0 & / -4 \\
 \frac{1}{2}x^3 &= -4 & / : \frac{1}{2} \\
 x^3 &= \frac{-4}{\frac{1}{2}} \\
 x &= \sqrt[3]{-8} \\
 x &= -2 \\
 \text{Polynomdivision: } (-2) & \\
 \begin{array}{r}
 (\frac{1}{2}x^3) : (x+2) = \frac{1}{2}x^2 - 1x + 2 \\
 \underline{-(\frac{1}{2}x^3)} \\
 -1x^2 \\
 \underline{-(-1x^2)} \\
 2x \\
 \underline{-(2x)} \\
 0
 \end{array}
 \end{aligned}$$

$$x_{1/2} = \frac{-1 \pm \sqrt{9}}{-2}$$

$$x_{1/2} = \frac{-1 \pm 3}{-2}$$

$$x_1 = \frac{-1+3}{-2} \quad x_2 = \frac{-1-3}{-2}$$

$$x_1 = -1 \quad x_2 = 2$$

$x_1 = -1$; 2-fache Nullstelle
 $x_2 = 2$; 1-fache Nullstelle

Aufgabe (13)

$$-1x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-1x^3 + 3x^2 - 4) : (x+1) = -1x^2 + 4x - 4 \\ -(-1x^3 - 1x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 + 4x) \\ \hline -4x - 4 \\ -(-4x - 4) \\ \hline 0 \end{array}$$

$$-1x^2 + 4x - 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{-2}$$

$$x_{1/2} = \frac{-4 \pm 0}{-2}$$

$$x_1 = \frac{-4+0}{-2} \quad x_2 = \frac{-4-0}{-2}$$

$$x_1 = 2 \quad x_2 = 2$$

$x_1 = -1$; 1-fache Nullstelle
 $x_2 = 2$; 2-fache Nullstelle

Aufgabe (14)

$$x(4x^2 + 5x - 6) = 0 \Rightarrow x = 0 \quad \vee \quad 4x^2 + 5x - 6 = 0$$

$$4x^2 + 5x - 6 = 0$$

$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$

$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$

$$x_{1/2} = \frac{-5 \pm 11}{8}$$

$$x_1 = \frac{-5+11}{8} \quad x_2 = \frac{-5-11}{8}$$

$$x_1 = \frac{3}{4} \quad x_2 = -2$$

$x_1 = -2$; 1-fache Nullstelle
 $x_2 = 0$; 1-fache Nullstelle
 $x_3 = \frac{3}{4}$; 1-fache Nullstelle

Aufgabe (15)

$$-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6 = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-\frac{1}{2}x^3 - \frac{1}{2}x^2 + 4x + 6) : (x + 2) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3 \\ -(-\frac{1}{2}x^3 - 1x^2) \\ \hline \frac{1}{2}x^2 + 4x + 6 \\ -(\frac{1}{2}x^2 + x) \\ \hline 3x + 6 \\ -(3x + 6) \\ \hline 0 \end{array}$$

$$-\frac{1}{2}x^2 + \frac{1}{2}x + 3 = 0$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{(\frac{1}{2})^2 - 4 \cdot (-\frac{1}{2}) \cdot 3}}{2 \cdot (-\frac{1}{2})}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm \sqrt{6\frac{1}{4}}}{-1}$$

$$x_{1/2} = \frac{-\frac{1}{2} \pm 2\frac{1}{2}}{-1}$$

$$x_1 = \frac{-\frac{1}{2} + 2\frac{1}{2}}{-1} \quad x_2 = \frac{-\frac{1}{2} - 2\frac{1}{2}}{-1}$$

$$x_1 = -2 \quad x_2 = 3$$

$$x_1 = -2; \quad \underline{\text{2-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (16)

$$x(x^2 - 4x + 3) = 0 \Rightarrow x = 0 \quad \vee \quad x^2 - 4x + 3 = 0$$

$$1x^2 - 4x + 3 = 0$$

$$x_{1/2} = \frac{+4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+4 \pm \sqrt{4}}{2}$$

$$x_{1/2} = \frac{4 \pm 2}{2}$$

$$x_1 = \frac{4 + 2}{2} \quad x_2 = \frac{4 - 2}{2}$$

$$x_1 = 3 \quad x_2 = 1$$

$$x_1 = 0; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_2 = 1; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_3 = 3; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (17)

$$-\frac{27}{55}x^3 - \frac{54}{55}x^2 + 5\frac{2}{5}x + 5\frac{49}{55} = 0$$

Numerische Suche :

$$\underline{x_1 = -4; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_2 = -1; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_3 = 3; \quad 1\text{-fache Nullstelle}$$

Aufgabe (18)

$$\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5} = 0$$

Nullstelle für Polynomdivision erraten: -3

$$\begin{array}{r} (\frac{1}{10}x^3 + \frac{3}{10}x^2 - 1\frac{3}{5}x - 4\frac{4}{5}) : (x + 3) = \frac{1}{10}x^2 - 0x - 1\frac{3}{5} \\ -(\frac{1}{10}x^3 + \frac{3}{10}x^2) \\ \hline -0x^2 - 1\frac{3}{5}x - 4\frac{4}{5} \\ -(-0x^2 - 0x) \\ \hline -1\frac{3}{5}x - 4\frac{4}{5} \\ -(-1\frac{3}{5}x - 4\frac{4}{5}) \\ \hline 0 \end{array}$$

$$\frac{1}{10}x^2 - 0x - 1\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+0 \pm \sqrt{(0)^2 - 4 \cdot \frac{1}{10} \cdot (-1\frac{3}{5})}}{2 \cdot \frac{1}{10}}$$

$$x_{1/2} = \frac{+0 \pm \sqrt{\frac{16}{25}}}{\frac{1}{5}}$$

$$x_{1/2} = \frac{0 \pm \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = \frac{0 + \frac{4}{5}}{\frac{1}{5}} \quad x_2 = \frac{0 - \frac{4}{5}}{\frac{1}{5}}$$

$$x_1 = 4 \quad x_2 = -4$$

$$\underline{x_1 = -4; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_2 = -3; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_3 = 4; \quad 1\text{-fache Nullstelle}$$

Aufgabe (19)

$$-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5} = 0$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r} (-5\frac{2}{5}x^3 - 37\frac{4}{5}x^2 - 75\frac{3}{5}x - 43\frac{1}{5}) : (x + 2) = -5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} \\ -(-5\frac{2}{5}x^3 - 10\frac{4}{5}x^2) \\ \hline -27x^2 - 75\frac{3}{5}x - 43\frac{1}{5} \\ -(-27x^2 - 54x) \\ \hline -21\frac{3}{5}x - 43\frac{1}{5} \\ -(-21\frac{3}{5}x - 43\frac{1}{5}) \\ \hline 0 \end{array}$$

$$-5\frac{2}{5}x^2 - 27x - 21\frac{3}{5} = 0$$

$$x_{1/2} = \frac{+27 \pm \sqrt{(-27)^2 - 4 \cdot (-5\frac{2}{5}) \cdot (-21\frac{3}{5})}}{2 \cdot (-5\frac{2}{5})}$$

$$x_{1/2} = \frac{+27 \pm \sqrt{262\frac{11}{25}}}{-10\frac{4}{5}}$$

$$x_{1/2} = \frac{27 \pm 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = \frac{27 + 16\frac{1}{5}}{-10\frac{4}{5}} \quad x_2 = \frac{27 - 16\frac{1}{5}}{-10\frac{4}{5}}$$

$$x_1 = -4 \quad x_2 = -1$$

$$x_1 = -4; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

$$x_3 = -1; \quad \text{1-fache Nullstelle}$$

Aufgabe (20)

$$x^2(-6\frac{3}{4}x - 13\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \vee \quad -6\frac{3}{4}x - 13\frac{1}{2} = 0$$

$$-6\frac{3}{4}x - 13\frac{1}{2} = 0 \quad / + 13\frac{1}{2}$$

$$-6\frac{3}{4}x = 13\frac{1}{2} \quad / : (-6\frac{3}{4})$$

$$x = \frac{13\frac{1}{2}}{-6\frac{3}{4}}$$

$$x = -2$$

$$x_1 = -2; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{2-fache Nullstelle}$$

Aufgabe (21)

$$\frac{2}{3}x^3 + 2x^2 - 2\frac{2}{3}x - 8 = 0$$

Numerische Suche :

$$x_1 = -3; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

$$x_3 = 2; \quad \text{1-fache Nullstelle}$$

Aufgabe (22)

$$x(-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14}) = 0 \Rightarrow x = 0 \quad \vee \quad -\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$-\frac{27}{28}x^2 - \frac{27}{28}x + 5\frac{11}{14} = 0$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{(-\frac{27}{28})^2 - 4 \cdot (-\frac{27}{28}) \cdot 5\frac{11}{14}}}{2 \cdot (-\frac{27}{28})}$$

$$x_{1/2} = \frac{+\frac{27}{28} \pm \sqrt{23,2}}{-1\frac{13}{14}}$$

$$x_{1/2} = \frac{\frac{27}{28} \pm 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = \frac{\frac{27}{28} + 4\frac{23}{28}}{-1\frac{13}{14}} \quad x_2 = \frac{\frac{27}{28} - 4\frac{23}{28}}{-1\frac{13}{14}}$$

$$x_1 = -3 \quad x_2 = 2$$

$$x_1 = -3; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{1-fache Nullstelle}$$

$$x_3 = 2; \quad \text{1-fache Nullstelle}$$

Aufgabe (23)

$$x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^3 + 3x^2 - 4) : (x - 1) = x^2 + 4x + 4 \\ -(x^3 - 1x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 - 4x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$\begin{aligned} 1x^2 + 4x + 4 &= 0 \\ x_{1/2} &= \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} \\ x_{1/2} &= \frac{-4 \pm \sqrt{0}}{2} \\ x_{1/2} &= \frac{-4 \pm 0}{2} \\ x_1 &= \frac{-4 + 0}{2} & x_2 &= \frac{-4 - 0}{2} \\ x_1 &= -2 & x_2 &= -2 \\ \underline{x_1 = -2; \quad 2\text{-fache Nullstelle}} \\ \underline{x_2 = 1; \quad 1\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (24)

$$\begin{aligned} x^2(-5\frac{1}{16}x + 10\frac{1}{8}) &= 0 \Rightarrow x = 0 \quad \vee \quad -5\frac{1}{16}x + 10\frac{1}{8} = 0 \\ -5\frac{1}{16}x + 10\frac{1}{8} &= 0 \quad / -10\frac{1}{8} \\ -5\frac{1}{16}x &= -10\frac{1}{8} \quad / : (-5\frac{1}{16}) \\ x &= \frac{-10\frac{1}{8}}{-5\frac{1}{16}} \\ x &= 2 \\ \underline{x_1 = 0; \quad 2\text{-fache Nullstelle}} \\ \underline{x_2 = 2; \quad 1\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (25)

$$\frac{1}{6}x^3 - \frac{1}{2}x^2 - 1\frac{2}{3}x + 4 = 0$$

Numerische Suche :

$$\begin{aligned} \underline{x_1 = -3; \quad 1\text{-fache Nullstelle}} \\ \underline{x_2 = 2; \quad 1\text{-fache Nullstelle}} \\ \underline{x_3 = 4; \quad 1\text{-fache Nullstelle}} \end{aligned}$$

Aufgabe (26)

$$x(-2x^2 + 12x - 18) = 0 \Rightarrow x = 0 \quad \vee \quad -2x^2 + 12x - 18 = 0$$

$$\begin{aligned}
 -2x^2 + 12x - 18 &= 0 \\
 x_{1/2} &= \frac{-12 \pm \sqrt{12^2 - 4 \cdot (-2) \cdot (-18)}}{2 \cdot (-2)} \\
 x_{1/2} &= \frac{-12 \pm \sqrt{0}}{-4} \\
 x_{1/2} &= \frac{-12 \pm 0}{-4} \\
 x_1 &= \frac{-12 + 0}{-4} & x_2 &= \frac{-12 - 0}{-4} \\
 x_1 &= 3 & x_2 &= 3 \\
 x_1 &= 0; & & \text{1-fache Nullstelle} \\
 x_2 &= 3; & & \text{2-fache Nullstelle}
 \end{aligned}$$

Aufgabe (27)

$$x(40\frac{1}{2}x^2 + 81x + 40\frac{1}{2}) = 0 \Rightarrow x = 0 \quad \vee \quad 40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} = 0$$

$$\begin{aligned}
 40\frac{1}{2}x^2 + 81x + 40\frac{1}{2} &= 0 \\
 x_{1/2} &= \frac{-81 \pm \sqrt{81^2 - 4 \cdot 40\frac{1}{2} \cdot 40\frac{1}{2}}}{2 \cdot 40\frac{1}{2}} \\
 x_{1/2} &= \frac{-81 \pm \sqrt{0}}{81} \\
 x_{1/2} &= \frac{-81 \pm 0}{81} \\
 x_1 &= \frac{-81 + 0}{81} & x_2 &= \frac{-81 - 0}{81} \\
 x_1 &= -1 & x_2 &= -1 \\
 x_1 &= -1; & & \text{2-fache Nullstelle} \\
 x_2 &= 0; & & \text{1-fache Nullstelle}
 \end{aligned}$$

Aufgabe (28)

$$54x^3 - 270x^2 + 432x - 216 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r}
 (54x^3 \quad -270x^2 \quad +432x \quad -216) : (x-1) = 54x^2 - 216x + 216 \\
 \underline{-(54x^3 \quad -54x^2)} \\
 \quad \quad -216x^2 \quad +432x \quad -216 \\
 \quad \quad \underline{-(-216x^2 \quad +216x)} \\
 \quad \quad \quad \quad 216x \quad -216 \\
 \quad \quad \quad \quad \underline{-(216x \quad -216)} \\
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\begin{aligned}
 54x^2 - 216x + 216 &= 0 \\
 x_{1/2} &= \frac{+216 \pm \sqrt{(-216)^2 - 4 \cdot 54 \cdot 216}}{2 \cdot 54} \\
 x_{1/2} &= \frac{+216 \pm \sqrt{0}}{108} \\
 x_{1/2} &= \frac{216 \pm 0}{108} \\
 x_1 &= \frac{216 + 0}{108} & x_2 &= \frac{216 - 0}{108} \\
 x_1 &= 2 & x_2 &= 2
 \end{aligned}$$

$$\underline{x_1 = 1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 2; \quad 2\text{-fache Nullstelle}}$$

Aufgabe (29)

$$x(1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35}) = 0 \Rightarrow x = 0 \quad \vee \quad 1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$1\frac{19}{35}x^2 - 10\frac{4}{5}x + 18\frac{18}{35} = 0$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{(-10\frac{4}{5})^2 - 4 \cdot 1\frac{19}{35} \cdot 18\frac{18}{35}}}{2 \cdot 1\frac{19}{35}}$$

$$x_{1/2} = \frac{+10\frac{4}{5} \pm \sqrt{2 \cdot 38}}{3\frac{3}{35}}$$

$$x_{1/2} = \frac{10\frac{4}{5} \pm 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = \frac{10\frac{4}{5} + 1\frac{19}{35}}{3\frac{3}{35}} \quad x_2 = \frac{10\frac{4}{5} - 1\frac{19}{35}}{3\frac{3}{35}}$$

$$x_1 = 4 \quad x_2 = 3$$

$$\underline{x_1 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 4; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (30)

$$x^2(-2x + 6) = 0 \Rightarrow x = 0 \quad \vee \quad -2x + 6 = 0$$

$$-2x + 6 = 0 \quad / -6$$

$$-2x = -6 \quad / : (-2)$$

$$x = \frac{-6}{-2}$$

$$x = 3$$

$$\underline{x_1 = 0; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (31)

$$x^2(-2x + 6) = 0 \Rightarrow x = 0 \quad \vee \quad -2x + 6 = 0$$

$$-2x + 6 = 0 \quad / -6$$

$$-2x = -6 \quad / : (-2)$$

$$x = \frac{-6}{-2}$$

$$x = 3$$

$$\underline{x_1 = 0; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 3; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (32)

$$x(5\frac{2}{5}x^2 + 27x + 32\frac{2}{5}) = 0 \Rightarrow x = 0 \quad \vee \quad 5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} = 0$$

$$5\frac{2}{5}x^2 + 27x + 32\frac{2}{5} = 0$$

$$x_{1/2} = \frac{-27 \pm \sqrt{27^2 - 4 \cdot 5\frac{2}{5} \cdot 32\frac{2}{5}}}{2 \cdot 5\frac{2}{5}}$$

$$x_{1/2} = \frac{-27 \pm \sqrt{29\frac{4}{25}}}{10\frac{4}{5}}$$

$$x_{1/2} = \frac{-27 \pm 5\frac{2}{5}}{10\frac{4}{5}}$$

$$x_1 = \frac{-27 + 5\frac{2}{5}}{10\frac{4}{5}} \quad x_2 = \frac{-27 - 5\frac{2}{5}}{10\frac{4}{5}}$$

$$x_1 = -2 \quad x_2 = -3$$

$$x_1 = -3; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

$$x_3 = 0; \quad \text{1-fache Nullstelle}$$

Aufgabe (33)

$$x(\frac{1}{3}x^2 - 1x - 1\frac{1}{3}) = 0 \Rightarrow x = 0 \quad \vee \quad \frac{1}{3}x^2 - 1x - 1\frac{1}{3} = 0$$

$$\frac{1}{3}x^2 - 1x - 1\frac{1}{3} = 0$$

$$x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot \frac{1}{3} \cdot (-1\frac{1}{3})}}{2 \cdot \frac{1}{3}}$$

$$x_{1/2} = \frac{+1 \pm \sqrt{2\frac{7}{9}}}{\frac{2}{3}}$$

$$x_{1/2} = \frac{1 \pm 1\frac{2}{3}}{\frac{2}{3}}$$

$$x_1 = \frac{1 + 1\frac{2}{3}}{\frac{2}{3}} \quad x_2 = \frac{1 - 1\frac{2}{3}}{\frac{2}{3}}$$

$$x_1 = 4 \quad x_2 = -1$$

$$x_1 = -1; \quad \text{1-fache Nullstelle}$$

$$x_2 = 0; \quad \text{1-fache Nullstelle}$$

$$x_3 = 4; \quad \text{1-fache Nullstelle}$$

Aufgabe (34)

$$-0,096x^3 - 0,193x^2 + 1\frac{19}{35}x + 3\frac{3}{35} = 0$$

NumerischeSuche :

$$x_1 = -4,02; \quad \text{1-fache Nullstelle}$$

$$x_2 = -2; \quad \text{1-fache Nullstelle}$$

$$x_3 = 4,01; \quad \text{1-fache Nullstelle}$$

Aufgabe (35)

$$-\frac{27}{56}x^3 - \frac{27}{28}x^2 + 2\frac{23}{56}x + 2\frac{25}{28} = 0$$

NumerischeSuche :

$$\underline{x_1 = -3; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_2 = -1; \quad 1\text{-fache Nullstelle}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}$$

Aufgabe (36)

$$-13\frac{1}{2}x^3 - 67\frac{1}{2}x^2 - 108x - 54 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54) : (x+1) = -13\frac{1}{2}x^2 - 54x - 54 \\ -(-13\frac{1}{2}x^3 \quad -13\frac{1}{2}x^2) \\ \hline \phantom{(-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54)} -54x^2 \quad -108x \quad -54 \\ \phantom{(-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54)} -(-54x^2 \quad -54x) \\ \hline \phantom{(-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54)} -54x \quad -54 \\ \phantom{(-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54)} -(-54x \quad -54) \\ \hline \phantom{(-13\frac{1}{2}x^3 \quad -67\frac{1}{2}x^2 \quad -108x \quad -54)} 0 \end{array}$$

$$-13\frac{1}{2}x^2 - 54x - 54 = 0$$

$$x_{1/2} = \frac{+54 \pm \sqrt{(-54)^2 - 4 \cdot (-13\frac{1}{2}) \cdot (-54)}}{2 \cdot (-13\frac{1}{2})}$$

$$x_{1/2} = \frac{+54 \pm \sqrt{0}}{-27}$$

$$x_{1/2} = \frac{54 \pm 0}{-27}$$

$$x_1 = \frac{54+0}{-27} \quad x_2 = \frac{54-0}{-27}$$

$$x_1 = -2 \quad x_2 = -2$$

$$\underline{x_1 = -2; \quad 2\text{-fache Nullstelle}$$

$$\underline{x_2 = -1; \quad 1\text{-fache Nullstelle}$$

Aufgabe (37)

$$x^3 - 2x^2 + 2x - 1 = 0$$

Nullstelle für Polynomdivision erraten: 1

$$\begin{array}{r} (x^3 \quad -2x^2 \quad +2x \quad -1) : (x-1) = x^2 - 1x + 1 \\ -(x^3 \quad -1x^2) \\ \hline -1x^2 \quad +2x \quad -1 \\ -(-1x^2 \quad +x) \\ \hline x \quad -1 \\ -(x \quad -1) \\ \hline 0 \end{array}$$

$$1x^2 - 1x + 1 = 0$$

$$x_{1/2} = \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+1 \pm \sqrt{-3}}{2}$$

Diskriminante negativ keine Lösung

4 Gleichungen höheren Grades

Gerader Exponent: $ax^n + c = 0$

$$ax^n + c = 0 \quad / -c$$

$$ax^n = -c \quad / : a$$

$$x_{1/2} = \pm \sqrt[n]{\frac{-c}{a}}$$

Diskriminante:

$$D = \frac{-c}{a}$$

$D = 0$ eine Lösung

$D > 0$ zwei Lösungen

$D < 0$ keine Lösung

$$-2x^4 + 162 = 0 \quad / -162$$

$$-2x^4 = -162 \quad / : (-2)$$

$$x^4 = \frac{-162}{-2}$$

$$x = \pm \sqrt[4]{81}$$

$$x_1 = 3 \quad x_2 = -3$$

Ungerader Exponent: $ax^n + c = 0$

Umformen:

$$ax^n + b = 0$$

$$ax^n + b = 0 \quad / -b$$

$$ax^n = -b \quad / : a$$

$$x^n = \frac{-b}{a}$$

$$x = \sqrt[n]{\frac{-b}{a}}$$

$$\frac{-b}{a} > 0 \quad x = \sqrt[n]{\frac{-b}{a}}$$

$$\frac{-b}{a} < 0 \quad x = -\sqrt[n]{\left|\frac{-b}{a}\right|}$$

$$5x^3 + 320 = 0 \quad / -320$$

$$5x^3 = -320 \quad / : 5$$

$$x^3 = -\frac{320}{5}$$

$$x = -\sqrt[3]{64}$$

$$x = -4$$

Biquadratische Gleichung

$$ax^4 + bx^2 + c = 0$$

$$\text{Substitution: } u = x^2 \quad u^2 = x^4$$

$$\text{Quadratische Gleichung: } au^2 + bu + c = 0$$

$$\text{Lösungen: } u_1 \quad u_2$$

$$\text{Resubstitution: } x^2 = u_1 \quad x^2 = u_2$$

$$x^4 - 10x^2 + 9 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 10u + 9 = 0$$

$$u_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+10 \pm \sqrt{64}}{2}$$

$$u_{1/2} = \frac{10 \pm 8}{2}$$

$$u_1 = \frac{10+8}{2} \quad u_2 = \frac{10-8}{2}$$

$$u_1 = 9 \quad u_2 = 1$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x_3 = 1 \quad x_4 = -1$$

4.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: Polynom vom Grad n

Gesucht:

Lösung der Gleichung

(1) $x^4 - 12x^3 + 54x^2 - 108x + 81 = 0$

(2) $x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$

(3) $2x^4 + 16x^3 + 48x^2 + 84x + 72 = 0$

(4) $-6x^4 + 72x^3 - 324x^2 + 648x - 486 = 0$

(5) $x^4 - 18x^2 + 81 = 0$

(6) $-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$

(7) $x^4 - 3x^3 = 0$

(8) $x^4 + x^3 - 9x^2 + 11x - 4 = 0$

(9) $-\frac{1}{6}x^4 + 2x^2 = 0$

(10) $\frac{1}{2}x^4 - 3x^3 + 5x^2 = 0$

(11) $-1x^4 + 3x^2 + 2x = 0$

(12) $-x^3 + 3x^2 - 4 = 0$

(13) $-2x^5 = 0$

(14) $-\frac{1}{4}x^5 + \frac{2}{3}x^4 = 0$

(15) $x^5 - 3x^4 = 0$

(16) $x^5 - 10x^3 + 9x = 0$

(17) $\frac{1}{2}x^5 + 2x^2 = 0$

(18) $-\frac{1}{6}x^5 + 2x^3 = 0$

(19) $\frac{1}{2}x^5 - 3x^4 + 5x^3 = 0$

(20) $-x^5 + 3x^3 + 2x^2 = 0$

(21) $-x^5 + 3x^4 - 4x^2 = 0$

(22) $4x^2 + 5x - 6 = 0$

4.2 Lösungen

Aufgabe (1)

$$x^4 - 12x^3 + 54x^2 - 108x + 81 = 0$$

$$x^4 - 12x^3 + 54x^2 - 108x + 81$$

Nullstelle für Polynomdivision erraten: 3

$$\begin{array}{r} (x^4 - 12x^3 + 54x^2 - 108x + 81) : (x - 3) = x^3 - 9x^2 + 27x - 27 \\ -(x^4 - 3x^3) \\ \hline -9x^3 + 54x^2 - 108x + 81 \\ -(-9x^3 + 27x^2) \\ \hline 27x^2 - 108x + 81 \\ -(27x^2 - 81x) \\ \hline -27x + 81 \\ -(-27x + 81) \\ \hline 0 \end{array}$$

$$x^3 - 9x^2 + 27x - 27 = 0$$

Nullstelle für Polynomdivision erraten: 3

$$\begin{array}{r} (x^3 - 9x^2 + 27x - 27) : (x - 3) = x^2 - 6x + 9 \\ -(x^3 - 3x^2) \\ \hline -6x^2 + 27x - 27 \\ -(-6x^2 + 18x) \\ \hline 9x - 27 \\ -(9x - 27) \\ \hline 0 \end{array}$$

$$1x^2 - 6x + 9 = 0$$

$$x_{1/2} = \frac{+6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$x_{1/2} = \frac{+6 \pm \sqrt{0}}{2}$$

$$x_{1/2} = \frac{6 \pm 0}{2}$$

$$x_1 = \frac{6 + 0}{2} \quad x_2 = \frac{6 - 0}{2}$$

$$x_1 = 3 \quad x_2 = 3$$

$$\underline{x_1 = 3; \quad 4\text{-fache Nullstelle}}$$

Aufgabe (2)

$$x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$$

$$x^4 + 4x^3 + 6x^2 + 4x + 1$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (x^4 + 4x^3 + 6x^2 + 4x + 1) : (x + 1) = x^3 + 3x^2 + 3x + 1 \\ -(x^4 + x^3) \\ \hline 3x^3 + 6x^2 + 4x + 1 \\ -(3x^3 + 3x^2) \\ \hline 3x^2 + 4x + 1 \\ -(3x^2 + 3x) \\ \hline x + 1 \\ -(x + 1) \\ \hline 0 \end{array}$$

$$x^3 + 3x^2 + 3x + 1 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (x^3 + 3x^2 + 3x + 1) : (x + 1) = x^2 + 2x + 1 \\
 \underline{-(x^3 + x^2)} \\
 2x^2 + 3x + 1 \\
 \underline{-(2x^2 + 2x)} \\
 x + 1 \\
 \underline{-(x + 1)} \\
 0
 \end{array}$$

$$\begin{aligned}
 1x^2 + 2x + 1 &= 0 \\
 x_{1/2} &= \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\
 x_{1/2} &= \frac{-2 \pm \sqrt{0}}{2} \\
 x_{1/2} &= \frac{-2 \pm 0}{2} \\
 x_1 &= \frac{-2 + 0}{2} & x_2 &= \frac{-2 - 0}{2} \\
 x_1 &= -1 & x_2 &= -1 \\
 x_1 &= -1; & & \text{4-fache Nullstelle}
 \end{aligned}$$

Aufgabe (3)

$$2x^4 + 16x^3 + 48x^2 + 84x + 72 = 0$$

$$2x^4 + 16x^3 + 48x^2 + 84x + 72$$

Nullstelle für Polynomdivision erraten: -2

$$\begin{array}{r}
 (2x^4 + 16x^3 + 48x^2 + 84x + 72) : (x + 2) = 2x^3 + 12x^2 + 24x + 36 \\
 \underline{-(2x^4 + 4x^3)} \\
 12x^3 + 48x^2 + 84x + 72 \\
 \underline{-(12x^3 + 24x^2)} \\
 24x^2 + 84x + 72 \\
 \underline{-(24x^2 + 48x)} \\
 36x + 72 \\
 \underline{-(36x + 72)} \\
 0
 \end{array}$$

$$2x^3 + 12x^2 + 24x + 36 = 0$$

Numerische Suche :

$$\begin{aligned}
 x_1 &= -4, 15; & & \text{1-fache Nullstelle} \\
 x_2 &= -2; & & \text{1-fache Nullstelle}
 \end{aligned}$$

Aufgabe (4)

$$-6x^4 + 72x^3 - 324x^2 + 648x - 486 = 0$$

$$-6x^4 + 72x^3 - 324x^2 + 648x - 486$$

Nullstelle für Polynomdivision erraten:3

$$\begin{array}{r}
 (-6x^4 + 72x^3 - 324x^2 + 648x - 486) : (x - 3) = -6x^3 + 54x^2 - 162x + 162 \\
 \underline{-(-6x^4 + 18x^3)} \\
 54x^3 - 324x^2 + 648x - 486 \\
 \underline{-(54x^3 - 162x^2)} \\
 -162x^2 + 648x - 486 \\
 \underline{-(-162x^2 + 486x)} \\
 162x - 486 \\
 \underline{-(162x - 486)} \\
 0
 \end{array}$$

$$-6x^3 + 54x^2 - 162x + 162 = 0$$

Nullstelle für Polynomdivision erraten: 3

$$\begin{array}{r} (-6x^3 + 54x^2 - 162x + 162) : (x - 3) = -6x^2 + 36x - 54 \\ -(-6x^3 + 18x^2) \\ \hline 36x^2 - 162x + 162 \\ -(36x^2 - 108x) \\ \hline -54x + 162 \\ -(-54x + 162) \\ \hline 0 \end{array}$$

$$-6x^2 + 36x - 54 = 0$$

$$x_{1/2} = \frac{-36 \pm \sqrt{36^2 - 4 \cdot (-6) \cdot (-54)}}{2 \cdot (-6)}$$

$$x_{1/2} = \frac{-36 \pm \sqrt{0}}{-12}$$

$$x_{1/2} = \frac{-36 \pm 0}{-12}$$

$$x_1 = \frac{-36 + 0}{-12} \quad x_2 = \frac{-36 - 0}{-12}$$

$$x_1 = 3 \quad x_2 = 3$$

$$x_1 = 3; \quad \underline{\underline{4\text{-fache Nullstelle}}}$$

Aufgabe (5)

$$x^4 - 18x^2 + 81 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 18u + 81 = 0$$

$$u_{1/2} = \frac{+18 \pm \sqrt{(-18)^2 - 4 \cdot 1 \cdot 81}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+18 \pm \sqrt{0}}{2}$$

$$u_{1/2} = \frac{18 \pm 0}{2}$$

$$u_1 = \frac{18 + 0}{2} \quad u_2 = \frac{18 - 0}{2}$$

$$u_1 = 9 \quad u_2 = 9$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x_1 = -3; \quad \underline{\underline{2\text{-fache Nullstelle}}}$$

$$x_2 = 3; \quad \underline{\underline{2\text{-fache Nullstelle}}}$$

Aufgabe (6)

$$-\frac{1}{4}x^4 + \frac{2}{3}x^3 = 0$$

$$x^3 \left(-\frac{1}{4}x + \frac{2}{3} \right) = 0 \Rightarrow x = 0 \quad \vee \quad -\frac{1}{4}x + \frac{2}{3} = 0$$

$$-\frac{1}{4}x + \frac{2}{3} = 0 \quad / -\frac{2}{3}$$

$$-\frac{1}{4}x = -\frac{2}{3} \quad / : \left(-\frac{1}{4} \right)$$

$$x = \frac{-\frac{2}{3}}{-\frac{1}{4}}$$

$$x = 2\frac{2}{3}$$

$$x_1 = 0; \quad \underline{\text{3-fache Nullstelle}}$$

$$x_2 = 2\frac{2}{3}; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (7)

$$x^4 - 3x^3 = 0$$

$$x^3(x - 3) = 0 \Rightarrow x = 0 \quad \vee \quad x - 3 = 0$$

$$x - 3 = 0 \quad / + 3$$

$$x = 3$$

$$x_1 = 0; \quad \underline{\text{3-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (8)

$$x^4 + x^3 - 9x^2 + 11x - 4 = 0$$

$$x^4 + x^3 - 9x^2 + 11x - 4$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^4 + x^3 - 9x^2 + 11x - 4) : (x - 1) = x^3 + 2x^2 - 7x + 4 \\ -(x^4 - 1x^3) \\ \hline 2x^3 - 9x^2 + 11x - 4 \\ -(2x^3 - 2x^2) \\ \hline -7x^2 + 11x - 4 \\ -(-7x^2 + 7x) \\ \hline 4x - 4 \\ -(4x - 4) \\ \hline 0 \end{array}$$

$$x^3 + 2x^2 - 7x + 4 = 0$$

Nullstelle für Polynomdivision erraten:1

$$\begin{array}{r} (x^3 + 2x^2 - 7x + 4) : (x - 1) = x^2 + 3x - 4 \\ -(x^3 - 1x^2) \\ \hline 3x^2 - 7x + 4 \\ -(3x^2 - 3x) \\ \hline -4x + 4 \\ -(-4x + 4) \\ \hline 0 \end{array}$$

$$1x^2 + 3x - 4 = 0$$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1}$$

$$x_{1/2} = \frac{-3 \pm \sqrt{25}}{2}$$

$$x_{1/2} = \frac{-3 \pm 5}{2}$$

$$x_{1/2} = \frac{-3 + 5}{2}$$

$$x_1 = 1 \quad x_2 = \frac{-3 - 5}{2}$$

$$x_1 = 1 \quad x_2 = -4$$

$$x_1 = -4; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_2 = 1; \quad \underline{\text{3-fache Nullstelle}}$$

Aufgabe (9)

$$\begin{aligned}
-\frac{1}{6}x^4 + 2x^2 &= 0 \\
x^2(-\frac{1}{6}x^2 + 2) &= 0 \Rightarrow x = 0 \quad \vee \quad -\frac{1}{6}x^2 + 2 = 0 \\
-\frac{1}{6}x^2 + 2 &= 0 \quad / -2 \\
-\frac{1}{6}x^2 &= -2 \quad / : (-\frac{1}{6}) \\
x^2 &= \frac{-2}{-\frac{1}{6}} \\
x &= \pm\sqrt{12} \\
x_1 &= 3,46 \quad x_2 = -3,46 \\
x_1 &= -3,46; \quad \text{1-fache Nullstelle} \\
x_2 &= 0; \quad \text{2-fache Nullstelle} \\
x_3 &= 3,46; \quad \text{1-fache Nullstelle}
\end{aligned}$$

Aufgabe (10)

$$\begin{aligned}
\frac{1}{2}x^4 - 3x^3 + 5x^2 &= 0 \\
x^2(\frac{1}{2}x^2 - 3x + 5) &= 0 \Rightarrow x = 0 \quad \vee \quad \frac{1}{2}x^2 - 3x + 5 = 0 \\
\frac{1}{2}x^2 - 3x + 5 &= 0 \\
x_{1/2} &= \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}} \\
x_{1/2} &= \frac{+3 \pm \sqrt{-1}}{1} \\
\text{Diskriminante negativ keine Lösung} \\
x_1 &= 0; \quad \text{2-fache Nullstelle}
\end{aligned}$$

Aufgabe (11)

$$\begin{aligned}
-1x^4 + 3x^2 + 2x &= 0 \\
x(-1x^3 + 3x + 2) &= 0 \Rightarrow x = 0 \quad \vee \quad -1x^3 + 3x + 2 = 0 \\
-1x^3 + 3x + 2 &= 0
\end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
(-1x^3 \quad +3x \quad +2) : (x+1) = -1x^2 + x + 2 \\
\underline{-(-1x^3 \quad -1x^2)} \\
 \quad x^2 \quad +3x \quad +2 \\
\underline{-(x^2 \quad +x)} \\
 \quad \quad 2x \quad +2 \\
\underline{-(2x \quad +2)} \\
 \quad \quad \quad \quad \quad 0
\end{array}$$

$$\begin{aligned}
-1x^2 + 1x + 2 &= 0 \\
x_{1/2} &= \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} \\
x_{1/2} &= \frac{-1 \pm \sqrt{9}}{-2} \\
x_{1/2} &= \frac{-1 \pm 3}{-2} \\
x_1 &= \frac{-1 + 3}{-2} \quad x_2 = \frac{-1 - 3}{-2} \\
x_1 &= -1 \quad x_2 = 2
\end{aligned}$$

$$\underline{x_1 = -1; \quad 2\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (12)

$$-1x^4 + 3x^3 - 4x = 0$$

$$x(-1x^3 + 3x^2 - 4) = 0 \Rightarrow x = 0 \quad \vee \quad -1x^3 + 3x^2 - 4 = 0$$

$$-1x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r} (-1x^3 + 3x^2 - 4) : (x+1) = -1x^2 + 4x - 4 \\ -(-1x^3 - 1x^2) \\ \hline 4x^2 - 4 \\ -(4x^2 + 4x) \\ \hline -4x - 4 \\ -(-4x - 4) \\ \hline 0 \end{array}$$

$$-1x^2 + 4x - 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{-2}$$

$$x_{1/2} = \frac{-4 \pm 0}{-2}$$

$$x_1 = \frac{-4 + 0}{-2} \quad x_2 = \frac{-4 - 0}{-2}$$

$$x_1 = 2 \quad x_2 = 2$$

$$\underline{x_1 = -1; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_3 = 2; \quad 2\text{-fache Nullstelle}}$$

Aufgabe (13)

$$-2x^5 = 0$$

$$x^5 = 0 \Rightarrow x = 0$$

$$\underline{x_1 = 0; \quad 5\text{-fache Nullstelle}}$$

Aufgabe (14)

$$-\frac{1}{4}x^5 + \frac{2}{3}x^4 = 0$$

$$x^4\left(-\frac{1}{4}x + \frac{2}{3}\right) = 0 \Rightarrow x = 0 \quad \vee \quad -\frac{1}{4}x + \frac{2}{3} = 0$$

$$-\frac{1}{4}x + \frac{2}{3} = 0 \quad / -\frac{2}{3}$$

$$-\frac{1}{4}x = -\frac{2}{3} \quad / : \left(-\frac{1}{4}\right)$$

$$x = \frac{-\frac{2}{3}}{-\frac{1}{4}}$$

$$x = 2\frac{2}{3}$$

$$\underline{x_1 = 0; \quad 4\text{-fache Nullstelle}}$$

$$\underline{x_2 = 2\frac{2}{3}; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (15)

$$x^5 - 3x^4 = 0$$

$$x^4(x - 3) = 0 \Rightarrow x = 0 \quad \vee \quad x - 3 = 0$$

$$x - 3 = 0 \quad / + 3$$

$$x = 3$$

$$x_1 = 0; \quad \underline{\text{4-fache Nullstelle}}$$

$$x_2 = 3; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (16)

$$x^5 - 10x^3 + 9x = 0$$

$$x(x^4 - 10x^2 + 9) = 0 \Rightarrow x = 0 \quad \vee \quad x^4 - 10x^2 + 9 = 0$$

$$u = x^2 \quad u^2 = x^4$$

$$1u^2 - 10u + 9 = 0$$

$$u_{1/2} = \frac{+10 \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$u_{1/2} = \frac{+10 \pm \sqrt{64}}{2}$$

$$u_{1/2} = \frac{10 \pm 8}{2}$$

$$u_1 = \frac{10 + 8}{2} \quad u_2 = \frac{10 - 8}{2}$$

$$u_1 = 9 \quad u_2 = 1$$

$$x^2 = 9$$

$$x = \pm\sqrt{9}$$

$$x_1 = 3 \quad x_2 = -3$$

$$x^2 = 1$$

$$x = \pm\sqrt{1}$$

$$x_1 = 1 \quad x_2 = -1$$

$$x_1 = -3; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_2 = -1; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_3 = 0; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_4 = 1; \quad \underline{\text{1-fache Nullstelle}}$$

$$x_5 = 3; \quad \underline{\text{1-fache Nullstelle}}$$

Aufgabe (17)

$$\frac{1}{2}x^5 + 2x^2 = 0$$

$$x^2(\frac{1}{2}x^3 + 2) = 0 \Rightarrow x = 0 \quad \vee \quad \frac{1}{2}x^3 + 2 = 0$$

$$\frac{1}{2}x^3 + 2 = 0$$

$$\frac{1}{2}x^3 + 2 = 0 \quad / - 2$$

$$\frac{1}{2}x^3 = -2 \quad / : \frac{1}{2}$$

$$x^3 = \frac{-2}{\frac{1}{2}}$$

$$x = \sqrt[3]{-4}$$

$$x = -1,59$$

Polynomdivision: $(-1, 59)$

$$\begin{array}{r}
 \left(\frac{1}{2}x^3 \quad +2\right) : (x+1,59) = \frac{1}{2}x^2 - 0,794x + 1,26 \\
 \underline{-(\frac{1}{2}x^3 \quad +0,794x^2)} \\
 \quad \quad \quad -0,794x^2 \quad +2 \\
 \quad \quad \quad \underline{-(-0,794x^2 \quad -1,26x)} \\
 \quad \quad \quad \quad \quad \quad 1,26x \quad +2 \\
 \quad \quad \quad \quad \quad \quad \underline{-(1,26x \quad +2)} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$\frac{1}{2}x^2 - 0,794x + 1,26 = 0$$

$$x_{1/2} = \frac{+0,794 \pm \sqrt{(-0,794)^2 - 4 \cdot \frac{1}{2} \cdot 1,26}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+0,794 \pm \sqrt{-1,89}}{1}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = -1,59; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 2\text{-fache Nullstelle}}$$

Aufgabe (18)

$$\begin{aligned}
 -\frac{1}{6}x^5 + 2x^3 &= 0 \\
 x^3(-\frac{1}{6}x^2 + 2) &= 0 \Rightarrow x = 0 \quad \vee \quad -\frac{1}{6}x^2 + 2 = 0 \\
 -\frac{1}{6}x^2 + 2 &= 0 \quad / -2 \\
 -\frac{1}{6}x^2 &= -2 \quad / : (-\frac{1}{6})
 \end{aligned}$$

$$x^2 = \frac{-2}{-\frac{1}{6}}$$

$$x = \pm\sqrt{12}$$

$$x_1 = 3,46 \quad x_2 = -3,46$$

$$\underline{x_1 = -3,46; \quad 1\text{-fache Nullstelle}}$$

$$\underline{x_2 = 0; \quad 3\text{-fache Nullstelle}}$$

$$\underline{x_3 = 3,46; \quad 1\text{-fache Nullstelle}}$$

Aufgabe (19)

$$\begin{aligned}
 \frac{1}{2}x^5 - 3x^4 + 5x^3 &= 0 \\
 x^3(\frac{1}{2}x^2 - 3x + 5) &= 0 \Rightarrow x = 0 \quad \vee \quad \frac{1}{2}x^2 - 3x + 5 = 0 \\
 \frac{1}{2}x^2 - 3x + 5 &= 0
 \end{aligned}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{(-3)^2 - 4 \cdot \frac{1}{2} \cdot 5}}{2 \cdot \frac{1}{2}}$$

$$x_{1/2} = \frac{+3 \pm \sqrt{-1}}{1}$$

Diskriminante negativ keine Lösung

$$\underline{x_1 = 0; \quad 3\text{-fache Nullstelle}}$$

Aufgabe (20)

$$\begin{aligned}
 -1x^5 + 3x^3 + 2x^2 &= 0 \\
 x^2(-1x^3 + 3x + 2) &= 0 \Rightarrow x = 0 \quad \vee \quad -1x^3 + 3x + 2 = 0 \\
 -1x^3 + 3x + 2 &= 0
 \end{aligned}$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-1x^3 \quad +3x \quad +2) : (x+1) = -1x^2 + x + 2 \\
 \underline{-(-1x^3 \quad -1x^2)} \\
 \quad \quad x^2 \quad +3x \quad +2 \\
 \quad \quad \underline{-(x^2 \quad +x)} \\
 \quad \quad \quad \quad 2x \quad +2 \\
 \quad \quad \quad \quad \underline{-(2x \quad +2)} \\
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$-1x^2 + 1x + 2 = 0$$

$$x_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-1 \pm \sqrt{9}}{-2}$$

$$x_{1/2} = \frac{-1 \pm 3}{-2}$$

$$x_1 = \frac{-1 + 3}{-2} \quad x_2 = \frac{-1 - 3}{-2}$$

$$x_1 = -1 \quad x_2 = 2$$

$$x_1 = -1; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_3 = 2; \quad \underline{1\text{-fache Nullstelle}}$$

Aufgabe (21)

$$-1x^5 + 3x^4 - 4x^2 = 0$$

$$x^2(-1x^3 + 3x^2 - 4) = 0 \Rightarrow x = 0 \quad \vee \quad -1x^3 + 3x^2 - 4 = 0$$

$$-1x^3 + 3x^2 - 4 = 0$$

Nullstelle für Polynomdivision erraten: -1

$$\begin{array}{r}
 (-1x^3 \quad +3x^2 \quad -4) : (x+1) = -1x^2 + 4x - 4 \\
 \underline{-(-1x^3 \quad -1x^2)} \\
 \quad \quad 4x^2 \quad -4 \\
 \quad \quad \underline{-(4x^2 \quad +4x)} \\
 \quad \quad \quad \quad -4x \quad -4 \\
 \quad \quad \quad \quad \underline{-(-4x \quad -4)} \\
 \quad \quad \quad \quad \quad \quad 0
 \end{array}$$

$$-1x^2 + 4x - 4 = 0$$

$$x_{1/2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot (-1) \cdot (-4)}}{2 \cdot (-1)}$$

$$x_{1/2} = \frac{-4 \pm \sqrt{0}}{-2}$$

$$x_{1/2} = \frac{-4 \pm 0}{-2}$$

$$x_1 = \frac{-4 + 0}{-2} \quad x_2 = \frac{-4 - 0}{-2}$$

$$x_1 = 2 \quad x_2 = 2$$

$$x_1 = -1; \quad \underline{1\text{-fache Nullstelle}}$$

$$x_2 = 0; \quad \underline{2\text{-fache Nullstelle}}$$

$$x_3 = 2; \quad \underline{2\text{-fache Nullstelle}}$$

Aufgabe (22)

$$4x^5 + 5x^4 - 6x^3 = 0$$

$$x^3(4x^2 + 5x - 6) = 0 \Rightarrow x = 0 \quad \vee \quad 4x^2 + 5x - 6 = 0$$

$$4x^2 + 5x - 6 = 0$$
$$x_{1/2} = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 4 \cdot (-6)}}{2 \cdot 4}$$
$$x_{1/2} = \frac{-5 \pm \sqrt{121}}{8}$$
$$x_{1/2} = \frac{-5 \pm 11}{8}$$
$$x_1 = \frac{-5 + 11}{8} \quad x_2 = \frac{-5 - 11}{8}$$
$$x_1 = \frac{3}{4} \quad x_2 = -2$$
$$\underline{x_1 = -2; \quad 1\text{-fache Nullstelle}}$$
$$\underline{x_2 = 0; \quad 3\text{-fache Nullstelle}}$$
$$\underline{x_3 = \frac{3}{4}; \quad 1\text{-fache Nullstelle}}$$

5 Exponentialgleichungen

$$a \cdot b^{(cx+d)} + f = 0$$

$$a \cdot b^{(cx+d)} + f = 0$$

$$a \cdot b^{(cx+d)} + f = 0 \quad / -f$$

$$a \cdot b^{(cx+d)} = -f \quad / : a$$

$$b^{(cx+d)} = \frac{-f}{a} \quad / \log_b(\dots)$$

$$\frac{-f}{a} > 0 \Rightarrow$$

$$\log_b(b^{(cx+d)}) = \log_b\left(\frac{-f}{a}\right)$$

$$\text{Logarithmengesetz: } \log_b b^n = n \log_b b = n$$

$$(cx+d) \log_b(b) = \log_b\left(\frac{-f}{a}\right)$$

$$cx+d = \log_b\left(\frac{-f}{a}\right) \quad / -d \quad / : c$$

$$x = \frac{\log_b\left(\frac{-f}{a}\right) - d}{c}$$

$$\frac{-f}{a} \leq 0 \Rightarrow \text{keine Lösung}$$

$$-2 \cdot 2^{(2x+3)} + 4 = 0$$

$$-2 \cdot 2^{(2x+3)} + 4 = 0 \quad / -4$$

$$-2 \cdot 2^{(2x+3)} = -4 \quad / : -2$$

$$2^{(2x+3)} = 2 \quad / \log_2$$

$$2x+3 = \log_2(2) \quad / -3 \quad / : 2$$

$$x = -1$$

$$\text{Basis: } e = 2,718\dots(\text{eulersche Zahl})$$

$$2 \cdot e^{(3x+4)} - 6 = 0$$

$$2 \cdot e^{(3x+4)} - 6 = 0 \quad / +6$$

$$2 \cdot e^{(3x+4)} = +6 \quad / : 2$$

$$e^{(3x+4)} = 3 \quad / \ln$$

$$3x+4 = \ln(3) \quad / -4 \quad / : 3$$

$$x = -0,967$$

5.1 $ab^{(cx+d)} + f = 0$

5.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ab^{(cx+d)} + f = 0$

Basis: b

Gesucht: Lösung der Gleichung

(1) $a = 2 \quad b = 3 \quad c = 2 \quad d = 2 \quad f = -1$

(2) $a = 2 \quad b = 3 \quad c = 4 \quad d = 5 \quad f = -6$

(3) $a = 5 \quad b = 4 \quad c = 2 \quad d = 1 \quad f = -10$

(4) $a = -2 \quad b = 3 \quad c = 2 \quad d = 1 \quad f = 10$

(5) $a = -2 \quad b = 3 \quad c = 2 \quad d = 1 \quad f = 18$

(6) $a = 4 \quad b = 2 \quad c = 6 \quad d = 5 \quad f = 4$

5.1.2 Lösungen

Aufgabe (1)

$$\begin{aligned}
2 \cdot 3^{(2x+2)} - 1 &= 0 \\
2 \cdot 3^{(2x+2)} - 1 &= 0 \quad / + 1 \\
2 \cdot 3^{(2x+2)} &= +1 \quad / : 2 \\
3^{(2x+2)} &= \frac{1}{2} \quad / \log_3 \\
2x + 2 &= \log_3\left(\frac{1}{2}\right) \quad / - 2 \quad / : 2 \\
x &= -1,32
\end{aligned}$$

Aufgabe (4)

$$\begin{aligned}
-2 \cdot 3^{(2x+1)} + 10 &= 0 \\
-2 \cdot 3^{(2x+1)} + 10 &= 0 \quad / - 10 \\
-2 \cdot 3^{(2x+1)} &= -10 \quad / : -2 \\
3^{(2x+1)} &= 5 \quad / \log_3 \\
2x + 1 &= \log_3(5) \quad / - 1 \quad / : 2 \\
x &= 0,232
\end{aligned}$$

Aufgabe (2)

$$\begin{aligned}
2 \cdot 3^{(4x+5)} - 6 &= 0 \\
2 \cdot 3^{(4x+5)} - 6 &= 0 \quad / + 6 \\
2 \cdot 3^{(4x+5)} &= +6 \quad / : 2 \\
3^{(4x+5)} &= 3 \quad / \log_3 \\
4x + 5 &= \log_3(3) \quad / - 5 \quad / : 4 \\
x &= -1
\end{aligned}$$

Aufgabe (5)

$$\begin{aligned}
-2 \cdot 3^{(2x+1)} + 18 &= 0 \\
-2 \cdot 3^{(2x+1)} + 18 &= 0 \quad / - 18 \\
-2 \cdot 3^{(2x+1)} &= -18 \quad / : -2 \\
3^{(2x+1)} &= 9 \quad / \log_3 \\
2x + 1 &= \log_3(9) \quad / - 1 \quad / : 2 \\
x &= \frac{1}{2}
\end{aligned}$$

Aufgabe (3)

$$\begin{aligned}
5 \cdot 4^{(2x+1)} - 10 &= 0 \\
5 \cdot 4^{(2x+1)} - 10 &= 0 \quad / + 10 \\
5 \cdot 4^{(2x+1)} &= +10 \quad / : 5 \\
4^{(2x+1)} &= 2 \quad / \log_4 \\
2x + 1 &= \log_4(2) \quad / - 1 \quad / : 2 \\
x &= -\frac{1}{4}
\end{aligned}$$

Aufgabe (6)

$$\begin{aligned}
4 \cdot 2^{(6x+5)} + 4 &= 0 \\
4 \cdot 2^{(6x+5)} + 4 &= 0 \quad / - 4 \\
4 \cdot 2^{(6x+5)} &= -4 \quad / : 4 \\
2^{(6x+5)} &= -1 \quad / \log_2 \\
-1 &\leq 0 \rightarrow \text{keine Lösung}
\end{aligned}$$

5.2 $ae^{(cx+d)} + f = 0$

5.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $ae^{(cx+d)} + f = 0$

Basis: $e = 2,718\dots$ (eulersche Zahl)

Gesucht: Lösung der Gleichung

$$\begin{array}{llll} (1) & a = 4 & c = 5 & d = 2 & f = 4 \\ (2) & a = 4 & c = 5 & d = 2 & f = -4 \\ (3) & a = 4 & c = 5 & d = 1 & f = -4 \end{array}$$

$$(4) \quad a = 4 \quad c = 5 \quad d = -4 \quad f = -4$$

5.2.2 Lösungen

Aufgabe (1)

$$\begin{aligned}
 4 \cdot e^{(5x+2)} + 4 &= 0 \\
 4 \cdot e^{(5x+2)} + 4 &= 0 \quad / - 4 \\
 4 \cdot e^{(5x+2)} &= -4 \quad / : 4 \\
 e^{(5x+2)} &= -1 \quad / \ln \\
 -1 &\leq 0 \rightarrow \text{keine Lösung}
 \end{aligned}$$

Aufgabe (3)

$$\begin{aligned}
 4 \cdot e^{(5x+1)} - 4 &= 0 \\
 4 \cdot e^{(5x+1)} - 4 &= 0 \quad / + 4 \\
 4 \cdot e^{(5x+1)} &= +4 \quad / : 4 \\
 e^{(5x+1)} &= 1 \quad / \ln \\
 5x + 1 &= \ln(1) \quad / - 1 \quad / : 5 \\
 x &= -\frac{1}{5}
 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned}
 4 \cdot e^{(5x+2)} - 4 &= 0 \\
 4 \cdot e^{(5x+2)} - 4 &= 0 \quad / + 4 \\
 4 \cdot e^{(5x+2)} &= +4 \quad / : 4 \\
 e^{(5x+2)} &= 1 \quad / \ln \\
 5x + 2 &= \ln(1) \quad / - 2 \quad / : 5 \\
 x &= -\frac{2}{5}
 \end{aligned}$$

Aufgabe (4)

$$\begin{aligned}
 4 \cdot e^{(5x-4)} - 4 &= 0 \\
 4 \cdot e^{(5x-4)} - 4 &= 0 \quad / + 4 \\
 4 \cdot e^{(5x-4)} &= +4 \quad / : 4 \\
 e^{(5x-4)} &= 1 \quad / \ln \\
 5x - 4 &= \ln(1) \quad / + 4 \quad / : 5 \\
 x &= \frac{4}{5}
 \end{aligned}$$

6 Logarithmusgleichungen

$$a \log_b (cx + d) + f = 0$$

$$a \log_b (cx + d) + f = 0$$

$$a \log_b (cx + d) + f = 0 \quad / - f$$

$$a \log_b (cx + d) = -f \quad / : a$$

$$\log_b (cx + d) = \frac{-f}{a} \quad / b$$

$$b^{(\log_b (cx+d))} = b^{\left(\frac{-f}{a}\right)}$$

$$cx + d = b^{\left(\frac{-f}{a}\right)} \quad / - d \quad / : c$$

$$x = \frac{b^{\left(\frac{-f}{a}\right)} - d}{c}$$

$$2 \cdot \log_3(4x + 5) - 4 = 0$$

$$2 \cdot \log_3(4x + 5) - 4 = 0 \quad / + 4$$

$$2 \cdot \log_3(4x + 5) = +4 \quad / : 2$$

$$\log_3(4x + 5) = 2 \quad / 3^{\cdot}$$

$$4x + 5 = 3^2 \quad / - 5 \quad / : 4$$

$$x = \frac{3^2 - 5}{4}$$

Basis: $e = 2,718\dots$ (Eulersche Zahl)

$$\log_e x = \ln x \quad 4 \cdot \ln(5x + 7) + 8 = 0$$

$$4 \cdot \ln(5x + 7) + 8 = 0 \quad / - 8$$

$$4 \cdot \ln(5x + 7) = -8 \quad / : 4$$

$$\ln(5x + 7) = -2 \quad / e^{\cdot}$$

$$5x + 7 = e^{-2} \quad / - 7 \quad / : 5$$

$$x = \frac{e^{-2} - 7}{5}$$

$$x = -1,37$$

$$\log_b x = 0$$

$$\log_b x = 0 \quad / b$$

$$x = b^0$$

$$x = 1$$

$$\lg x = 0 \quad / 10$$

$$x = 10^0$$

$$x = 1$$

$$\ln x = 0 \quad / e$$

$$x = e^0$$

$$x = 1$$

6.1 $a \log_b (cx + d) + f = 0$

6.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \log_b (cx + d) + f = 0$

Gesucht: Lösung der Gleichung

- | | | | | | | | | | | | |
|-----|--------|--------|---------|---------|--------|-----|-------|-------------------|--------|--------|--------|
| (1) | a = 2 | b = 10 | c = 5 | d = 10 | f = -2 | (6) | a = 2 | b = 3 | c = 4 | d = 1 | f = 4 |
| (2) | a = 3 | b = 4 | c = 5 | d = -10 | f = -2 | (7) | a = 2 | b = 3 | c = 4 | d = 1 | f = -4 |
| (3) | a = 10 | b = 5 | c = -10 | d = 2 | f = 5 | (8) | a = 2 | b = 4 | c = 4 | d = 1 | f = -4 |
| (4) | a = 3 | b = 4 | c = -2 | d = 4 | f = 6 | (9) | a = 2 | b = $\frac{1}{2}$ | c = -2 | d = -1 | f = -4 |
| (5) | a = 2 | b = 3 | c = 4 | d = 1 | f = 3 | | | | | | |

6.1.2 Lösungen

Aufgabe (1)

$$\begin{aligned} 2 \cdot \log_{10}(5x + 10) - 2 &= 0 \\ 2 \cdot \log_{10}(5x + 10) - 2 &= 0 \quad / + 2 \\ 2 \cdot \log_{10}(5x + 10) &= +2 \quad / : 2 \\ \log_{10}(5x + 10) &= 1 \quad / 10 \cdot \\ 5x + 10 &= 10^1 \quad / - 10 \quad / : 5 \\ x &= \frac{10^1 - 10}{5} \\ x &= 0 \end{aligned}$$

Aufgabe (2)

$$\begin{aligned} 3 \cdot \log_4(5x - 10) - 2 &= 0 \\ 3 \cdot \log_4(5x - 10) - 2 &= 0 \quad / + 2 \\ 3 \cdot \log_4(5x - 10) &= +2 \quad / : 3 \\ \log_4(5x - 10) &= \frac{2}{3} \quad / 4 \cdot \\ 5x - 10 &= 4^{\frac{2}{3}} \quad / + 10 \quad / : 5 \\ x &= \frac{4^{\frac{2}{3}} + 10}{5} \\ x &= 2,5 \end{aligned}$$

Aufgabe (3)

$$\begin{aligned} 10 \cdot \log_5(-10x + 2) + 5 &= 0 \\ 10 \cdot \log_5(-10x + 2) + 5 &= 0 \quad / - 5 \\ 10 \cdot \log_5(-10x + 2) &= -5 \quad / : 10 \\ \log_5(-10x + 2) &= -\frac{1}{2} \quad / 5 \cdot \\ -10x + 2 &= 5^{-\frac{1}{2}} \quad / - 2 \quad / : -10 \\ x &= \frac{5^{-\frac{1}{2}} - 2}{-10} \\ x &= 0,155 \end{aligned}$$

Aufgabe (4)

$$\begin{aligned} 3 \cdot \log_4(-2x + 4) + 6 &= 0 \\ 3 \cdot \log_4(-2x + 4) + 6 &= 0 \quad / - 6 \\ 3 \cdot \log_4(-2x + 4) &= -6 \quad / : 3 \\ \log_4(-2x + 4) &= -2 \quad / 4 \cdot \\ -2x + 4 &= 4^{-2} \quad / - 4 \quad / : -2 \\ x &= \frac{4^{-2} - 4}{-2} \\ x &= 1\frac{31}{32} \end{aligned}$$

Aufgabe (5)

$$\begin{aligned} 2 \cdot \log_3(4x + 1) + 3 &= 0 \\ 2 \cdot \log_3(4x + 1) + 3 &= 0 \quad / - 3 \\ 2 \cdot \log_3(4x + 1) &= -3 \quad / : 2 \\ \log_3(4x + 1) &= -1\frac{1}{2} \quad / 3 \cdot \end{aligned}$$

$$\begin{aligned} 4x + 1 &= 3^{-1\frac{1}{2}} \quad / - 1 \quad / : 4 \\ x &= \frac{3^{-1\frac{1}{2}} - 1}{4} \\ x &= -0,202 \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} 2 \cdot \log_3(4x + 1) + 4 &= 0 \\ 2 \cdot \log_3(4x + 1) + 4 &= 0 \quad / - 4 \\ 2 \cdot \log_3(4x + 1) &= -4 \quad / : 2 \\ \log_3(4x + 1) &= -2 \quad / 3 \cdot \\ 4x + 1 &= 3^{-2} \quad / - 1 \quad / : 4 \\ x &= \frac{3^{-2} - 1}{4} \\ x &= -\frac{2}{9} \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} 2 \cdot \log_3(4x + 1) - 4 &= 0 \\ 2 \cdot \log_3(4x + 1) - 4 &= 0 \quad / + 4 \\ 2 \cdot \log_3(4x + 1) &= +4 \quad / : 2 \\ \log_3(4x + 1) &= 2 \quad / 3 \cdot \\ 4x + 1 &= 3^2 \quad / - 1 \quad / : 4 \\ x &= \frac{3^2 - 1}{4} \\ x &= 2 \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} 2 \cdot \log_4(4x + 1) - 4 &= 0 \\ 2 \cdot \log_4(4x + 1) - 4 &= 0 \quad / + 4 \\ 2 \cdot \log_4(4x + 1) &= +4 \quad / : 2 \\ \log_4(4x + 1) &= 2 \quad / 4 \cdot \\ 4x + 1 &= 4^2 \quad / - 1 \quad / : 4 \\ x &= \frac{4^2 - 1}{4} \\ x &= 3\frac{3}{4} \end{aligned}$$

Aufgabe (9)

$$\begin{aligned} 2 \cdot \log_{\frac{1}{2}}(-2x - 1) - 4 &= 0 \\ 2 \cdot \log_{\frac{1}{2}}(-2x - 1) - 4 &= 0 \quad / + 4 \\ 2 \cdot \log_{\frac{1}{2}}(-2x - 1) &= +4 \quad / : 2 \\ \log_{\frac{1}{2}}(-2x - 1) &= 2 \quad / \frac{1}{2} \cdot \\ -2x - 1 &= \frac{1}{2}^2 \quad / + 1 \quad / : -2 \\ x &= \frac{\frac{1}{2}^2 + 1}{-2} \\ x &= -\frac{5}{8} \end{aligned}$$

6.2 $a \ln(cx + d) + f = 0$

6.2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben: $a \ln(cx + d) + f = 0$

Gesucht: Lösung der Gleichung

(1) $a = 2$ $c = 3$ $d = 4$ $f = -4$
(2) $a = -2$ $c = 3$ $d = 4$ $f = 4$
(3) $a = 2$ $c = 3$ $d = 4$ $f = 4$
(4) $a = -\frac{1}{2}$ $c = 4$ $d = -2$ $f = -2$

(5) $a = 2$ $c = -2$ $d = 3$ $f = 4$
(6) $a = \frac{1}{4}$ $c = 2$ $d = -1$ $f = -3$

6.2.2 Lösungen

Aufgabe (1)

$$\begin{aligned}
2 \cdot \ln(3x + 4) - 4 &= 0 \\
2 \cdot \ln(3x + 4) - 4 &= 0 & / + 4 \\
2 \cdot \ln(3x + 4) &= +4 & / : 2 \\
\ln(3x + 4) &= 2 & / e^{\cdot} \\
3x + 4 &= e^2 & / - 4 & / : 3 \\
x &= \frac{e^2 - 4}{3} \\
x &= 1,13
\end{aligned}$$

Aufgabe (4)

$$\begin{aligned}
-\frac{1}{2} \cdot \ln(4x - 2) - 2 &= 0 \\
-\frac{1}{2} \cdot \ln(4x - 2) - 2 &= 0 & / + 2 \\
-\frac{1}{2} \cdot \ln(4x - 2) &= +2 & / : -\frac{1}{2} \\
\ln(4x - 2) &= -4 & / e^{\cdot} \\
4x - 2 &= e^{-4} & / + 2 & / : 4 \\
x &= \frac{e^{-4} + 2}{4} \\
x &= 0,505
\end{aligned}$$

Aufgabe (2)

$$\begin{aligned}
-2 \cdot \ln(3x + 4) + 4 &= 0 \\
-2 \cdot \ln(3x + 4) + 4 &= 0 & / - 4 \\
-2 \cdot \ln(3x + 4) &= -4 & / : -2 \\
\ln(3x + 4) &= 2 & / e^{\cdot} \\
3x + 4 &= e^2 & / - 4 & / : 3 \\
x &= \frac{e^2 - 4}{3} \\
x &= 1,13
\end{aligned}$$

Aufgabe (5)

$$\begin{aligned}
2 \cdot \ln(-2x + 3) + 4 &= 0 \\
2 \cdot \ln(-2x + 3) + 4 &= 0 & / - 4 \\
2 \cdot \ln(-2x + 3) &= -4 & / : 2 \\
\ln(-2x + 3) &= -2 & / e^{\cdot} \\
-2x + 3 &= e^{-2} & / - 3 & / : -2 \\
x &= \frac{e^{-2} - 3}{-2} \\
x &= 1,43
\end{aligned}$$

Aufgabe (3)

$$\begin{aligned}
2 \cdot \ln(3x + 4) + 4 &= 0 \\
2 \cdot \ln(3x + 4) + 4 &= 0 & / - 4 \\
2 \cdot \ln(3x + 4) &= -4 & / : 2 \\
\ln(3x + 4) &= -2 & / e^{\cdot} \\
3x + 4 &= e^{-2} & / - 4 & / : 3 \\
x &= \frac{e^{-2} - 4}{3} \\
x &= -1,29
\end{aligned}$$

Aufgabe (6)

$$\begin{aligned}
\frac{1}{4} \cdot \ln(2x - 1) - 3 &= 0 \\
\frac{1}{4} \cdot \ln(2x - 1) - 3 &= 0 & / + 3 \\
\frac{1}{4} \cdot \ln(2x - 1) &= +3 & / : \frac{1}{4} \\
\ln(2x - 1) &= 12 & / e^{\cdot} \\
2x - 1 &= e^{12} & / + 1 & / : 2 \\
x &= \frac{e^{12} + 1}{2} \\
x &= 8,14 \cdot 10^4
\end{aligned}$$